Australian electoral reform and two concepts of representation

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Abstract:

The most distinctive Australian contribution to institutional design is the construction of electoral systems. Before Federation, remote colonies were an ideal seedbed for radical ideas on representation. Those ideas appealed to a microcosmic concept of representation. Since Federation, politicians have mostly stressed the rival (and partly incompatible) principal-agent concept of representation. I analyse the work of E. J. Nanson (1850--1936; Professor of Mathematics, University of Melbourne, 1875--1922) in this context. Nanson was one of only two anglophones in the 19th century to understand social choice theory (the other being Lewis Carroll). His fundamental papers were written in what was then one of the smallest and most isolated anglophone universities in the world. Nanson's failure to influence Australian institutional design at the foundation of the Commonwealth, and the subsequent adoption of Nanson's recommendations for Senate elections, both throw light on the incompatible conceptions of representation. So does the 1983 amendment of Senate procedures.

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1 Two rival concepts

Since the sixteenth century, two rival meanings of the English verb 'to represent' and its cognate noun 'representation' have been at war. Both of them are etymologically well grounded. Neither can be said to be wrong. But, in democratic theory, they are incompatible. They cannot simultaneously be used as normative justifications of any electoral system. Therefore, for two centuries, advocates of each class of system have talked past one another into an intellectual void. Australian politics provides notable examples of this.

The first strand of meaning of 'to represent' includes the following senses, as classified by the *Oxford English Dictionary*. In each case an illustrative quotation is given from the dictionary

> 1. a   To bring into presence: *esp.* to present (oneself or another) *to* or *before* a person
>
> > 1585 T. WASHINGTON tr. Nicholay's Voy. IV. i. 114 In the day time they did represent themselues before the Gouernours
>
> 8. a.   To take or fill the place of (another) in some respect or for some purpose; to be a substitute in some capacity for (a person or body); to act for (another) by a deputed right
>
> > 1651 HOBBES Leviath. II. xix. 95 It is manifest, that men who are in absolute liberty may..give Authority to One man, to represent them every one.
>
> 8. b.   *spec.* To be accredited deputy or substitute for (a number of persons) in a legislative or deliberative assembly; to be member of Parliament for (a certain constituency)….
>
> > 1778 BURKE Corr. (1844) II. 216, I do not wish to represent Bristol, or to represent any place, but upon terms that shall be honourable.

Hobbes and Burke shared this conception of representation. In Hobbes's world, citizens are all the authors of everything their representative does. In Burke's world, the MP is an agent (but not a delegate) of the principals (constituents) who sent him to Parliament. This is what is now called a *principal-agent* world. How principals can, or should, control their agents, and how far agents can, or should, faithfully represent their principals' views, are large issues, contested across the whole of social science since Hobbes and Burke, but fortunately irrelevant to this paper.

The second strand begins with portraits:

> 4. b.   *spec*. To exhibit by means of painting, sculpture, etc.; to portray, depict, delineate.
>
>> a1586 SIDNEY Arcadia I. (1605) 9 The Painter meaning to represent the present condition of the young ladie
>
> [*proportional*, sense 2.b]:   proportional representation, a system of parliamentary representation based on numerical (rather than regional) divisions of the electorate, spec. one by which each party is represented in proportion to the numerical strength of the vote it receives….
>
>> 1884 Pall Mall G. 19 Dec. 3/1 Proportional Representation finds little favour with the caucuses.

The wisdom of the *Pall Mall Gazette* generally applies in Australia as well. Caucuses do not like proportional representation {PR]. But representation in this sense is clearly *microcosmic*. In 1789, the concept was used simultaneously by Mirabeau in the French Revolution and by John Adams in the American Revolution. Mirabeau said in 1789, 'The Estates are to the nation as a map is to its physical extent: whether in whole or in part, the copy must always have the same proportions as the original' (quoted in Nohlen 1984; my translation). In one of his tracts on PR, E. J. Nanson (1900, p.13) quoted this saying of Mirabeau, and added that only Tasmania implemented it. The microcosmic conception of representation is obviously allied to PR. The principal-agent conception is allied to notions of majority rule. As the participants in the Australian debate about voting systems around 1900 were well aware, these are radically different conceptions. One cannot say that one is correct and the other wrong. But it is hard, perhaps impossible, for any parliament to form simultaneously a government and a mirror of the nation. Nor for legislators to be simultaneously delegates and trustees.

The first known implementation of microcosmic PR was for the Common Council of Adelaide in 1839:

> In order to counteract the tendency to exasperated party feeling which is sometimes found to exist in small communities, as well as to make timely provision against the arbitrary power which under popular governments the majority exercise over the minority, we … recommend, that the municipal elections may be so conducted that a majority of the rate-payers may not have the power to exclude the minority from returning their due proportion of members to the Common Council (Great Britain, *Parliamentary Papers* 1839 XVII: Third Annual Report of the Colonial Commissioners for South Australia, p. 19, quoted by Hart 1992, p.8).

Thus from the beginning, the microcosmic conception was also anti-majoritarian. That is why it was so fervently championed (to no good effect) by J. S. Mill - as a device to thwart the tyranny of the majority. Mill's protégé, Thomas Hare, produced several editions of his *The election of representatives, parliamentary and municipal* which circulated widely in Australia. The fourth edition (Hare 1873) is important because of its multifarious and undigested appendices. In one, what Australians call the 'Hare' scheme can be independently traced to the Danish mathematician C.G. Andrae (who understood its mathematical properties better than did Hare). Another reveals that what is usually called preferential voting in Australia (and Alternative Vote in the UK) was proposed by the Harvard professor W. R. Ware.

Thanks to Catherine Spence and Andrew Inglis Clark, the Hare scheme of PR was adopted in Tasmania in the late 19th century, and has formed a point of reference for Single Transferable Vote (alias Hare-Clark) ever since. At the same time, the Ware scheme of preferential voting was spreading through the remaining states, with a characteristic modification in Queensland designed in 1892 to restrict the counting of preferences to the top two parties in contention, with the aim of freezing out the rising Labour Party (Reilly 1997).

The Queensland example highlights two important and related truths.

1. Incumbent politicians usually prefer the principal-agent to the microcosm conception of representation. They are the sitting agents, and they would rather their principals continued to choose them.
2. Unless the electoral system is constitutionally entrenched, the incumbent agents can block any electoral system change, and will usually do so, unless they anticipate that voter preferences are changing in such a way that the incumbents would be better off under a different electoral system.

These truths have been repeatedly demonstrated in Australian politics. 19th-century ideas of representation in Australia were predominantly microcosmic. Colonial authorities making rules for remote settlements promoted proportional representation. As the colonies developed responsible government, so the attractions to incumbents of microcosmic representation faded. Yet the seed of microcosmic representation had been planted - not exactly to flourish but at least to survive in the 20th century.

The next sections of this paper review the activities of the E.J. Nanson (1850-1936; Professor of Mathematics, University of Melbourne 1875-1922) at the time when Commonwealth constitutional choices were being made. First, though, his extraordinary and lonely intellectual achievement needs to be put in context.

