TURNING WATER INTO WINE

NEW METHODS OF CALCULATING FARM OUTPUT AND NEW INSIGHTS INTO RISING CROP YIELDS DURING THE AGRICULTURAL REVOLUTION[1]

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ABSTRACT

Constructing an agricultural output series requires a rational economic basis on which to convert one crop into another, and a conversion method using only the information which we have at our disposal. The traditional method fails on both counts. We develop two alternative methods. The first is extremely parsimonious but imprecise; the second is less parsimonious but allows us to isolate the effect of different crop rotations on wheat yields. We compare the three methods using the farm dataset compiled by Arthur Young on his tours of England and Wales during the Agricultural Revolution and we show that new fallow crops substantially increased the wheat yield.

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1. Introduction.

The agricultural sector produces a wide range of crops which are not immediately comparable - not only grains, but also root crops, fallows and industrial crops. In order to make comparisons between different production units (whether they are farms, regions or countries) we need to amalgamate all the disparate outputs into one output series. It is of only limited interest to say that Britain produces more wheat but France produces more wine. Moreover, any production function analysis is far easier if we only have to deal with one dependent variable.[2] Hence it is common to express all the outputs in terms of their equivalent amount of wheat, due largely to the historical importance of wheat in the population's diet. The Traditional Method (TM) uses the following formula:

$$\frac{Q^{Be}.P^{Be}}{P^{W}} = \text{wheat-equivalent output of beans} = TM \qquad (1)$$

where Q^{Be} is the quantity of beans in bushels, P^{Be} is the price of beans per bushel, P^{W} is the price of wheat per bushel, and TM is the wheat output calculated using the Traditional Method.

The Traditional Method of conversion is based on the notion of arbitrage. The farmer could sell his beans and use the revenue to purchase a certain amount of wheat. If the market is valuing wheat and beans correctly, then this conversion through the market reflects the rate at which wheat can be physically converted into beans. This is merely a recognition of the fact that in equilibrium the marginal rate of transformation of wheat for beans must be equal to the price ratio:

$$\frac{Y^{W}}{Y^{Be}} = \frac{P^{Be}}{P^{W}} \qquad (2)$$

where Y^{W} is the yield of wheat in bushels per acre, Y^{Be} is the yield of beans in bushels per acre, P^{Be} is the price of beans per bushel, and P^{W} is the price of wheat per bushel.[3]

There are two problems with the Traditional Method. One problem is that it is biased whenever the acreage is not divided equally between all the crops (ie virtually always). The other problem is that it requires a great deal of information (ie the output and prices of all the crops grown on the farm). We will first consider the problem of data.

In order to use the Traditional Method we need to know the output and price of every crop. But often we do not know precisely how the arable acreage is divided between crops and we do not know the yield of every crop. It is common in historical sources to record data on wheat but nothing else. Similarly, price data is often scarce and may not be directly relevant to the locality which we are considering. For example, historians analysing the

[2]An alternative approach is to use a revenue function, which employs price and output data to calculate total revenue and uses total revenue as the dependent variable. Since this method also requires all the relevant output prices to be known, it raises the same data problems for empirical work. The other alternative is to use a cost function - but this is no less problematic because it requires the prices of all the inputs to be known (rather than the prices of all the outputs). The input prices are usually not available and there are grave problems regarding the valuation of capital inputs, even when all the relevant prices are known.

[3] The price ratio appears to be the 'wrong way up' because the marginal rate of transformation is always defined as negative. See Layard and Walters, *Microeconomic Theory.*

data compiled by Arthur Young in 1770 have had to resort to using county-level price data compiled by other authorities.[4] These price series may - or may not - reflect the relative prices faced by farmers in remote areas. More fundamentally, missing markets can systematically deprive us of the price data required to employ the Traditional Method. In particular, there is no market for bare fallows and consequently no method of valuing their output: hence the Traditional Method assumes that the output of fallows is zero. But this is clearly false - if the value of fallows were zero then farmers would stop fallowing the land. The value of fallows is partly reflected in higher crop yields, and we show below that this is *imperfectly* captured by the Traditional Method. But there is also a benefit from running animals on the fallow lands: this return is entirely ignored in the Traditional Method. A similar problem occurs in peasant agriculture, which is typical of early modern Europe and many developing countries today. The economic definition of a peasant is someone who is imperfectly integrated into the market,[5] so price signals and arbitrage may not be effective. Yet a rational farmer will virtually always respond to relative yields, so it is more useful to have an output measure which does not depend on market prices.[6] Finally, the New Method is much easier to calculate - which, according to Occam's razor, is a sufficient reason for preferring it.

