Deus ex machina: the inconsistency of optimal (monetary) delegation*

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Abstract. A prominent solution to the time inconsistency problem inherent to monetary policymaking consists of delegating monetary policy to an independent central bank by an appropriately designed inflation contract or target. This paper shows that delegation is *not* a solution to this problem: *optimal* delegation requires commitment and is not time-consistent, while time-consistent delegation is suboptimal. We prove these results formally in two popular models of monetary policy: a backward- and a forward-looking one. Introducing costs of reappointing the central banker can only solve this problem if the government is infinitely averse to changing central bank's contract. Our results have immediate implications in terms of (i) explaining inflation performance; (ii) giving a more prominent role to central bank independence and reputation building in fighting inflation.

JEL codes: *E31, E52, E61, C61, C73.*

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**Deus ex machina* (Latin, 'God from the machinery'): Device in Greek theatre in which problems were resolved at the end of a play by the intervention of a God who was apparently brought down from Olympus. In fact he was moved by 'machinery' (a crane). It now refers to any contrived interposition in a novel, play, or film, and in general to any external, unexpected, last-minute resolution of a difficulty. *Oxford Paperback Encyclopedia.*

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*"We conclude that there is **no** way control theory can be made applicable to economic planning when expectations are rational."*(Kydland and Prescott (1977), Abstract, emphasis in original).

The optimal design of domestic monetary institutions has been the focus of an impressive amount of literature over the past three decades. A large part of this literature is concerned with solving the 'time inconsistency' problem of monetary policy, first identified by Kydland and Prescott (1977). The basic insight is very simple: an authority conducting monetary policy cannot commit -with respect to a rational private sector- to follow an optimal state-contingent policy rule since it has an incentive to deviate after expectations are formed. The time-consistent equilibrium that will arise will be one where policy is chosen discretionarily, and is suboptimal from the society's point of view. For example, when the socially optimal rate of output is higher than the natural one due to some real distortion, the government would stimulate output by 'surprise inflation'. This trade-off has been explored by Barro and Gordon (1983a,b) and is known as the 'inflationary bias' of monetary policy. Moreover, the more recent literature has found that the gains from monetary policy commitment go well beyond the initial elimination of this 'average inflation bias'. Specifically, in models where shocks give rise to a trade-off between the variability of inflation versus output, Lockwood (1995), Svensson (1997) and Clarida, Gali, and Gertler (1999) find that (independently from the inflationary bias) a *stabilization bias* also emerges in the absence of commitment: in response to shocks creating this trade-off, there is too much output stabilization and too little inflation stabilization in the time-consistent, discretionary equilibrium. Importantly, all these papers demonstrate that even when the inflationary bias is absent (e.g. because the government targets the natural rate of output), the stabilization bias remains and gains from commitment occur even when the inflation bias is absent (see also Woodford (2003) for a recent review)[1].

A prominent way purported to solve the time inconsistency problem is found by Walsh (1995) and Persson and Tabellini (1993) in a static framework. It consists of delegating monetary policy to an independent central bank, e.g. by means of an 'optimal contract' provided by the government[2]. This optimally designed contract induces an incentive scheme to the central bank (as e.g. linear penalties for excessive inflation) that makes it, despite acting discretionarily, implement the optimal policy as the unique dynamically consistent equilibrium[3]. These results have been extended by i.a. Lockwood (1995) and Svensson (1997) to a backward-looking dynamic context featuring an endogenous state variable, showing that optimal contracts can eliminate both the inflation bias and the stabilization bias. More recent research in the forward-looking 'New Keynesian' vein, such as Jensen (2002) and

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[1]Although similar in implications, the stabilization bias is different according to whether the structural model is backward-looking (as in Lockwood and Svensson) or forward-looking (as in Clarida et al and Woodford). We cover both cases.

[2]Earlier delegation schemes, such as Rogoff's 1985 proposal to make the central bank more inflation-averse than the government, do not solve the time inconsitency problem since they induce a tradeoff between reducing the average inflation bias and inducing a suboptimal response to shocks.