2   E. J. Nanson and principal-agent representation

The axiomatic theory of social choice is one of the central discoveries of modern social science. Arrow's Theorem (Arrow 1951) proves that no social welfare function - that is,

procedure for deriving a social ordering (or choice) from a set of individual orderings - can simultaneously satisfy a number of minimally demanding normative requirements of fairness and logicality. In one frequently quoted version of the theorem, the incompatible conditions go by the acronym CUPID for <u>C</u>ollective Rationality, <u>U</u>niversal Domain, the weak <u>P</u>areto principle, <u>I</u>ndependence of Irrelevant Alternatives, and non-<u>D</u>ictatorship. (See further any introductory text on social choice, e.g., Hargreaves-Heap et al. 1992, Part III). Two important sets of corollary theorems follow. One (Gibbard 1973; Satterthwaite 1975) proves that all determinate choice functions are either dictatorial or manipulable. The other (McKelvey 1979; Schofield 1985) proves that when the number of issue dimensions is three or more, chaotic outcomes from majority rule are pervasive. 'Chaotic outcomes' here mean outcomes involving global cycling, so that there is a path of successive majority votes from any outcome to any other (A beats B, which beats C, which (beats D, …,) which beats A. Besides these impossibility results, social choice has some well-known existence results. They include May's (1952) proof of the unique properties of the simple majority decision rule for two alternatives, and Black's (1948, 1949, 1958) median voter theorem, often attributed to Downs (1957).

Many of these results were foreshadowed in the distant past. Full details have been published elsewhere (McLean and London 1990; McLean and Hewitt 1994; McLean and Urken 1995; McLean 1995a, b). The following is a brief summary. Strategic voting (in which people misrepresent their preferences and/or manipulate the procedure in order to give their favourite option a better chance) is first discussed by Pliny the Younger in AD 105. Exhaustive pairwise voting as a way of finding the best out of more than two candidates is proposed by Ramon Lull in 1299. What we call the Borda count (because previously assumed to have been first proposed by J.-C. de Borda in 1770) was described by Nicholas of Cusa in 1435. A Borda count is a ranking rule in which each voter awards a score of *a* for a last place and increment of *b* for every place above that. Normally, *a* is taken as either 0 or 1, and *b* as 1. The voters' scores are summed and the candidate with the highest score wins.

The late 18th century saw a great flowering of axiomatic social choice in French mathematical literature, to which Borda, Condorcet, and the lesser-known figures S. Lhuilier and P. Daunou made important contributions. Condorcet, among much else, proposed that the winner in a multi-candidate contest should be that candidate who beat each of the others in an exhaustive set of pairwise comparisons. Such a candidate is a 'Condorcet winner'; the opposite is a 'Condorcet loser'. During this 'first golden age of social choice' Condorcet discovered the discrepancy between the Borda and Condorcet rules for selecting a winner, and the existence of majority-rule cycles (where A may beat B, who beats C, who beats A; in this case the Condorcet winner does not exist). The principle of Independence of Irrelevant Alternatives, which plays an important role in Arrow's Theorem, was characterised and defended by both Condorcet and Daunou. (McLean 1995a, b).

By 1810, all this had been lost again. The paradoxes of social choice were rediscovered independently by C. L. Dodgson (Lewis Carroll) in the 1870s and 1880s. Meanwhile, Borda and Condorcet found the only English reader who understood them in E. J.

Nanson. Nanson proposed a voting rule which combined the merits of the Condorcet and Borda rules, and had it implemented for the internal academic elections of Melbourne University. Its characteristics are sometimes discussed in the professional literature of social choice (Niou 1987; Fishburn 1990; Nurmi 1992, 1995; Saari 1995), where it has friends: 'Nanson ... continues to make fascinating reading more than a century after its publication' (Fishburn 1990, p. 101). Indeed, there has been a mini-boom in Nanson studies. Since (but not because of) the publication of McLean (1996), the *Social Science Citation* Index records no fewer than 14 citations of Nanson's work, mostly to the Nanson rule and Nanson (1882). Many of these citations are due to Don Saari of Northwestern University and his associates (e.g., Saari, 2000 a, b; Saari and Merlin 2000; Merlin, 2000). However, outside these circles it is unknown.

Nanson wrote one important paper on principal/agent representation, 'Methods of Election' (Nanson 1882; republished in full in McLean and Urken 1995, pp. 321--59). This is the paper that has made his modern reputation. In it he shows that he is one of only two English-language writers before 1951 to understand the principles set out by Borda and Condorcet, and the issues between them. (The other was C. L. Dodgson, better known as Lewis Carroll, for whom see Black 1958; McLean and Urken 1995, pp. 279--320; McLean, McMillan and Monroe 1996. Dodgson, unlike Nanson, had never read Condorcet.). Nanson's paper speaks for itself and is now readily available, so the following summary is extremely brief.

He opens by explaining that he is dealing with 'an election ... to select, if possible, some candidate who shall, in the opinion of a majority of the electors, be most fit for the post'. Thus, as he reiterates at the end, his paper is not about proportional representation. He relies throughout on what is now called the Condorcet criterion, that if a candidate exists who beats each of the others in exhaustive pairwise comparisons, he (she, it) should be chosen. He shows how no voting method in regular use satisfies the Condorcet criterion. One such method is 'Ware's method', (i.e., preferential voting): the procedure in which each voter ranks the candidates and, if no candidate gains over half of the first preference votes, candidates with the smallest numbers of first preferences are successively eliminated until somebody does get a majority. As Nanson shows, this method may fail to select a Condorcet winner.

Nanson states that Condorcet had a theoretical method of finding the winner, but that it was so poorly characterised in Condorcet's writing that 'it is quite hopeless to find out what Condorcet meant' and, in any case, the counting involved would be impossibly laborious (Condorcet's general method has, after a further century, been formally characterised - see Young 1988). He offers a method that combines the merits of Borda and Condorcet. This is, initially, to have the voters rank the candidates, and conduct a Borda count. After the initial Borda count, all candidates who failed to achieve more than the average score of all candidates are eliminated, and the Borda count is rerun among the survivors. This continues as often as necessary until a single winner is selected. Nanson proves that, if a Condorcet winner exists, this method, unlike the ordinary Borda count, is guaranteed to choose her (it, him).