The solution to the problem of lack of price data is to convert beans etc. into their wheat equivalent using only the yield of wheat per acre and the acreage of beans etc.. We call this technique the New Method and we define it as:

$$A^{Be}.Y^{W} = \text{wheat-equivalent output of beans} = NM \qquad (3)$$

where A^{Be} is the acreage of beans and NM is the wheat output calculated using the New Method. It is easy to show that if equation (2) holds, then the New Method and the Traditional Method are equal. See Appendix I for a proof of this proposition. The intuition for this equality is straightforward. Arbitrage can occur not only through prices but also through quantities. The farmer can convert wheat into beans either by selling it onto the market or by sowing a field with beans instead of wheat.[7] In either case the rational farmer will equalise the returns from the two crops.

The New Method solves the information problem. We do not need to know the acreage or output of every individual crop on the farm - to find total output we simply multiply the *total farm acreage* by the yield of wheat per acre. (If we do not know the yield of wheat then we can convert all the outputs into an alternative crop, provided that we know the yield of at least one crop). Nor is there any need for price data on any crop. The ability of the New Method to economise on information is crucial. In the Arthur Young dataset, we can use the New Method to calculate total output for 308 arable farms. If we use the Traditional Method then we can calculate total output for 13 farms. If we ignore the problem of fallows then we can use the Traditional Method to calculate total output for 160 farms, although we show below that this leads to a serious bias in the output series.

Unfortunately, there are a number of complications which need to be addressed. In the next section we show that the New Method is not as straightforward to implement as it appears from the mathematical proof due to externalities between crops. We also show that the Traditional Method suffers from the same flaw. The presence of externalities biases upwards both output series. In section 3 we solve the externality problem by reintroducing price data and developing the Adjusted Method. In section 4 we compare the efficiency of the three output series. In section 5 we show how the Adjusted Method can be used to isolate the impact of different crop rotations on wheat yields. In section 6 we state our conclusions.

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[4] For example, Allen relies on *The London Gazette* (1770) in 'The Distributional Consequences,' (1982). The *London Gazette* price is an unweighted average of the price of each product in the four major market towns in each county. Since the *London Gazette* does not give all the relevant prices, Allen has to assume that peas and beans sold for the same price. He also has to ignore fallow crops such as turnips and clover.

[5] Ellis F, *Peasant Economics* (CUP, 1991).

[6] Subsistence farmers may not respond to relative yields if they are very poor; they might pursue an alternative maximand, such as equalising the marginal nutritional output of different crops.

[7] Physical arbitrage can occur as long as there is some land available on the farm which can support both beans and wheat. In practice, both crops were grown on most farms and crop rotations were sufficiently flexible to allow this kind of arbitrage.

2. Externalities Between Crops.

It is well known that there are externalities between crops. Growing root and fallow crops between cereal crops enriches the soil and raises the subsequent grain yields. Even growing weeds (ie bare fallowing the land) leads to higher overall output than growing wheat year after year in the same field. This is the driving force behind crop rotation, which has been an established agricultural practice for thousands of years.[8] But the presence of externalities between crops makes our analysis much more complicated. The farmer will no longer equalise the yield and price ratios between beans and wheat (for example) because growing beans will raise the subsequent wheat yield. He will be willing to suffer a lower return on the bean crop until the foregone profit equals the extra profit from a richer wheat crop. This implies that if we simply examine the physical yields per acre of beans and wheat then we will underestimate the *true* yield from growing beans. We will correspondingly *overestimate* the true yield from growing wheat. This is problematic because it means that when we multiply the total acreage by the wheat yield, as the New Method requires, we will overestimate total output.

It is important to note that the Traditional Method of conversion falls into the same trap. It underestimates the bean yield and gives the total wheat-equivalent bean output as simply:

$$\frac{A^{Be}.Y^{Be}.P^{Be}}{P^{W}} = \text{wheat-equivalent bean output} \qquad (4)$$

At the same time it misallocates some of the bean yield to the increased wheat yield.

$$A^{W}.Y^{W} = \text{total output of wheat} \qquad (5)$$

If the arable acreage is divided equally between all the crops in the rotation then this is not a problem: the overestimate for wheat will be exactly offset by the underestimate for the other crops. We will simply have misallocated the returns to different crops within the rotation. But in fact, the data from Arthur Young shows that it was very rare for farmers to grow the same amount of each crop. They generally had a slight bias towards cereals (55 per cent cereals versus 45 per cent roots and fallows); and within cereals there was a bias in favour of wheat (wheat occupied 22 per cent of total arable acreage as opposed to 18 per cent for barley and 16 per cent for oats). This creates an overestimate of total output, and the overestimate rises as production becomes more skewed in favour of wheat. This proposition is demonstrated in Appendix II.