[3]Svensson (1997) shows how these contracts can actually be thought of in terms of real-world inflation targeting regimes (such as those of New Zealand, Canada, Sweden, UK and others), whereby the central bank is assigned a lower inflation target than society's.

Walsh (2003), has also studied institutional design as a way to ameliorate the stabilization bias that occurs in that framework.

This paper shows that optimal delegation schemes deemed to solve time inconsistency are subject to exactly the same time inconsistency problem as optimal policy. The incentive to deviate from optimal policy could only disappear when it comes to delegating if something -such as a *Deus ex machina* in a Greek tragedy- made the government implement an optimal policy, which was not compatible with its incentives in the first place. We solve for the -time consistent- delegation parameters chosen by the government recursively based on its rationality, and show that these are different from the 'optimal' delegation parameters in an intuitive way. Specifically, the contract that the government chooses optimally in our setup leads to implementation of the discretionary equilibrium, which is consistent with government's initial incentives. While optimal delegation is indeed desirable, nothing insures its implementability. Whether the government is subject to a time inconsistency problem or not is independent of whether it chooses monetary policy directly or it designs an incentive scheme for the central bank to which it delegates policy. Moreover, this finding is independent of whether the structural model is backward- or forward-looking. Finally, we show that making it costly for the government to revise the institutional arrangement does not solve the problem, but merely postpones it.

Our results echo the original message of Kydland and Prescott and have immediate implications. Empirically, they question the alleged causality between 'inflation targeting' regimes and the success in fighting inflation. Theoretically, they imply a stronger case for central bank independence and a more prominent role for reputation building as a way to achieve a desired low-inflation equilibrium. It should be noted that while our results are reminiscent of a verbal argument put forth by McCallum (1995) and formalized by Jensen (1997), a few important and subtle differences are apparent in both substance and interpretation. Firstly, while both McCallum's discussion and Jensen's analysis imply that the optimal contract will not be *enforced*, our results are stronger: *optimal* delegation is not time-consistent, and therefore will not be chosen in the first place; indeed, equilibrium -time-consistent, subgame perfect- delegation takes the form that would ensure implementation of the suboptimal equilibrium. It is in this sense that we view our results as supporting central bank independence and reputation building. Secondly, McCallum's argument is entirely verbal[4], whereas we present a formal proof in two dynamic models featuring both an (average and state-dependent) inflation bias and a stabilization bias. Thirdly, whereas McCallum's discussion and Jensen's analysis pertain to solving the inflation bias problem in a static model, our results apply more generally to models in which gains from commitment occur despite the absence of an inflation bias[5] ; specifically, both our models contain a stabilization bias: one is backward looking and contains an endogenous state, while the other is forward looking. Finally, while we derive our results in models of monetary policy, the main idea is very general; the same intuition would carry over to

[4]Jensen (1997) formalizes McCallum's argument, but he also focuses on *enforcement* of the optimal contract, and not on the time (in)consistency of delegation. Moreover, his model is also static: enforcement is just another stage within a period game that is repeated over time, and the only distortion is the average inflation bias.

[5]Blinder (1997) argues convincingly that policymakers do not try to push output above the natural level, and hence the inflation bias is simply not a problem.

any model in which there is a time inconsistency problem and delegation schemes have been proposed as a way to solve it (see Persson and Tabellini (2000) for a review of the literature).

In the remainder we proceed as follows: Section 1 solves for the commitment and discretion equilibrium in a backward-looking model with an endogenous state variable (a dynamic version of the Barro-Gordon model). Section 2 finds the optimal delegation parameters (specifically, inflation contract) and claims that optimal delegation is, just as optimal policy with commitment, time inconsistent. Section 3 substantiates this claim, modelling the delegation stage explicitly. First, we show that optimal delegation only occurs under the assumption of commitment; then, we solve for 'time-consistent' or 'subgame perfect' contracts and show that they lead to the suboptimal discretionary equilibrium. Section 4 extends these results to a forward-looking New Keynesian model. Section 5 studies whether costs of changing the contract can alleviate the problem and Section 6 concludes.