Nanson's achievement in writing this paper in Melbourne in 1882 is remarkable. At the time there were three academic libraries in the city: the Melbourne Public Library (now the State Library of Victoria); the library of the Royal Society of Victoria; and the library of the university. Of these, only the first was of any use for Nanson's work. The university library was badly neglected. A decade earlier, the university had been spending more on its grounds than on its library (Blainey 1957, pp. 37, 50, 112). The library of the Royal Society of Victoria was essentially a collection of scientific periodicals acquired in exchange for the society's own transactions. It contained few books and no old books.[1] The Melbourne Public Library had (has) a set of the 1804 edition of Condorcet, from which Nanson (1882) quotes various passages. No library in Victoria held a copy of Condorcet's *Essai* (Condorcet 1785), nor a set of the *Histoire et Mémoires de l'Académie Royale des Sciences* in which Borda (1784) appears with Condorcet's commentary, nor yet an edition of Laplace's *Théorie analytique des probabilités* (Laplace 1814).[2] Yet these are the texts that Nanson, alone in the English-speaking world, read and understood. How did he do it? He doubtless read Todhunter (1865 - contemporary copies in both the public (State) and Melbourne University libraries). But Todhunter completely fails to understand Condorcet. And Nanson cites passages from both Condorcet and Laplace that are not in Todhunter. Perhaps Nanson brought some of these works when he arrived from Cambridge in 1875; perhaps he visited England or France; conceivably, there were copies elsewhere in Australia. We have only negative evidence.[3]

This highlights Nanson's loneliness. Although his chair was well paid, he did not enjoy teaching elementary mathematics to ill-motivated students. His teaching notes suggest that his courses were tedious.[4] The university went from boom to slump with the rest of the Melbourne economy as gold came and went. Nanson had a house, provided by the university, but complained constantly about its condition.[5] A crisis in 1900, when the accountant and the bursar lost the university's money at the races, led the state government to appoint a Royal Commission which gives a very detailed picture of life at the university. In 1904 it was tiny, with 14 professors, 18 lecturers, and 526 students. There was only one other mathematician on the faculty. Nanson is criticised both implicitly and explicitly in the Royal Commission report (*PP* Victoria 1904, vol. 2, pp 28, 30, 44). In an appendix to the report, the university's staff listed their publications. Some of the scientists gave full lists of papers. Nanson supplied only a single paragraph listing journals in which his work had appeared, without full citations. Although his junior lecturer was a Fellow of the Royal Society, there is nothing from these records of Nanson's teaching and research to hint that he was a distinguished mathematician.

Nanson's paper of 1882 thus seemed to come from nowhere, and to go nowhere. It was discussed at the March 1883 meeting of the Royal Society of Victoria, five months after its delivery, but the Society's minutes contain only the one-sentence statement that it was discussed.[6] However, he did succeed in getting his method adopted for elections for a student society at Trinity College, the Anglican constituent college of Melbourne University (McLean and Urken 1995, p. 338), and for elections to the Assembly and Canonry of the Anglican diocese of Melbourne[7]. The method was formalised in the statutes of the University of Melbourne between 1926 and 1982, for the election of

members of its Council and of academic committees. In 1968 it was also adopted by the University of Adelaide for the election of members of its University Council. The Adelaide implementation has been programmed and tested, and is still in use.[8] The only other known adoption of the Nanson rule anywhere in the world was for city elections in Marquette, Michigan, in the 1920s (Hoag and Hallett 1926, p. 491). The Melbourne rules, as given below, changed little between 1926 (see Baldwin 1926) and 1982.

21.   The voter shall indicate the order of his preference by writing numbers opposite the names of all or some or one of the candidates. A number opposite the name of a candidate shall indicate a preference for that candidate over each candidate opposite whose name a higher number or no number is written and the same number or no number opposite the names of two or more candidates shall indicate that the voter considers these candidates of equal merit.

22.   The number of preferences for each candidate over each other candidate shall be ascertained. In each case where on a voting paper no preference is expressed as between two candidates half a preference is to be credited to each of the two candidates. Where no preference is expressed as between more than two candidates the candidates so bracketed shall be dealt with two at a time. The number of preferences shall be arranged in tabular form in which one column (vertical) and one row (horizontal) are assigned to each candidate the number of preferences (for instance) for candidate P over candidate Q being written down in column P row Q.

23.   The numbers in each column shall be summed. The column with the lowest sum and the corresponding row shall be excluded and the remaining numbers in each column shall again be summed. The column with the lowest sum at this stage and the corresponding row shall be excluded and this process of summing and exclusion shall be repeated until only two columns are left. Of the candidates to whom these columns refer that one who has the majority of preferences over the other shall be declared elected.

24.   If a further vacancy is to be filled the column and row assigned to each elected candidate shall be excluded and the process of election carried out in precisely the same manner as before.

25.   If at any stage two columns (for instance those assigned to B and C) have the same sum and there is no other column with so small a sum then the column C shall be excluded if B has a majority of the preferences as between B and C but if B has exactly half of the preferences as between B and C the returning officer shall decide which column is to be excluded. If at any stage three or more columns have the same sum and there is no other column with a lower sum the returning officer shall decided which column is to be excluded.

26.   The returning officer shall subject to the provisions herein contained determine conclusively all questions of detail concerning the election. (Melbourne University, *Statutes*, 1982 edition, section 8:2. Original punctuation).

Some practical issues of counting under the Nanson rule are discussed by Baldwin (1926), who quotes some important supplementary rules for scrutineers, but it has attracted little notice elsewhere. The most distinctive feature of the rules is the construction in rules 22 and 23 of an $N \times N$ matrix (for N candidates) in which the votes of each candidate against each other are entered. This may be labelled a 'Dodgson matrix', because a version was proposed by C. L. Dodgson, and an example is in Table 4 below. In the 1960s a program to count votes under the Nanson rule was written in the university's administrative offices - probably in COBOL. This must have been one of the earliest vote-aggregation programs in existence. In 1983, after an enquiry by the Registrar of the University, the Nanson method was retained for some internal elections, but abandoned for Council elections in favour of STV using the Hare quota and the Gregory method for transfers (see below). This is implemented by means of a PASCAL program written by the Registrar. The reason for abandoning the Nanson system was that it was perceived to advantage inoffensive but not outstanding candidates as against those who attracted strong support.[9] Had he been alive in 1983, Nanson might have retorted that the former were more likely to be Condorcet winners than the latter.

3   *Nanson and microcosmic representation*

As far back as 1880, Nanson had published a scholarly description of proportional representation. In 1899 and 1900 he published two pamphlets aimed at politically interested readers generally, and designed to influence the electoral systems for elections to the first Commonwealth parliament. The publications of 1880, 1899, and 1900 cover similar ground but are not identical; taken as a set and even individually they contain tensions and contradictions. Finally in 1910 he published a short paper in the proceedings of that year's meeting of the Australasian Association for the Advancement of Science.

Nanson (1880) reviews the electoral systems classed by the Swiss electoral reformer Ernest Naville as 'Empirical' and 'Scientific'. The first class comprises the single [non-transferable] vote, the cumulative vote, and what Naville called the 'restricted vote'. These were used or discussed in some contemporary British elections (for details see Hart 1992; McMillan 1995). The 'restricted' or 'limited' vote is a system in which there are n seats to fill, and each voter may cast a maximum of m ($m < n$) votes, but may not lump more than one vote on to any one candidate. Where $m = 1$, this becomes the single non-transferable vote. The cumulative vote denotes any system in which voters may cast m votes ($m > 1$), but may lump any number up to m of them on to one candidate. Nanson rejects all these systems on the grounds that

> electors are compelled to submit to the dictation of party leaders, and adopt some process of organisation under pain of compromising their party.... Besides this, if

it should happen that the calculations of its leaders are wrong, the party does not obtain its fair share of representation (Nanson 1880, p. 34).