3. The Solution.

The solution to the externality problem in constructing the output series is to adjust downwards the wheat yield. We purge it of the effect of other crops in the rotation to find the True Wheat Yield. We can then simply multiply the arable acreage by the True Wheat Yield to obtain a correct estimate of total output. We call this the Adjusted Method. This adjustment process has the disadvantage that we need to use price data in order to scale down the wheat yields. However, the process also reveals a great deal of new information. In particular, it allows us to isolate the impact on wheat yields of every other crop in the rotation - for the first time, we can quantify the importance of new crop technology on wheat yields during the Agricultural Revolution.

Let us take the case of a six course rotation of wheat, beans, barley, peas, oats and turnips. We first set up the arbitrage conditions for wheat and the other crops. The example of beans is given below. Let the True Wheat Yield be Y^{W*} and let the extra wheat yield resulting from growing beans be Y^{WBe}. We know that the revenue from the True Wheat Yield ($Y^{W*}.P^{W}$) must be equal to the direct revenue from the crop of beans ($P^{Be}.Y^{Be}$), plus

[8] For example, see Virgil, *Georgics*. For a more modern analysis see Shiel R, 'Improving soil productivity in the pre-fertiliser era,' in *Land, Labour and Livestock* (ed. Campbell B and M Overton; MUP, 1991).

the indirect revenue resulting from higher wheat yields ($Y^{WBe}.P^{W}$). Otherwise the farmer would be better off growing less wheat and more beans. Then,

$$Y^{W*}.P^{W} = P^{Be}.Y^{Be} + Y^{WBe}.P^{W}$$

$$\Rightarrow \quad Y^{W*} = \frac{P^{Be}.Y^{Be}}{P^{W}} + Y^{WBe}$$

$$\Rightarrow \quad Y^{WBe} = Y^{W*} - \frac{P^{Be}.Y^{Be}}{P^{W}} \qquad (6)$$

But we also have an identity. The True Wheat Yield (Y^{W*}) equals the total wheat yield (that is, the *measured* wheat yield, Y^{W}) minus the wheat yield due to beans (Y^{WBe}) and all the other crops. Then,

$$Y^{W*} = Y^{W} - Y^{WBe} - Y^{WB} - Y^{WP} - Y^{WO} - Y^{WT} \qquad (7)$$

By substituting equation (6) and the other arbitrage conditions into equation (7) we easily find the True Wheat Yield:

$$Y^{W*} = \frac{Y^{W} + \frac{P^{Be}.Y^{Be}}{P^{W}} + \frac{P^{B}.Y^{B}}{P^{W}} + \frac{P^{P}.Y^{P}}{P^{W}} + \frac{P^{O}.Y^{O}}{P^{W}} + \frac{P^{T}.Y^{T}}{P^{W}}}{6} \qquad (8)$$

We can now proceed to calculate total wheat equivalent output by multiplying the total acreage of all the crops involved by Y^{W*}. This is the Adjusted Method (AM). So total wheat-equivalent output (Q^{W*}) is:

$$Q^{W*} = Y^{W*}.(A^{W} + A^{Be} + A^{B} + A^{P} + A^{O} + A^{T}) = AM \qquad (9)$$

Using the Adjusted Method, we calculated the total wheat-equivalent output for all the farms in the Arthur Young dataset. (Obviously, we made the calculations using the specific crop rotation relevant to each farm; the 160 farms for which we have all the required data used 44 different rotations between them). We also calculated total wheat-equivalent output using the New Method and the Traditional Method for the same sample of farms. As noted above, the existence of bare fallows - and the complete absence of fallow price data - raise serious problems for the Traditional and Adjusted Methods. Moreover, there is no farm price data for clover (only yield data). So we had to assume that the price of clover was uniform across all farms at 438 pence per load - this approximate valuation is derived from Arthur Young.[9] Due to these problems we have made three different comparisons of the three Methods, based on different samples of farms. Table 1 below gives the estimated output using all three methods for *only those farms (13) which had no clover and no fallows*. Table 2 gives the results for *only those farms (31) which had no fallow* (ie it includes some farms which grew clover). Table 3 gives the results for *all farms (160)*.