1. Commitment and discretion in a dynamic backward-looking model

The first model we use is a dynamic version of the Barro-Gordon (1983a,b) model that incorporates autoregressive dynamics (persistence, or 'hysteresis') in output and hence features an endogenous state variable[6]. This has strong implications for the delegation problem, as the optimal delegation parameters become state-contingent (these issues are studied in detail e.g. in Svensson (1997) or Lockwood (1995)). In this Section we just reproduce the main results in our context for future use. We assume, following the aforementioned studies, that the expectations-augmented aggregate supply curve is:

$$y_t = \rho y_{t-1} + \alpha \left(\pi_t - \pi_t^e\right) + \varepsilon_t \tag{1.1}$$

In the above, y_t is the log of output, π_t is the inflation rate, π_t^e the inflation expected by the private sector and ε_t an iid supply shock with mean zero and variance σ^2. The natural rate of output has been normalized to zero for convenience, hence y_t can be regarded as deviations from the natural rate. ρ is a constant parameter in the $[0, 1)$ interval capturing autoregressive dynamics in output. Suppose the private sector forms inflation expectations according to the rational expectations rule:

$$\pi_t^e = E\left[\pi_t \mid \digamma_{t-1}\right] \equiv E_{t-1}\pi_t \tag{1.2}$$

The $E\left[. \mid \digamma_{t-1}\right]$ is the conditional expectation taken with respect to the information set $\digamma_{t-1}$, containing all the information available at time $t-1$, i.e. $\digamma_{t-1} = \{y_i, \pi_i, \varepsilon_i, \rho, \alpha\}_{i=1}^{t-1}$. Equation (1.1) will act as a constraint on the state variable in the future period, of the form: $y_t = \Gamma^d(y_{t-1}, \varepsilon_t)$. In the commitment case (to be discussed below), both equations (1.1) and (1.2) act as constraints, i.e. $y_t = \Gamma^c(y_{t-1}, \varepsilon_t)$.

The government's preferences are identical to those of society's and are assumed to concern inflation and output deviations from some optimal levels. Following the literature, these are supposed to be given by the following period loss function,

[6]Persistence in output or unemployment, or 'hysteresis', is a well-known stylised fact (see Blanchard and Summers, 1986). Theoretical models can be built in which this result is explained (a review of this research can be found in Lockwood and Philipopoulos, 1994).

where (π^*, y^*) is the socially optimal equilibrium and λ the weight on output stabilization[7]:

$$L_t = \frac{1}{2}\left[(\pi_t - \pi^*)^2 + \lambda (y_t - y^*)^2\right] \tag{1.3}$$

Note that the assumption that $y^* > 0$ (which is the natural rate) gives rise to the inflation bias described before. Suppose further for simplicity that the government can perfectly control the inflation rate and that the timing of events at each t is as follows: (0) *either* government *commits to an optimal rule* or *delegates policy to an independent central bank*, depending on the cases considered below; (i) expectations π_t^e are formed by (1.2); (ii) shocks ε_t are realized; (iii) π_t is chosen, if commitment has not taken place previously; (iv) y_t is fully determined.

When at (0) the government commits to a **state-contingent optimal rule** the policy is a solution to the problem:

$$\inf_{\{\pi_t\},\{\pi_t^e\}} E_0\left[\sum_{t=1}^{\infty}\beta^{t-1}L_t\right] \equiv v(y_0, \varepsilon_0), \quad s.t.(1.1),(1.2),(1.3). \tag{1.4}$$

Note that due to commitment the government can be regarded as choosing the inflation expectations since these are determined at (i) by government's decision at (0). Note that in this case uncertainty is not resolved when decision is being taken and the dynamic constraint correspondence (function) is $y_t = \Gamma^c(y_{t-1}, \varepsilon_t)$, comprising both (1.1) and (1.2). The Bellman equation associated to problem (1.4) is (where a superscript 'c' stands for 'commitment' throughout):