Nanson probably had in mind the then notorious behaviour of the Birmingham Liberals. From 1867 to 1885, the city of Birmingham was a three-member seat in which each elector had two votes. The Liberals divided the city into a number of zones, in each of which they asked their supporters to vote for a different pair of Liberal candidates. The Liberals won all three seats in every election (McLean, McMillan, and Monroe 1996 pp. xxvi--xxvii). Nanson was right to say that parties may miscalculate their support and thus win fewer seats than they otherwise could, but he did not see so deeply into this puzzle as Lewis Carroll (see below).

Nanson proceeds to discuss Naville's class of 'scientific' systems, but in fact he takes seriously only one of them, the system variously known as Single Transferable Vote (STV) and Hare-Clark. It will be called STV in the remainder of this paper because both Hare's and Inglis Clark's versions suffer from weaknesses that Nanson identified. Hare's proposal was quite widely known in Australia at this time, and the pattern of citations by Australian authors, not only to Hare, but to those whom Hare mentions, suggests that they mostly used the 4th edition of Hare's tract, Hare (1873).[10]

Nanson discusses two weaknesses of the Hare system: its incorrect quota, and its unsatisfactory procedure for transferring votes when a candidate is elected with a surplus. Where there are S seats to fill and V votes have been cast, the Hare quota is V/S. But as Nanson (1880, 1899, 1900) points out, this is too high. The correct quota is the 'necessary and sufficient number' (Nanson 1900, p.23) that ensures that exactly S candidates will be elected. This number, as had been proved by H. R. Droop in 1869 (see also Droop 1881), is $V/(S + 1)$, rounded up to the next integer. Nanson (1900, pp. 15--20) explains why the Hare-Clark system, when implemented in Tasmania in 1896, had led to perverse results. Because A. I. Clark, who implemented it, used the Hare quota V/S, early candidates had to cross a higher threshold than later ones, who could scramble in with a Droop quota or less. Thus, in Nanson's view, Hare-Clark gave rise to 'no end of scheming to get in on even less than the true low quota'. (The worked example in Tables 2-4 below may help illuminate Nanson's point).

Nevertheless, Nanson explains where the intuition for the Hare quota comes from:

> The object of the preferential quota system is to enable the voters to divide themselves into as many absolutely equal unanimous electorates ... as there are seats to be filled (Nanson 1899, p.4)

The other intractable problem with Hare schemes is the distribution of the surpluses of successful candidates. Andrae had proposed the simplest viable solution, namely random selection of ballot papers to be transferred. Where the quota in use (whichever quota it is) is denoted Q, and a candidate i has obtained $Q + s_i$ votes, the candidate's proportionate surplus is $s_i/(Q + s_i)$. Andrae proposed that a proportion $s_i/(Q + s_i)$ of the candidate's first-preference ballots should be selected at random and distributed to the

available second-place candidate named on each. Mathematically, Andrae's proposal is sound, but no other contemporary reformer liked it.[11] One practical problem (see Nanson 1900, pp. 15--20) was that if a candidate's scrutineers were involved, as in Tasmania, they would try to ensure that it was their candidate's papers that were selected for transfer.

Nanson's first transfer procedure (1880, pp. 40--52) was horrendously complex and quite impracticable. By 1899 he was advocating 'a simple device, due to a Melbourne citizen, Mr J. B. Gregory' (Nanson 1900, p.15). The Gregory principle was that all the ballots for a candidate elected with a surplus would be transferred in parcels made up according to which candidate was the second available preference. Each parcel would remain intact, but its contents would be weighted at the fraction $s_i/(Q + s_i)$. If a parcel had to be passed on a second or subsequent time, it would be weighted again; however,

> [w]hen a candidate with a surplus has more than one parcel the Gregory principle is applied, not to the whole of his parcels, but only to the last parcel which came to him, to the parcel which actually raised him above the quota (Nanson 1899, p.4).

Nanson (1899, 1900) insisted that tracking the movement and weighting of these parcels was a simple task of double-entry bookkeeping. (The Gregory principle has been used by the Australian Electoral Commission since 1984 to count Senate votes.) However, Nanson had second thoughts on all of this. His 1910 paper is quite odd. He proposes a proportional system for legislative elections as follows. Each legislator would have a parliamentary vote weighted by the number of popular votes obtained. In each district there would be as many seats as parties contesting the district, with each elected member having a vote weighted by the number of votes he or she achieved. This scheme, Nanson said, would be

> practically the Hare scheme without the quota and without the tedious process of surplus distribution. It is, in fact, the Hare scheme shorn of all its tedious complicated details ... but ... it furnishes a much more refined and accurate representation of the weights of parties (Nanson 1910, p.568).

Here Nanson acknowledges that the difficulties of the Hare scheme are greater than he had earlier admitted. Unfortunately, his proposed alternative is unworkable. Nanson does not consider either the problem of operating a legislature with huge numbers of MPs wielding differently weighted votes, nor the perverse incentives that his scheme would create for Monster Raving Loonies to enter, nor that the number of seats in each district could not be fixed until the number of candidates was known.

Nanson (1900) is the only one of the four papers to discuss the preferential vote in single-member districts. As noted above, a variant of this had been introduced in Queensland in 1893 under the title of the 'contingent vote'. Nanson (1900, p.11) characterises the Queensland scheme as the compression of a French double-ballot election into one round. In the French system, anybody may stand in the first ballot; but if no candidate gets over

50% of the votes cast, then the first two candidates only go through to a second ballot. The Queensland contingent voting scheme worked as the standard preferential voting (Alternative Vote) scheme now does, except that the ballots of supporters of eliminated candidates could be transferred only to whichever of the two survivors they ranked more highly. Thus, if a Queensland ballot paper did not rank either of the two top candidates in the first round, it could not be transferred. It would be 'far superior', Nanson argued, to proceed to a full preferential system in which votes from eliminated candidates could be transferred without restriction.

Nanson noted that in Victoria's own legislature the tendency was towards single-member electorates. He seems to have assumed (correctly) that political pressures would enforce single-member electorates for the Commonwealth House of Representatives, although he argued vociferously against single-member electorates for the Commonwealth Senate. He claimed that, if full preferential voting were introduced,

> Victoria has now the chance of securing the rule of the majority in each of her single electorates (Nanson 1900, p.10)

On the face of it, this is a very odd statement. Nanson had demonstrated in 1882 that preferential voting may fail to secure majority rule. A Condorcet winner may be eliminated. The most that can be said for preferential voting is that it cannot elect a Condorcet loser, since the winner in the final round has at least won a majority against one candidate. Nanson's statement is also contradicted within the same pamphlet, as when he states (p.18) that single electorates 'cannot prevent the minority ruling the country'. Why then did Nanson make it? Our surmise is that he guessed that both in Victoria and nationally, the lower house would be elected from single-member electorates. Nanson was convinced that for this purpose preferential voting was at least better than first past the post, which *can* elect a Condorcet loser - indeed, can elect a candidate ranked last by an absolute majority of the voters. Such a candidate is in the bottom subset of the set of Condorcet losers. It was nevertheless rash and misleading for Nanson to make a statement he knew to be false.