Table 1. 'Pure' sample (13 observations).

Method	Mean Output (bu)	SE of Mean Output (bu)	Variance

[9] Young A, *Eastern Tour*, p164.

Traditional	7395	2674	92967007
Adjusted	7354	2679	93300724
New	10781	4065	214828235

Table 2. 'Clover but no fallow' sample (31 observations).

Method	Mean Output (bu)	SE of Mean Output (bu)	Variance
Traditional	5084	1355	56931703
Adjusted	5068	1356	56970643
New	6311	1865	107779694

Table 3. 'Full' sample (160 observations).

Method	Mean Output (bu)	SE of Mean Output (bu)	Variance
Traditional	2630	386	23897823
Adjusted	2602	383	23446804
New	3346	511	41798811

In all cases the three output series are very highly correlated (between 96 and 99 per cent using Pearson's Product Moment Correlation Coefficient). The upward bias of the Traditional Method is apparent in all the calculations, but appears to be slight. The New Method clearly provides an over-estimate, but is it not clear by how much. The data from Table 1 suggest that the New Method is over-estimating by 47 per cent. But in Table 3 (the full sample) we know that the Adjusted Method is giving an underestimate due to the fallow problem **-** so the New Method must be over-estimating by less than 29 per cent. We will now consider in more detail the properties of each output series.

4. The Bias, Efficiency and Heteroskedasticity of the Three Methods.

It is generally desirable to use an estimator which is *unbiased* **-** that is, an estimator which gives a prediction which is correct on average. It is also desirable to have an *efficient* estimator **-** that is, an estimator which has a low variance. These two properties are combined in the Mean Squared Error (MSE), which is defined as:

$$\text{MSE} = \text{variance} + (\text{bias})^2$$

The estimator with the lowest Mean Squared Error is generally the preferred estimator. In Table 4 below we give the MSE for each of the three Methods, based on the 'pure' sample. The Traditional Method and the Adjusted Method perform virtually identically, and they are both much better than the New Method.

Table 4.

Method	Mean Squared Error	Variance	Bias
Traditional	92967791	92967007	28
Adjusted	93300724	93300724	0
New	226572564	214828235	3427

It would useful to know the MSE for the full sample (ie for those farms which use clover and a bare fallow) because this is the most likely situation faced by a researcher. However, when fallows are included the Adjusted

Method and the Traditional Method are both biased downwards. Moreover, the Traditional Method is inherently biased upwards (as shown in Appendix II) so it is impossible to know even the direction of the overall bias. Since *all* the Methods are biased in the presence of bare fallows, we cannot calculate the MSE because we cannot quantify the bias.

In fact, although the Mean Squared Error is the usual tool for discriminating between estimators, it is not clear that it is appropriate in this instance. Let us consider the effect of bias. Precise estimates are sometimes essential in empirical work. If we are comparing the absolute level of output - for example, making international productivity comparisons - then we need a measure which is not biased. Or at least the bias must be the same in all cases, a condition which is very unlikely to hold in practice.[10] But biased estimates are not *necessarily* a problem in empirical work. For many applications, particularly the estimation of production functions, we normally take natural logarithms of all the variables. The overestimate of total output inherent in the Traditional and New Methods then biases the constant downwards but leaves all the coefficients unchanged. This can be shown as follows. The relationship between 'true' total output and the factor inputs K_i has the following form:

$$\ln Q^{W*} = C + \alpha \ln K_i + \mu$$

If we use an estimate of total output which is consistently too high by around 47 per cent (such as the New Method, as suggested by Table 1) then the equation takes the form:

$$\ln(Q^W/1.47) = C + \alpha \ln K_i + \mu$$

$$\Rightarrow \quad \ln Q^W = C - \ln \frac{1}{1.47} + \alpha \ln K_i + \mu \qquad (10)$$

The estimate of α is unaffected by the bias of the output measure.