(1.5)
$$v^c(y_{t-1}) = \inf_{\{\pi_t\},\{\pi_t^e\}} E_{t-1}\left[\frac{1}{2}\left[(\pi_t - \pi^*)^2 + \lambda (y_t - y^*)^2\right] + \beta v^c(y_t)\right], \text{s.t.}(1.1),(1.2).$$

We substitute constraint (1.1) directly into the loss function, and attach the Lagrange multiplier θ_t to (1.2) to get the first order conditions for the right-hand side of (1.5) (do not assume a functional form for $v^c(.)$ for the moment), with respect to π_t and π_t^e respectively :

$$\begin{aligned}
\pi_t - \pi^* + \lambda\alpha (y_t - y^*) + \alpha\beta\frac{\partial v^c(y_t)}{\partial y_t} - \theta_t &= 0; \\
-E_{t-1}\left[\lambda\alpha (y_t - y^*) + \alpha\beta\frac{\partial v^c(y_t)}{\partial y_t}\right] + \theta_t &= 0.
\end{aligned}$$

Eliminating the Lagrange multiplier we obtain:

$$\pi_t - \pi^* + \lambda\alpha (y_t - y^*) + \alpha\beta\frac{\partial v^c(y_t)}{\partial y_t} - E_{t-1}\left[\lambda\alpha (y_t - y^*) + \alpha\beta\frac{\partial v^c(y_t)}{\partial y_t}\right] = 0, \tag{1.6}$$

and taking expectations of (1.6) at $t-1$ we pin down expected inflation:

$$\pi_t^e = \pi^*.$$

In order to solve for the Bellman equation we guess that the value function is quadratic (as the problem is linear-quadratic), i.e.:

$$v^c(y_t) = \gamma_0^c + \gamma_1^c y_t + \frac{\gamma_2^c}{2}y_t^2, \text{ hence } \frac{\partial v^c(y_t)}{\partial y_t} = \gamma_1^c + \gamma_2^c y_t. \tag{1.7}$$

[7]This loss function can easily be derived from the utility function of the representative household in a fully microfounded model with imperfect price adjustment - see e.g. Woodford (2003).

Substituting (1.7) and the expressions for y_t and π_t^e into the first order condition (1.6) we obtain the optimal state-contingent policy rule with commitment (taking the value function as given):

$$\pi_t^c = \pi^* - \frac{\alpha\left(\lambda + \beta\gamma_2^c\right)}{1 + \alpha^2\left(\lambda + \beta\gamma_2^c\right)}\varepsilon_t. \tag{1.8}$$

Substitute (1.8) into (1.1) to get the state variable equation:

$$y_t^c = \rho y_{t-1} + \frac{1}{1 + \alpha^2\left(\lambda + \beta\gamma_2^c\right)}\varepsilon_t. \tag{1.9}$$

Now the functional Bellman equation can be solved by substituting in (1.5) the value function (1.7) and π_t^c, y_t^c obtained above and identifying coefficients on y_{t-1}, y_{t-1}^2 and the constant. However, as we are only interested in γ_1 and γ_2 we use the Envelope Theorem on problem (1.5); i.e., since (π_t^c, y_t^c) is a minimum and y_{t-1} can be treated as a parameter we have:

$$\begin{aligned}
\frac{\partial v\left(y_{t-1}\right)}{\partial y_{t-1}} &= E_{t-1}\left[\frac{\partial L_t\left(\pi_t^c, y_t^c\right)}{\partial y_{t-1}} + \beta\frac{\partial v^c\left(y_t^c\right)}{\partial y_{t-1}}\right] \Rightarrow \\
\gamma_1^c + \gamma_2^c y_{t-1} &= E_{t-1}\left[\rho\lambda\left(\left(y_t - y^*\right)\right) + \beta\rho\left(\gamma_1^c + \gamma_2^c y_t\right)\right] \Rightarrow \\
\gamma_1^c + \gamma_2^c y_{t-1} &= \rho^2\lambda y_{t-1} - \rho\lambda y^* + \beta\rho\gamma_1^c + \beta\rho^2\gamma_2^c y_{t-1}.
\end{aligned}$$