5   *His partial and ambiguous posthumous triumphs*

Nanson was active at a rare constitutional moment. Methods for Commonwealth elections were being settled. The methods used for the first election would affect both the party composition and the individual preferences of those elected, who would have the leading role in any amendment of the electoral rules (which are not entrenched in the Constitution itself). Like Condorcet a century earlier, Nanson had an opportunity to influence political debate on the shape of the constitution. Like Condorcet, he failed. Unlike Condorcet, he did not die for his beliefs. (For Condorcet's constitutional moment see McLean and Hewitt 1994; Badinter and Badinter 1988). But his work was marred by inconsistencies worse than Condorcet's.

Nanson failed for both political and intellectual reasons. The political reasons were essentially reasons of vested interest, of which Nanson was well aware. Legislative

majorities must vote electoral systems; legislative majorities are elected under the existing system, whatever it is; normally, some members of the majority will do worse under any change and many members are uncertain how change will affect them; therefore endogenous change in electoral institutions is extremely rare. The framing of the Commonwealth Constitution could have been a moment when this chain was broken, for as Nanson (1900, p.10) observed, 'there are, as yet, no vested interests to be preserved'. But the chain was not broken. Each State elected its delegation to the first Commonwealth Parliament by its own choice of method. South Australia and Tasmania chose their state delegations to the House by an at-large election (using Hare-Clark in Tasmania, but block voting in South Australia}, and Queensland used the contingent vote, but the majority of members of the first House were chosen by first-past-the-post in single-member districts. The first Commonwealth Electoral Bill was introduced in the House in 1901 but made no progress. It was reintroduced in the Senate in 1902. The bill contained provisions, drafted by Nanson, for STV to be used in the Senate and preferential voting in the House. These provisions were dropped as the bill progressed in favour of first-past-the-post in the House and the block vote for the Senate. A Royal Commission in 1913-14 recommended a return to the systems advocated by Nanson. However, when they returned, they did so for political reasons, not reasons of principle. The politicking was particularly naked in the case of the introduction of preferential voting in the House. In the 1918 by-election in Corangamite (Vic.), three candidates, each calling themselves Nationalists of one variety or another, were nominated as against one Labor candidate. The incumbent Nationalist government, already rattled by similar events earlier in the year, rushed a bill introducing preferential voting through parliament in time to implement it at the by-election. One of the Nationalists was duly elected, where the Labor candidate would certainly have been elected under first-past-the-post (Graham 1962, p.174). Preferential voting was introduced to the Senate in 1919, but without provision for proportional representation, so that in ensuing Senate elections up to 1946, the majority winner obtained a hugely disproportionate seat share, winning every single seat in 1925 and 1943 (Table 1).

[Table 1 about here]

This occurred because after the first winner had been elected, the same votes were counted again to elect the next, and so on until all the places were filled. Thus a majoritarian procedure was used in a proportional situation, with predictably perverse results. However, the introduction of STV for Senate elections in 1948 arose less from disquiet at this than from calculation of the immediate partisan advantage of the then incumbent Labor government (for evidence from both sides attesting to this see Reid and Forrest 1989, p. 120; for a general review of rationales for STV in the Senate see Uhr 1995; for Labor and the Senate since 1900 see Galligan 1995, ch.4). Uhr (1999) has recently amended the story somewhat. One of the primary motivations for the change was the perceived need to increase the size of the House of Representatives. But constitutionally (Commonwealth Constitution, s.24), this entailed an increase in the size of the Senate. This enabled the proponents of reform within the Labor caucus to assure Labor senators that they would keep their seats even if Labor lost seats overall, as it duly did. Labor lost in 1949, but not as badly as it would have done under the previous Senate

electoral rules. However, PR for the Senate has denied the Labor Party a majority in it ever since.

Nanson's political failure was predictable; but at least it was not total. He was one of those who put STV and preferential voting on the political agenda so that when the Commonwealth Parliament did choose for change, it chose from Nanson's menu. And his election method gained a foothold in two Australian universities. For the historian of social choice, Nanson's intellectual failure is the more puzzling and the more disappointing. His pathbreaking paper of 1882 showed exactly what was wrong with the practical election methods he espoused in his papers of 1899 and 1900, and in his political activity. As stated above, preferential voting in single-member electorates appears in the 1882 paper as 'Ware's method', after W. R. Ware of Harvard, whom Nanson had found mentioned in Hare (1873). Nanson correctly points out that the Ware method may fail to elect a Condorcet winner, although it cannot elect a Condorcet loser. In the final paragraph of his 1882 paper, Nanson points out that, although the paper is not concerned with the election of representatives, nevertheless,

it follows from the principles which have been established in this paper that the process of 'elimination', which has been adopted by all the exponents of the Hare system, is not satisfactory (Nanson 1882, p. 240; McLean and Urken 1995, p. 359).

It is not satisfactory because any elimination system except Nanson's own may discard a Condorcet winner. Yet Nanson appeared to ignore his own lesson when writing his tracts of 1899 and 1900. The arbitrariness of elimination under STV fuels modern discussion of its weaknesses, beginning with the demonstration of its non-monotonicity by Doron and Kronick (1977). A system is non-monotonic if a candidate can become more popular and yet lose a chance of being elected. This can happen under STV because the order of eliminations is all-important. Two of Nanson's contemporaries, the brothers T. R. and H. P. C. Ashworth, glimpsed this. They wrote a book (Ashworth 1900) on electoral reform. In Reid and Forrest (1989) they are bracketed with Nanson, yet they differed sharply from him. They believed that 'the Hare system would be absolutely destructive to party government' (p. viii). To some supporters of Hare this was, of course, an advantage. But the Ashworths, who were principal-agent theorists of representation, believed that good government required a bipartisan system. They proposed that every candidate should be required to choose in advance to stand as either a Ministerialist or an Oppositionist, and proposed a scheme of PR subject to this constraint. The details of their scheme, which was not practicable, are irrelevant here. However, their criticism of Nanson scored one hit. They concocted an example to bolster their claim that the 'process of elimination adopted by all the Hare methods has no mathematical justification' (Ashworth 1900, p. 154). This was, as they go on to say, exactly what Nanson had said in 1882, and the Ashworths' example shows why.

Their example is as given in Table 2.

[Table 2 about here]

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The Ashworths assume a Hare quota of 9 for their example. This leads to the election of A, B, and C. With the (better) Droop quota, C and two out of A, B, and E would be elected (see Table 3).