Let us turn to the question of the variance of the three estimates. The New Method has far higher variance than the other two Methods because there is more measurement error. This is generally undesirable because it makes it less likely that the variable will appear to be significant in a regression (even when it is truly significant). However, in this case the problem is less serious than normal because we are estimating an output series - which will generally be the *dependent* variable in a regression. It is well known that measurement errors in the dependent variable (for example, those caused by excessive variance) do not bias the estimated coefficients of the independent variables.[11]

A more serious problem is heteroskedasticity. A Spearman Rank Correlation test shows that *all* of the Methods are likely to create heteroskedasticity in a regression, as the following example demonstrates. Let us assume that all farms devote more acres to wheat than any other crop and that the degree of bias is the same on all farms (all farms grow 40 per cent wheat, 30 per cent oats and 30 per cent turnips). We have seen that when output is skewed in favour of wheat, then the total wheat equivalent output will be overestimated. But the *absolute* size of the overestimate will rise as the acreage of the farm increases. This creates heteroskedasticity in the errors and biases upwards the t-statistics in a regression. Take the case of land inputs and total output in Figure 1 below. Output rises as the size of the farm increases, but there is also a rise in the measurement errors of the output series.

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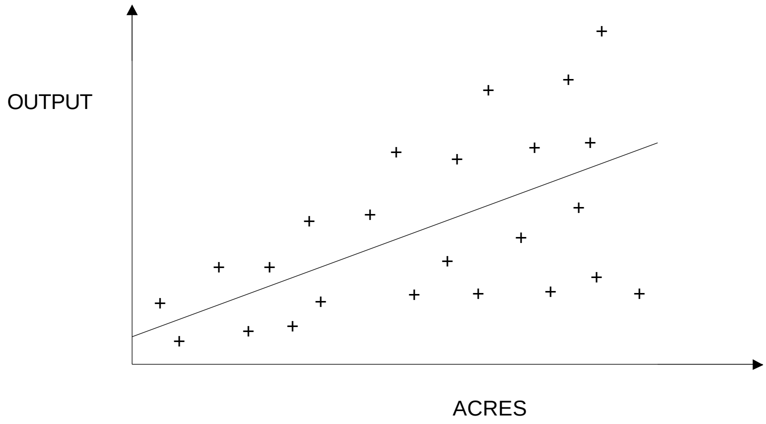
[10] With the Traditional Method the bias is induced by the over-representation of wheat in arable acreage: but the proportion of wheat in total acreage varies drastically between regions and countries. With the New Method the bias is caused by externalities between crops, which vary drastically from farm to farm. So if an unbiased estimator is required then the Adjusted Method must be used.

[11] Kennedy P, *A Guide To Econometrics*.

Figure 1.



We ran the following regression for all three output series:

$$Q^W = C + \alpha ACREAGE + \mu \qquad (11)$$

Although the precise formulation will obviously be more detailed and feature more variables, any production function will be based on the relation between land area and output and all the other variables will be very highly correlated with land input. Hence any heteroskedasticity problem revealed by the above regression signals a general problem. The results of the Spearman Rank Correlation Tests are given in Table 5 below.

Table 5.

	Correlation between ACREAGE and μ
Traditional Method	0.71
Adjusted Method	0.69
New Method	0.63

As expected, there is evidence of heteroskedasticity in all the regressions and the problem is of roughly equal magnitude for all of the output series. Fortunately, the problem of heteroskedasticity can usually be solved fairly easily by transforming the offending variable.[12]

In terms of their bias and efficiency, none of the output measures performs noticeably better than the other two. The New Method is the most biased, but bias is often not a problem. Moreover, although the Traditional and Adjusted Methods are less biased they have the additional problem of ignoring the fallows (which introduces more random measurement error). In practice, the absence of price or output data may compel us to use the New Method. And when price and output data are available, the Adjusted Method will be employed because it enables us to assess the impact of different crops on grain yields. We demonstrate this calculation in the next section.

5. Quantifying the Externalities between Crops.

Having made the calculations required by the Adjusted Method, it is a simple matter to calculate the contribution of each crop in the rotation to the wheat yield. For example, we substituted (8) into (6) to find the beneficial effect of beans:

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[12] Koutsoyiannis A, *Theory Of Econometrics* (MacMillan, 1977), p187.

$$Y^{WBe} = \frac{Y^{W} + \frac{P^{Be}.Y^{Be}}{P^{W}} + \frac{P^{B}.Y^{B}}{P^{W}} + \frac{P^{P}.Y^{P}}{P^{W}} + \frac{P^{O}.Y^{O}}{P^{W}} + \frac{P^{T}.Y^{T}}{P^{W}}}{6} - \frac{P^{Be}.Y^{Be}}{P^{W}} \qquad (12)$$

When applied to the Arthur Young dataset, this method gives the results reproduced in Tables 6, 7 and 8 (again we have used three different sub-samples in order to see the effect of the clover problem and the fallow problem).