Identifying coefficients and solving for γ_1, γ_2 we get:

$$\gamma_1^c = \frac{\rho\lambda}{\beta\rho - 1}y^*,\ \gamma_2^c = \frac{\rho^2\lambda}{1 - \beta\rho^2},$$

and substituting in (1.8) we get the *optimal policy rule under commitment*[8]:

$$\pi_t^c = \pi^* - \frac{\alpha\lambda}{1 + \lambda\alpha^2 - \beta\rho^2}\varepsilon_t. \tag{1.10}$$

This equilibrium is, however, not time consistent: the policymaker has incentives to deviate and stimulate output by inflating. The policy rule consistent with the incentives of the government can be obtained by solving for the **Markov Perfect Equilibrium** (see e.g. Fudenberg and Tirole 1991) or **discretionary equilibrium**. In this situation, at stage (0) nothing happens, and the government minimizes the loss at (iii), after shocks are realized and expectations are formed, taking expectations as given:

$$\inf_{\{\pi_t\}} E_0\left[\sum_{t=1}^{\infty}\beta^{t-1}L_t\right] \equiv v\left(y_0, \varepsilon_0\right), \tag{1.11}$$
$$\text{s.t.}(1.1), (1.3), \pi_t^e \text{ given.}$$

The Bellman equation associated with problem (1.11) is (a 'd' superscript stands for 'discretion'):

$$v^d\left(y_{t-1}\right) = E_{t-1}\inf_{\{\pi_t\}}\left[\frac{1}{2}\left[\left(\pi_t - \pi^*\right)^2 + \lambda\left(y_t - y^*\right)^2\right] + \beta v^d\left(y_t\right)\right], \text{s.t.}(1.1), \tag{1.12}$$

where the inf operator has been moved inside the expectations operator because when minimisation is done the supply shock realization ε_t is known. The first

[8]This is indeed a solution as the conditions of the stochastic verification principle (cf. Theorem 9.2 and Exercise 9.4 in Stokey and Lucas, 1989, Theorem 1 in Montrucchio, 2002) are satisfied in this simple case (shocks ε_t are iid with finite variance).

order condition with respect to π_t, assuming a quadratic value function $v^d(y_t) = \gamma_0^d + \gamma_1^d y_t + \gamma_2^d y_t^2$, is:

$$\pi_t - \pi^* + \alpha\left(\beta\gamma_1^d - \lambda y^*\right) + \alpha\left(\beta\gamma_2^d + \lambda\right) y_t = 0. \tag{1.13}$$

Taking expectations at $t-1$ of (1.13) we obtain expected inflation as a function of the value function parameters under discretion:

$$\pi_t^e = \pi^* - \alpha\left(\beta\gamma_1^d - \lambda y^*\right) - \alpha\rho\left(\beta\gamma_2^d + \lambda\right) y_{t-1}. \tag{1.14}$$

We can already state a general result by comparing (1.14) with expected inflation under commitment: the 'inflation bias' of the discretionary equilibrium features both an average term (second term in (1.14)) and a -state-contingent- term dependent on past output realizations (third term in (1.14)). Substituting (1.14) back into the first order condition (1.13) and using (1.1) we get the discretionary policy rule and hence output for a given value function:

$$\begin{aligned} \pi_t^d &= \pi^* - \alpha\left(\beta\gamma_1^d - \lambda y^*\right) - \frac{\alpha\left(\lambda + \beta\gamma_2^d\right)}{1 + \alpha^2\left(\lambda + \beta\gamma_2^d\right)}\varepsilon_t - \alpha\rho\left(\beta\gamma_2^d + \lambda\right) y_{t-1}; \\ y_t^d &= \rho y_{t-1} + \frac{1}{1 + \alpha^2\left(\lambda + \beta\gamma_2^d\right)}\varepsilon_t. \end{aligned} \tag{1.15}$$