[Table 3 about here]

In either case, D and F would both be eliminated. Yet, as the Ashworths say, D, E, and F are each majority winners over each of A, B, and C. The best way to establish this is by use of a Dodgson matrix (Table 4)

[Table 4 about here]

The Dodgson matrix is compiled and read as follows. From the profile of preferences (Table 2) we read off the results of each pairwise comparison. Those who prefer each candidate are put in the cell 'For' that candidate and 'against' the other. Thus for example, from Table 2 we see that 16 voters rank B above A and 13 rank A above B; therefore 16 is entered in the cell 'For B, against A', and 13 in the cell 'For A, against B'. The procedure is the same as in rules 22--23 of the Melbourne University implementation of the Nanson rule (see above). When the matrix is complete, the row totals are scanned twice. First, the row sum for each candidate is calculated and entered in the column 'Borda score of row candidate'. This number directly gives the Borda rank-order count for each candidate. In the given example, the candidates in descending order are D, E, F, A, B, C. Secondly the row is scanned to count the number of wins for each candidate. A candidate wins a pairwise contest if the number in any row cell is larger than that in the diagonally opposite cell. Thus, the number in the cell 'For B, against A' exceeds the number in the cell 'For A, against B'; therefore B beats A. This is in fact B's only victory, so B's Copeland score (that is, simply, the number of victories, disregarding the margin of victory) is 1. On the Copeland score, the candidates in descending order are D, E, F, and then a three-way tie among A, B, and C.

There are rival conceptions of 'winning' in majority rule. The Copeland score is a more general version of the procedure of finding a Condorcet winner. Where there is a candidate with a unique highest Copeland score, that candidate is the Condorcet winner, and beats all others in pairwise comparison - D in the given example. The Borda and Condorcet (Copeland) orderings do not always coincide. But nobody has ever suggested that there is any other acceptable notion of 'winning' in a multi-candidate contest than one of those two. The Ashworths' example demonstrates that STV (Hare-Clark) fails to select the 'winners' on either criterion.

Nanson and Dodgson were roughly contemporary (Nanson was 18 years younger). They were both English-trained mathematicians (Nanson at Cambridge, Dodgson at Oxford). There is no evidence that they knew one another's mathematical work, although they had at least one professional acquaintance (Isaac Todhunter) in common. If they had read each other's work in voting theory, Dodgson would have found a route to Borda and Condorcet, instead of having to find out their discoveries all over again. And Nanson would have discovered from Dodgson's *Principles of Parliamentary Representation* (see

McLean, McMillan and Monroe 1996) that the single non-transferable vote has merits he did not suspect. If parties behave rationally, it will bring about proportionate representation without either Hare-style eliminations or the impractical weighted votes favoured by the Ashworths (and, later, by Nanson himself).

Nanson's procedure for selecting a majority winner is relatively simple and robust and, as stated above, was programmed as early as the 1960s in Melbourne. While Nanson himself had considerable trouble with recommending procedures for counting ballot papers expressing indifference, they are quite easy to handle once the Nanson procedure is mapped on to a Dodgson N xN matrix. As a majority rule procedure, it is due for a revival. As a procedure for proportional representation, it would not work for the same reason as preferential voting failed to work in the Senate between the wars: if there are blocks of voters, then any procedure which recounts the same votes for the election of the first, second, ... , nth successful candidate has the same effect as block plurality voting: the largest single faction will tend to secure all the seats. Nanson could neither reconcile his views on majority rule with his enthusiasm for PR, nor carry the day in the rough world of Australian politics at the foundation of the Commonwealth. But the first task is impossible because the two notions of representation are incompatible; the second would have defeated many people tougher than Nanson.

Since the introduction of PR for the Senate, Australian politicians have continued their profoundly ambiguous attitude to theories of representation. At one level, the Government of the day is always attracted to the principal-agent theory (and wishes it could get its way in the Senate). The Opposition of the day is always attracted to the microcosm theory (and is delighted that it can block some Government measures in the Senate). Recent evidence on the inconsistencies of both main parties on this point is in Sawer (1999). But the interests of the two main coalitions may not coincide with those of the electorate. As Sawer (1999) points out, 'A significant number of Australians vote differently for the two houses of the federal parliament and, of those, a percentage are quite explicit in seeing a minor party vote in the Senate as an insurance policy against overweening government.' This pattern has also been noted in Germany and New Zealand.

What makes the issue yet more complex is that, as in other bicameral territories, the House of Representatives represents population, and the Senate represents territory. Therefore, the majority party in the House may always be obstructed by what is not only a minority party, but a party representing the population of one or more small states, in the Senate. This sharpens the debate between principal-agent and microcosm advocates. It should also force Australians (and their candid friends elsewhere) to review the federalism that Australian political tradition has so long sought to suppress (Galligan 1995, *passim*). It also throws light on the strangely under-reported (at least outside Australia) but fundamental changes of 1983 in the Senate voting procedures (Farrell 2002). The Senate then changed from an STV chamber to a 'list PR' chamber. That profoundly changes, although without necessarily diminishing, the way it operates as a microcosm. It would probably have shocked T.W. Hill, Catherine Spence, and Andrew Inglis Clark.

The state of Australian constitutionalism was nicely summarised by Geoffrey Brennan (1999), himself summarising another contributor to the 1999 symposium on 50 years of Senate PR:

> If one believes—as Harry Evans' paper suggests—that good government is, like the amateur golfer's swing, a mass of compensating errors, then a good case might be made for the use of PR in the Senate without requiring one to decide on whether PR is, in a global sense, a better electoral system than the single-member electoral district system that characterises the House of Representatives…. PR in the Senate has been by no means the worst feature of our political institutional array. It has certainly been one of the most interesting and distinctive features. On that basis, if no other, it deserves the standard birthday treatment, and a cheerful round of `Happy Birthday'.

References

Arrow, K. J. (1951) *Social choice and individual values* New York: Wiley

Ashworth, T. R. and H. P. C. (1900) *Proportional representation applied to party government: a new electoral system* Melbourne: George Robertson & Co.

Badinter, R. and Badinter, E. (1988) *Condorcet (1743--1794): un intellectuel en politique* Paris: Fayard

Baldwin, J. M. (1926), 'The technique of the Nanson preferential majority system of election', *Proceedings of the Royal Society of Victoria* n.s. 39: 42--52

Black, D. (1948), 'On the rationale of group decision-making', *Journal of Political Economy* 56 pp. 23--34

Black, D. (1949), 'The elasticity of committee decisions with alterations in the members' preference schedules', *South African Journal of Economics* 17, pp. 88--102

Black, D. (1958), *The theory of committees and elections* Cambridge: Cambridge University Press

Blainey, G. (1957), *A centenary history of the University of Melbourne* Melbourne: Melbourne University Press

Borda, J.-C. de (1784) 'Mémoire sur les élections au scrutin' *Mémoires de l'Académie Royale des Sciences année 1781* pp. 657--65

Brennan, G. (1999), 'The Senate and Proportional Representation: Some Concluding Observations', ch.19 of *Papers on Parliament* (Canberra: Department of the Senate, 1999) No. 34, December, edited by M. Sawer and S. Miskin, available on-line at http://www.aph.gov.au/senate/pubs/pops/Pop%2034/chapter19.htm.