Table 6.

	Mean (bu per acre)	SE of Mean (bu per acre)	SD (bu per acre)	Increase in True Wheat Yield (%)
Measured Wheat Yield	25.31	1.82	6.55	
True Wheat Yield	18.20	1.15	4.16	
Wheat Yield Due To Barley	-2.47	1.04	3.30	-13.57
Wheat Yield Due To Oats	-0.45	0.63	2.08	-2.47
Wheat Yield Due To Beans	-2.22	2.09	4.18	-12.2
Wheat Yield Due To Clover				
Wheat Yield Due To Peas	3.37	0.79	2.36	18.52
Wheat Yield Due To Turnips	9.15	1.27	4.22	50.27

The results contained in Table 6 above are the most reliable and plausible. Cereal crops tend to compete with wheat for nutrients and thereby reduce overall wheat yields - hence the effects of barley and oats are negative. By contrast, root crops tend to fix nitrogen in the soil and raise wheat yields.[13] The perverse sign on beans is very strange but the overall ranking of crops is reasonable. The new root crops such as turnips were more effective than traditional legumes such as beans and peas, so it is likely that turnips were the most beneficial crop and beans the least.[14] The superiority of turnips was caused by the different mechanism which they used to recycle nitrogen. Rather than fixing nitrogen directly from the atmosphere, the turnips were consumed by animals and the nitrogen returned to the soil in the form of animal dung. Moreover, turnips had two positive effects on wheat yields, rather than just one. Not only did they lead to an increase in nitrogen levels, they were also a 'cleaning crop'. When turnips were grown, it was common to weed between the rows of vegetables; this substantially reduced the content of weeds in the following wheat crop and led to higher yields.

The ranking of crops is similar in Tables 7 and 8 below. The negative coefficient on clover is surprising. This may be an artefact of our assumptions about the price of clover - if we assumed a lower clover price then the estimated effect of clover would rise. Alternatively, it may genuinely be that clover was reducing output. The problem of 'clover-sickness' was not well-understood in the eighteenth century, and it may be the case that those farmers who adopted clover were sowing it too often. This actually *reduces* the yields of grain.

Table 7.

	Mean (bu per acre)	SE of Mean (bu per acre)	SD (bu per acre)	Increase in True Wheat Yield (%)
Measured Wheat Yield	23.03	1.31	7.29	
True Wheat Yield	17.00	0.62	3.45	
Wheat Yield Due To Barley	-1.89	0.50	2.47	-11.12

[13] Neal E and K Neal, *Biology For Today* (Blandford, 1983).
[14] Shiel R, 'Improving soil productivity'.

Wheat Yield Due To Oats	1.15	0.54	2.47	6.76
Wheat Yield Due To Beans	-1.08	0.95	3.41	-6.35
Wheat Yield Due To Clover	-0.80	0.49	1.90	-4.71
Wheat Yield Due To Peas	3.47	0.74	2.22	20.41
Wheat Yield Due To Turnips	9.17	0.66	3.24	53.94

Table 8.

	Mean (bu per acre)	SE of Mean (bu per acre)	SD (bu per acre)	Increase in True Wheat Yield (%)
Measured Wheat Yield	23.02	0.49	6.16	
True Wheat Yield	17.42	0.32	4.11	
Wheat Yield Due To Barley	0.47	0.32	3.37	2.69
Wheat Yield Due To Oats	1.75	0.28	3.16	10.05
Wheat Yield Due To Beans	0.70	0.62	3.80	4.02
Wheat Yield Due To Clover	-1.35	0.37	2.05	-7.75
Wheat Yield Due To Peas	2.72	0.52	3.75	15.61
Wheat Yield Due To Turnips	7.80	0.37	3.01	44.78

We can see in Table 8 that the failure to include the bare fallow in our calculations has biased upwards our estimates of the externalities of the other crops. Due to the bare fallow, the yield of grain is higher than predicted by the prices and yields of the other crops. Consequently, our calculations have misallocated the benefits of the bare fallow to the other crops - hence the impact of barley and oats appear to be positive. This demonstrates very clearly the importance of having *all* the relevant information.