Taking into account that $\left(\pi_t^d, y_t^d\right)$ is a minimum, y_{t-1} is a parameter when minimisation is done and now it affects π_t^d and π_t^e, we apply the Envelope Theorem to (1.12) to get:

$$\frac{\partial v^d(y_{t-1})}{\partial y_{t-1}} = E_{t-1}\left[\frac{\partial L_t\left(\pi_t^d, y_t^d\right)}{\partial y_{t-1}} + \beta\frac{\partial v^d\left(y_t^d\right)}{\partial y_{t-1}}\right]$$

$$\Rightarrow \gamma_1^d + \gamma_2^d y_{t-1} = E_{t-1}\left[-\alpha\rho\left(\beta\gamma_2^d + \lambda\right)(\pi_t - \pi^*) + \rho\lambda(y_t - y^*) + \beta\rho\left(\gamma_1^d + \gamma_2^d y_t\right)\right]$$

Using (1.15) to substitute the obtained solutions for π_t^d, y_t^d we obtain:

$$\gamma_1^d + \gamma_2^d y_{t-1} = \rho\left(\beta\gamma_1^d - \lambda y^*\right)\left[1 + \alpha^2\left(\beta\gamma_2^d + \lambda\right)\right] + \left[1 + \alpha^2\left(\beta\gamma_2^d + \lambda\right)\right]\rho^2\left(\beta\gamma_2^d + \lambda\right) y_{t-1} \tag{1.16}$$

Identifying the coefficient on y_{t-1} in (1.16) we get a second-degree equation in γ_2^d :

$$\alpha^2\rho^2\beta^2\left(\gamma_2^d\right)^2 + \left(2\alpha^2\rho^2\beta\lambda + \rho^2\beta - 1\right)\gamma_2^d + \rho^2\lambda\left(1 + \alpha^2\lambda\right) = 0,$$

whose solutions are:

$$\gamma_{2_\pm}^d = \frac{1 - 2\alpha^2\rho^2\beta\lambda - \rho^2\beta \pm \sqrt{\left(\rho^2\beta - 1\right)^2 - 4\alpha^2\rho^2\beta\lambda}}{2\alpha^2\rho^2\beta^2}.$$

The solutions $\gamma_{2_\pm}^d$ are real if the existence condition

$$\left(\rho^2\beta - 1\right)^2 - 4\alpha^2\rho^2\beta\lambda \geq 0$$

is satisfied. The relevant solution is $\gamma_{2_-}^d = \frac{1-2\alpha^2\rho^2\beta\lambda-\rho^2\beta-\sqrt{(\rho^2\beta-1)^2-4\alpha^2\rho^2\beta\lambda}}{2\alpha^2\rho^2\beta^2}$, denoted from now on as γ_2^d[9]. Given a solution for γ_2^d, a solution for γ_1^d is obtained

[9]As this satisfies the verification principle condition $\lim_{t\to\infty}\beta^t E_0\left[v^d(y_t)\right]$, it is sufficient: this describes the unique value function. The other solution would necessarily violate the above condition.

by identifying the constant term giving:

$$\gamma_1^d = -\frac{\alpha^2\rho\left(\beta\gamma_2^d+\lambda\right)\lambda y^*}{1-\alpha^2\rho\beta\left(\beta\gamma_2^d+\lambda\right)}.$$

Substituting back these values we get the discretionary policy rule and output as in (1.15) as functions of the initial parameters. Note that since γ_2^d is different from γ_2^c, the inflation solution features also a *stabilization bias* (shock stabilization is sub-optimal, by looking at coefficients on ε_t).

2. Optimal contracts and their implementability

We are now in a position to reproduce the standard result concerning optimal delegation in a dynamic model (see e.g. Svensson (1997)), although our framework is slightly more general, as discussed below[10]. Suppose that at stage (0) the government delegates monetary policy to an independent central bank by means of a linear contract in inflation:

$$C_t = c_t\left(\pi_t - \pi^*\right).$$

Then, at stage (iii), the central bank will face the loss function (1.3) plus the additional linear term in inflation above and will minimise this new loss function discretionarily. The question this literature asks is how to implement the commitment equilibrium (y^c, π^c) with discretionary policymaking by optimally designing the c_t, i.e. the marginal penalties (rewards) for additional inflation[11]. It turns out this is indeed possible for some value of c_t. Note that c_t will not be constant (a constant contract is suboptimal in the dynamic setup) but will be a function of shocks and past output $c_t\left(y_{t-1}, \varepsilon_t\right)$. Now suppose delegation has taken place at stage (0) and the central bank minimizes the new loss function $L_t(.) + C_t$ taking delegation as given. We solve for the equilibrium as a function of contracts and see for what value of the latter is the resulting equilibrium identical to the commitment one (the approach usually taken in the literature to solve for optimal delegation parameters).