Condorcet, M. J. A. N de Caritat, marquis de (1785), *Essai sur l'application de l'analyse à la probabilité des décisions rendues à la pluralité des voix* Paris: Imprimerie Royale

Dodgson, C. L. (1876), 'A method of taking votes on more than two issues' Oxford: privately printed

Doron, G., and Kronick, R. (1977), 'Single Transferable Vote: an example of a perverse social choice function', *American Journal of Political Science* 21: 303--11.

Downs, A. (1957), *An economic theory of democracy* New York: Harper & Row

Droop, H. R. (1869), 'On the political and social effects of different methods of electing representatives', *Papers Read before the Juridical Society* 3: 469--507

Droop, H. R. (1881), 'On methods of electing representatives', *Journal of the Statistical Society* 44: 141--202

Farrell, D. (2002), 'Stretching the boundaries of 'STV': the 1983 reform of the electoral system to the Australian Senate', paper for APSA Jubilee conference, Canberra, October.

Fischer, A. J. (1978), 'The random sampling of votes in the Australian Senate', Adelaide: University of Adelaide Economics Department Papers, No. 7.

Fischer, A. J. (1980), 'Sampling errors in the electoral process for the Australian Senate', *Australian Journal of Statistics* 20, 24-39

Fischer, A.J. (1988), 'Electoral distortion under STV random sampling procedures: a comment', *British Journal of Political Science* 18: 142-3.

Fishburn, P.C. (1990) 'A note on "A note on the Nanson rule"', *Public Choice* 64: 101--2

Galligan, B. (1995), *A Federal Republic: Australia's Constitutional System of Government* Cambridge University Press 1995

Gibbard, A. (1973), 'Manipulation of voting schemes: a general result', *Econometrica* 41: 587--601

Graham, B. D. (1962), 'The choice of voting methods in Federal politics, 1902--1918', *Australian Journal of Politics and History* 8: 164--81

Hare, T. (1873), *The election of representatives, parliamentary and municipal* 4th edn London: Longmans

Hargreaves Heap, S., Hollis, M., Lyons, B., Sugden, R., and Weale, A. (1992), *The Theory of Choice: a critical guide* Oxford: Blackwell

Hart, J. (1992), *Proportional representation: critics of the British electoral system 1820--1945* Oxford: Clarendon Press

Hoag, C. G., and Hallett, G. H. (1926), *Proportional representation* New York: Macmillan

Hughes, C. A. and Graham, B. D. (1968) *A handbook of Australian government and politics 1890--1964* Canberra: ANU Press

Laplace, P.-S. de (1814) *Théorie analytique des probabilités* Paris: V.Courcier

McKelvey, R. D. (1979), 'General conditions for global intransitivities in formal voting models', *Econometrica* 47: 1085--1111

McLean, I. and London, J. (1990), 'The Borda and Condorcet principles: three medieval applications' *Social Choice and Welfare* **7**: 99--108

McLean, I. and Hewitt, F. (1994), *Condorcet: foundations of social choice and political theory* Aldershot: Edward Elgar, 1994

McLean, I. and Urken, A. B. (1995), *Classics of social choice* Ann Arbor: University of Michigan Press

McLean, I. (1995a), 'The first golden age of social choice 1784--1803', in W. Barnett, H. Moulin, M. Salles, and N. Schofield eds, *Social choice, welfare, and ethics: proceedings of the 8th International Symposium in Economic Theory and Econometrics* (Cambridge: Cambridge University Press), pp. 13--33.

McLean, I. (1995b), 'Independence of Irrelevant Alternatives before Arrow', *Mathematical Social Sciences* **30**: 107--26

McLean, I. (1996), 'E.J. Nanson, social choice, and electoral reform', *Australian Journal of Political Science* 31(3), 369-85.

McLean, I., McMillan, A., and Monroe, B. L. (1996), *A mathematical approach to proportional representation: Duncan Black on Lewis Carroll* Boston, MA: Kluwer Academic Publishers

McMillan, A. (1995), 'The limited vote in Britain', University of Warwick M. Phil. dissertation

May, K. O. (1952), 'A set of independent necessary and sufficient conditions for simple majority decision', *Econometrica* 20: 680--4

Merlin, V., M. Tataru, et al. (2000). "On the probability that all decision rules select the same winner." Journal of Mathematical Economics 33(2): 183-207

Nanson, E. J. (1880), 'Proportional representation', *Transactions and Proceedings of the Royal Society of Victoria* 17: 34--52

Nanson, E. J. (1882), 'Methods of election', *Transactions and Proceedings of the Royal Society of Victoria* 19: 197--240

Nanson, E. J. (1899), *Electoral reform: an exposition of the theory and practice of proportional representation* Melbourne: T. E. Verga

Nanson, E. J. (1900), *The real value of a vote and how to get it at the coming Federal elections* Melbourne: J. T. Picken

Nanson, E. J. (1910), 'The proxy vote', paper read before the Australasian Association for the Advancement of Science. Brisbane: Government Printer

Niou, E. M. S. (1987), 'A note on Nanson's rule', *Public Choice* 54: 191--3

Nohlen, D. (1984), 'Two incompatible principles of representation' in A. Lijphart and B. Grofman eds, *Choosing an electoral system: issues and alternatives* (New York: Praeger), pp. 83--9.

Nurmi, H. (1992), 'An assessment of voting system simulations', *Public Choice* 73 459--87

Nurmi, H. (1995), 'On the difficulty of making social choices', *Theory and Decision* 38: 99--119

Reid, G. S. and Forrest, M. (1989) *Australia's Commonwealth Parliament 1901--1988: ten perspectives* Melbourne: Melbourne University Press

Reilly, Ben (1997). 'The Plant Report and the Supplementary Vote - not so unique after all', *Represntation* 34(2): 95-102.

Riker, W.H. (1982) *Liberalism against Populism* San Francisco: W. H. Freeman

Saari, D. G. (1995), 'A chaotic exploration of aggregation paradoxes', *SIAM Review* 37: 37—52

Saari, D. G. (1996), 'The generic existence of a core for q-rules', forthcoming in *Journal of Economic Theory*

Saari, D. G. (2000a). "Mathematical structure of voting paradoxes I. Pairwise votes." Economic Theory 15(1): 1-53.

Saari, D. G. (2000b). "Mathematical structure of voting paradoxes II. Positional voting." Economic Theory 15(1): 55-102.