6. Conclusions.

In this paper we have developed two new methods of aggregating different agricultural outputs into a single crop (such as wheat). This aggregation is a prerequisite for comparing output across production units and for estimating agricultural production functions. The new techniques are called New Method and the Adjusted Method. Whilst both of these methods have their drawbacks, they are both superior to the Traditional Method (which simply weights all the crops by their price ratio with wheat).

The New Method is very simple and parsimonious - we only need to know the total acreage and the yield of one crop (preferably wheat). This is a great practical advantage in empirical work, where price information is scarce - usually non-existent for fallows - and agents may not be fully integrated into the local market. The New Method is nonetheless not entirely satisfactory because it is biased upwards and creates heteroskedasticity - so it is therefore only suitable for calculations performed in natural logarithms, such as production function analysis. However, we have shown that the Traditional Method is also biased and creates heteroskedasticity - so the New Method is strictly preferred to the Traditional Method.

The Adjusted Method is not inherently biased because it uses price data to adjust downwards the estimate of the wheat yield. The introduction of price data makes the calculation of total output much more precise but much more difficult. However, the Adjusted Method also reveals a great deal of new information about the contribution of different crops to the wheat yield. It is therefore to be preferred to both the New Method and the Traditional Method whenever the relevant data are available.

Employing the Adjusted Method on the dataset compiled by Arthur Young in 1770, we find that the impact of most crops in the rotation is broadly as we would expect - but the magnitudes are quite surprising. Cereal crops such as barley substantially reduced the yield of wheat (14 per cent) whereas root crops such as peas substantially increased the yield of wheat (19 per cent). The positive impact of new fallow crops seems to have been very great - turnips raised wheat yields by 50 per cent. This quantifies and supports the hypothesis that new crop technology was an important innovation leading to higher output during the Agricultural Revolution.

Appendix I.

Here we prove that the Traditional Method and the New Method are equivalent. We use the following definitions.

P^W=price of wheat
P^{Be}=price of beans
A^{Be}=acreage of beans
Y^W=yield per acre of wheat
Q^{Be}=quantity of beans
Q^{TM}=quantity of wheat by the Traditional Method
Q^{NM}=quantity of wheat by the New Method

We begin with the Traditional Method. Calculate the value of the bean crop by multiplying the quantity of beans by its price. Divide this amount of money by the price of wheat to find the quantity of wheat which is equal in value to the crop of beans.

$$\frac{P^{Be}.Q^{Be}}{P^W} = Q^{TM} \qquad \text{(1 repeated)}$$

The alternative method is to simply multiply the acreage of beans by the yield per acre of wheat to find the wheat equivalent output:

$$A^{Be}.Y^W = Q^{NM} \qquad \text{(3 repeated)}$$

We now prove that equation (1) equals equation (3). Note first that the quantity of beans is only the acreage of beans multiplied by the yield of beans per acre:

$$Q^{Be} = A^{Be}.Y^{Be} \qquad (13)$$

Substituting equation (13) back into equation (1) (the Traditional Method) gives:

$$\frac{P^B.A^{Be}.Y^{Be}}{P^W} = Q^{TM} \qquad (14)$$

Dividing equations (3) and (14) by $A^{Be}.Y^{Be}$ gives:

$$\frac{P^{Be}}{P^W} = \frac{Q^{TM}}{A^{Be}.Y^{Be}} \qquad (15)$$

and,

$$\frac{Y^W}{Y^{Be}} = \frac{Q^{NM}}{A^{Be}.Y^{Be}} \qquad (16)$$

But P^{Be}/P^{W} is the ratio of prices, and Y^{W}/Y^{Be} is the Marginal Rate of Transformation (that is, the rate at which we can transform wheat into beans by switching resources from one crop to the other). The neo-classical production condition requires that the Marginal Rate of Transformation always equals the price ratio in equilibrium: [15]

Lemma 1.

$$\frac{Y^{W}}{Y^{Be}} = \frac{P^{Be}}{P^{W}} \qquad \text{(2 repeated)}$$

[15] Layard R and A Walters, *Microeconomic Theory* (McGraw-Hill, 1978).

So as a corollary,

$$\frac{Q^{TM}}{A^{B}.Y^{B}} = \frac{Q^{NM}}{A^{B}.Y^{B}} \qquad (17)$$

$$=> \qquad Q^{TM} = Q^{NM}$$

We can put this result more intuitively. We can rearrange Lemma 1 to give:

$$Y^{W}.P^{W} = Y^{Be}.P^{Be}$$

The revenue from an acre of beans must be equal to the revenue from an acre of wheat, otherwise the farmer would devote more acres to the product with a higher revenue. He would keep transferring acres to the higher value product until the price fell and costs rose to such an extent that the two crops gave the same revenue per acre. So all we need to ensure equilibrium is arbitrage. Provided that there is an area of land which can produce both beans and wheat, arbitrage can occur. In practice, crops were rotated so that in one year there was wheat planted in a field and the next year it was beans, so there was more than enough land to ensure arbitrage on every farm.