As this is still a Markov Perfect Equilibrium the Bellman equation of the central bank will be (where 'b' superscript stands for 'bank'):

$$v^b\left(y_{t-1}\right) = E_{t-1}\inf_{\{\pi_t\}}\left[\frac{1}{2}\left[\left(\pi_t-\pi^*\right)^2+\lambda\left(y_t-y^*\right)^2\right]+c_t\left(\pi_t-\pi^*\right)+\beta v^b\left(y_t\right)\right], s.t.(1.1)$$

Assuming again $v^b\left(y_t\right) = \gamma_0^b + \gamma_1^b y_t + \gamma_2^b y_t^2$ and taking the first order condition we get:

$$\pi_t - \pi^* + \alpha\left(\beta\gamma_1^b - \lambda y^*\right) + \alpha\left(\beta\gamma_2^b+\lambda\right)y_t + c_t = 0. \tag{2.1}$$

Taking again expectations of (2.1) at $t-1$ we get the expected inflation in this regime as a function of the expected contract $c_t^e = E_{t-1}c_t$ under this regime (note that the expected contract appears as we allow c_t to be made contingent on ε_t).

$$\left(\pi_t^b\right)^e = \pi^* - \alpha\left(\beta\gamma_1^b - \lambda y^*\right) - \alpha\rho\left(\beta\gamma_2^b+\lambda\right)y_{t-1} - c_t^e \tag{2.2}$$

[10] Specifically, we allow the contract to be made contingent upon the exogenous state (shock) and find that the optimal contract is independent of the shock in equilibrium.

[11] Svensson (1997) also presents results for the case with delegation to an inflation-targeting central bank, i.e. one where π^* in the loss function is replaced with a value π^b and the latter is object of optimal design. The results are largely the same, although different in this dynamic case, so we focus on contracts as the essence of the argument is the same.

Substituting (2.2) back into (2.1) and using (1.1), we get inflation and output under this policy regime given the value function:

$$\begin{aligned}(2.3)\quad \pi_t^b &= \pi^* - \alpha\left(\beta\gamma_1^b - \lambda y^*\right) - \frac{\alpha\left(\lambda+\beta\gamma_2^b\right)}{1+\alpha^2\left(\lambda+\beta\gamma_2^b\right)}\varepsilon_t - \alpha\rho\left(\beta\gamma_2^b+\lambda\right)y_{t-1} \\ &\quad -\frac{\alpha^2\left(\lambda+\beta\gamma_2^b\right)}{1+\alpha^2\left(\lambda+\beta\gamma_2^b\right)}c_t^e - \frac{1}{1+\alpha^2\left(\lambda+\beta\gamma_2^b\right)}c_t \\ y_t^b &= \rho y_{t-1} + \frac{1}{1+\alpha^2\left(\lambda+\beta\gamma_2^b\right)}\varepsilon_t - \frac{\alpha}{1+\alpha^2\left(\lambda+\beta\gamma_2^b\right)}\left(c_t - c_t^e\right)\end{aligned}$$