Saari, D. G. and V. R. Merlin (2000). "A geometric examination of Kemeny's rule." Social Choice and Welfare 17(3): 403-438

Satterthwaite, M. (1975), 'Strategy-proofness and Arrow's conditions', *Journal of Economic Theory* 10: 187--217

Sawer, M. (1999), 'Overview: Institutional Design and the Role of the Senate', ch. 1 of *Papers on Parliament* (Canberra: Department of the Senate, 1999) No. 34, December,

edited by M. Sawer and S. Miskin, available on-line at
http://www.aph.gov.au/senate/pubs/pops/Pop%2034/chapter1.htm

Schofield, N. (1985), *Social choice and democracy* Berlin: Springer-Verlag

Todhunter, I. (1865), *A history of the mathematical theory of probability from the time of Pascal to that of Laplace* London: Macmillan

Uhr, J. (1995), 'Proportional representation in the Australian Senate: recovering the rationale', *Australian Journal of Political Science* 30 (Special Issue), 127--41

Uhr, J. (1999), 'Why We Chose Proportional Representation', ch. 2 of *Papers on Parliament* (Canberra: Department of the Senate, 1999) No. 34, December, edited by M. Sawer and S. Miskin, available on-line at
http://www.aph.gov.au/senate/pubs/pops/Pop%2034/front.htm.

Young, H. P. (1988), 'Condorcet's theory of voting', *American Political Science Review* 82: 1231--44

Tables

Table 1

Australian Senate election results while block preferential voting was in force

Election	Anti-Labor coalition[*]:		Labor:	
	Share of vote, %	Seats	Share of vote, %	Seats
1919	46.4	18	42.8	1
1922	36.2	8	45.7	11
1925	54.8	22	45.0	0
1928	49.5	12	49.0	7
1931	55.4	15	29.2	3
1934	48.2	18	28.1	0
1937	44.8	3	48.5	16
1940	50.4	16	37.5	3
1943	38.1	0	55.1	19
1946	43.3	3	52.1	16

[*]   Nationalist 1919--22; Nationalist + Country 1925--8; UAP + Country 1931--43; Liberal +

Country 1946. Rest of vote share, to 100%, went to groupings which won no seats at any election in the

period.

Source: Hughes and Graham (1968), pp. 320--75.

Table 2

Failure of STV (Hare-Clark) to elect Borda or Condorcet winners: the Ashworths' example

N. of voters **Preference:**	7	6	5	4	4	3
Highest	A	E	C	B	D	F
	D	F	E	D	C	B
	E	D	B	F	E	A
	F	A	D	A	F	E
	B	C	F	C	B	C
Lowest	C	B	A	E	A	D

3 seats to fill
29 votes cast
(Hare) quota = 9; Droop quota = 8

Outcome using Hare quota:

Stage I: F eliminated
Stage II: D eliminated
Stage III: C elected with no surplus
Stage IV: E eliminated
Stage V: A elected with surplus of 4
Stage VI: B (the only remaining candidate) elected on transfer of all A's surplus.

Outcome using Droop quota:

Stage I: F eliminated
Stage II: D eliminated
Stage III: C elected with a surplus of 1
Stage IV: Three-way tie for last place among A, E, and B (see Table 3)

Source: Ashworth (1900), p. 154.

Table 3

Preference profile for Table 1 under Hare-Clark, using Droop quota, after Stage III of the counting

Preference:	N. of voters	7*	6**	1***	7****
Highest		A	E	E	B
		E	A	B	A
Lowest		B	B	A	E

*   the 7 voters in column 1 of table 2

**   the 6 voters in column 2 of table 2

***   the 9 voters in columns 3 and 5 of table 2, each vote weighted at 1/9 on transferring C's surplus

****   the 7 voters in columns 4 and 6 of table 2

Table 4

Dodgson matrix, with Borda and Copeland scores, for the example in Table 2

Votes against:	A	B	C	D	E	F	Borda score of row candidate	Copeland score of row candidate
Votes for:								
A		13	20	10	14	7	**64**	**1**
B	16		14	12	7	9	**58**	**1**
C	9	15		8	13	9	**54**	**1**
D	19	17	21		15	20	**92**	**5**
E	15	22	16	14		22	**89**	**4**
F	22	20	20	9	7		**78**	**3**

Each of D, E, and F beats each of A, B, and C in pairwise comparison (Ashworth 1900, p. 155). A, B, and C are in a cycle for last place (A beats C, C beats B, B beats A). This is shown by their Copeland scores of 1 each. The Copeland score records the number of majority wins for each candidate. The row sum of scores for each candidate is both the aggregate of that candidate's pairwise votes against each of the others in turn and that candidate's Borda score (scaled at 0 for a last place, 1 for a second-last place, ..., $n - 1$ for a top place. The mathematical equivalence of these two numbers was proved by Borda (1784; McLean and Urken 1995, pp. 86--8)). D is thus the winner by either the Borda (rank-order) or the Condorcet (exhaustive pairwise comparison) procedure.

Notes

[1] See MS catalogue, 1891, in the archives of the Society: Latrobe Collection, State Library of Victoria, MSF 11663.

[2] All holdings of these titles in the libraries mentioned above were checked. No copies arrived in Melbourne before 1882.

[3] There is a collection of Nanson's papers in the Melbourne University Archives, comprising about 20 box files of professional materials removed from his house on the University's 'Professors' Row' on his death. It includes some of his mathematical notes and some textbooks; all are elementary. We have found no evidence that Nanson spent any long time away from Melbourne between 1875 and 1882.

[4] 'Multiply 3,478,913 by 254,787, divide the product by 537,215,791, and express the quotient and remainder in words'. From one of his teaching notebooks, marked 'Arithmetic: October Term 1878'. Nanson MSS, Melbourne University Archives.

[5] Melbourne University Archives, List of Administrative Records, *passim*: e.g., 1884/28 (EJN unsuccessfully requests a water closet for his house in place of an earth closet); 1890/32 (EJN reports on the insanitary state of his house; he later calls in the state health authorities, who declare it unfit for habitation).

[6] Royal Society of Victoria, Minute Book: Latrobe Collection, State Library of Victoria, MSF 11663, vol. VIII.

[7] See three undated pamphlets, at least the first two of which are by Nanson: (1) 'Preferential Voting'; (2) 'Remarks on the Report on the New Electoral System' and (3) 'The Recent Canonry Election: Analysis of Votes'. Nanson MSS, University Archives, Melbourne. The Report which pamphlet no. 2 attacks was hostile to the Nanson system, so it may have been abandoned soon after its introduction.

[8] University of Adelaide, 'The Voting System' 4-page leaflet, dated 19/8/92. I am very grateful to Emeritus Professor J. H. Bennett for drawing this to my attention.

[9] Information in this paragraph is from an interview with Jim Potter, Registrar, Melbourne University, 28.3.1996.

[10] The State and University libraries in Melbourne between them hold contemporary copies of the 2nd, 3rd, and 4th editions of Hare.

[11] The relative merits of random selection and the Gregory method for carrying out Senate vote transfers were discussed by the Australian Electoral Commission in the years preceding the 1984 legislation that now governs its operations. It selected the Gregory method. On the properties of random sampling in this application see Fischer (1978, 1980, 1988).