Appendix II.

Here we prove that the Traditional Method overestimates total output. Let us take the case of a simple four crop rotation. We use the following definitions:

Q^{TM} = total output by the Traditional Method
Y^{W*} = the 'true' yield of wheat (ie the measured yield minus that part of the yield which is due to the presence of other crops in the rotation)
Y^{WBe}= the yield of wheat due to the bean crop
Y^{WO} = the yield of wheat due to the oat crop
Y^{WT} = the yield of wheat due to the turnip crop
Y^{W} = the measured yield of wheat
Y^{Be*} = the 'true' yield of beans (ie the measured yield plus that part of the yield which is incorrectly ascribed to wheat)
Y^{O*} = the 'true' yield of oats
Y^{T*} = the 'true' yield of turnips

So,

$$Y^{W} = Y^{W*} + Y^{WBe} + Y^{WO} + Y^{WT} \qquad (18)$$

And,

$$Y^{T} = Y^{T*} + Y^{WT} \qquad (19)$$

(And the output of oats and beans can be broken down in the same way). The Traditional Method calculates total output as follows.

$$Q^{TM} = A^{W}.Y^{W} + \frac{A^{Be}.Y^{Be}.P^{Be}}{P^{W}} + \frac{A^{O}.Y^{O}.P^{O}}{P^{W}} + \frac{A^{T}.Y^{T}.P^{T}}{P^{W}} \qquad (20)$$

Substituting (18) and (19) into (20) and rearranging:

$$Q^{TM} = A^{W}(Y^{W*} + Y^{WBe} + Y^{WO} + Y^{WT}) + A^{Be}\left(\frac{P^{Be}.Y^{Be*}}{P^{W}} - \frac{P^{W}.Y^{WBe}}{P^{W}}\right)$$

$$+ A^{O}\left(\frac{P^{O}.Y^{O*}}{P^{W}} - \frac{P^{W}.Y^{WO}}{P^{W}}\right) + A^{T}\left(\frac{P^{T}.Y^{T*}}{P^{W}} - \frac{P^{W}.Y^{T}}{P^{W}}\right)$$

$$Q^{TM} = A^{W}.Y^{W*} + \frac{A^{Be}.Y^{Be*}.P^{Be}}{P^{W}} + \frac{A^{O}.Y^{O*}.P^{O}}{P^{W}} + \frac{A^{T}.Y^{T*}.P^{T}}{P^{W}}$$

$$+ Y^{WBe}(A^{W} - A^{Be}) + Y^{WO}(A^{W} - A^{O}) + Y^{WT}(A^{W} - A^{T}) \qquad (21)$$

We can see that equation (21) clearly overestimates total wheat-equivalent output. We ought to take the true output of each crop and adjust it by the relevant price ratio, which would be:

$$Q^{TM} = A^{W}.Y^{W*} + \underline{A^{Be}.Y^{Be*}.P^{Be}} + \underline{A^{O}.Y^{O*}.P^{O}} + \underline{A^{T}.Y^{T*}.P^{T}} \qquad (22) \qquad P^{W}$$

$$P^{W} \qquad\qquad P^{W}$$

But in equation (21) we have the additional terms:

$$Y^{WBe}(A^{W} - A^{Be}) + Y^{WO}(A^{W} - A^{O}) + Y^{WT}(A^{W} - A^{T})$$

Whenever the acreage of wheat is greater than (less than) the other crops, the total output will be overestimated (underestimated). By contrast, the new formula can be rearranged easily to give equation (22). We know from the neo-classical equilibrium condition that:

$$Y^{W*} = \frac{P^{Be}}{P^{W}} .Y^{be*} \qquad (23)$$

and similarly for the other crops. If we substitute equation (23) into equation (30) then we find directly that:

$$Q^{W*} = A^{W}.Y^{W*} + A^{Be}.Y^{W*} + A^{O}.Y^{W*} + A^{T}.Y^{W*} \qquad \text{(9 repeated)}$$

This is further confirmation of the accuracy of the new method.

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