To see precisely which contracts implement the commitment equilibrium, we need to find the parameters of the value function; we proceed as before by using the Envelope Theorem, the only additional complication being that we need to take into account that y_{t-1} influences now also c_t and c_t^e. Note that by definition and linearity of the model c_t and c_t^e differ only by a term in ε_t, so $\frac{\partial c_t}{\partial y_{t-1}} = \frac{\partial c_t^e}{\partial y_{t-1}}$ (hence y_{t-1} does not influence y_t^b through the contract and the influence on inflation is as below). Hence, as before, apply the Envelope Theorem to the Bellman equation to get:

$$\frac{\partial v^b\left(y_{t-1}\right)}{\partial y_{t-1}} = E_{t-1}\left[\frac{\partial L_t\left(\pi_t^b, y_t^b\right)}{\partial y_{t-1}} + \frac{\partial\left[c_t\left(\pi_t^b - \pi^*\right)\right]}{\partial y_{t-1}} + \beta\frac{\partial v^b\left(y_t^b\right)}{\partial y_{t-1}}\right] \Rightarrow$$

$$\begin{aligned}\gamma_1^b + \gamma_2^b y_{t-1} &= E_{t-1}\{[-\alpha\rho\left(\beta\gamma_2^b+\lambda\right) - \frac{\partial c_t^e}{\partial y_{t-1}}]\left(\pi_t - \pi^*\right) + \rho\lambda\left(y_t - y^*\right) + \\ (2.4)\qquad &\quad +\frac{\partial c_t^e}{\partial y_{t-1}}\left(\pi_t - \pi^*\right) - [\alpha\rho\left(\beta\gamma_2^b+\lambda\right) + \frac{\partial c_t^e}{\partial y_{t-1}}]c_t + \beta\rho\left(\gamma_1^b + \gamma_2^b y_t\right)\}\end{aligned}$$

We can already look at the optimal contract. By definition, this should be such that $\pi_t^e = \pi^*$ as in the optimal rule. But this can only happen in this equilibrium if, from (2.2):

$$c_t^e = \alpha\left(\lambda y^* - \beta\gamma_1^b\right) - \alpha\rho\left(\beta\gamma_2^b+\lambda\right)y_{t-1} \qquad (2.5)$$

Using (2.5) and $\pi_t^e = \pi^*$, (2.4) becomes after replacing and taking expectations:

$$\gamma_1^b + \gamma_2^b y_{t-1} = \rho^2\lambda y_{t-1} - \rho\lambda y^* + \beta\rho\gamma_1^b + \beta\rho^2\gamma_2^b y_{t-1},$$

which gives the same value function as in the commitment case, i.e. (a superscript 'o' stands for optimal):

$$\gamma_1^{bo} = \gamma_1^c = \frac{\rho\lambda}{\beta\rho-1}y^*, \gamma_2^{bo} = \gamma_2^c = \frac{\rho^2\lambda}{1-\beta\rho^2}$$

We need to look at the 'unexpected' part of the contract, but it is easily seen that for $\gamma_i^{bo} = \gamma_i^c$ the shock stabilization coefficients in π_t^b above are the same as in the optimal rule π_t^c, hence the contract need not be made contingent upon ε_t. Note that in contrast to the literature we did not assume this, but found it by optimality. The government delegates to the central bank such that the value function of the latter becomes identical to its own value function when committing. Hence the optimal contract, such that the commitment equilibrium is implemented in the discretionary case is:

$$c_t^o = \frac{\alpha\lambda}{1-\beta\rho}y^* - \frac{\alpha\lambda}{1-\beta\rho^2}\rho y_{t-1}, \forall t \geq 1 \qquad (2.6)$$

A natural question which we have formulated in the introduction arises now: why would a government that has incentives to deviate from the optimal policy rule π_t^c choose the contract c_t^o given by (2.6)? The critical step is taken when passing from equation (2.4) to (2.5). Clearly, what makes the government choose a contract leading to $\pi_t^e = \pi^*$ are not its incentives, since these incentives make it deviate from the optimal policy in the first place (Kydland and Prescott, 1977). What would make it act as such would be a kind of benevolence or commitment mechanism that would be hard to justify given that it did not have it before. In the next Section, we develop this argument formally and show that optimal contracts can only be obtained -as an optimal choice by the government at the delegation stage- under commitment; this is precisely *the same type of commitment required to implement the optimal policy rule.* We then solve for the 'time-consistent' (i.e. credible or Markov perfect) contracts and show that they lead to implementation of the suboptimal outcome.

3. Government's incentives: optimal vs. credible (time consistent) contracts

We model the choice of the contract recursively, based on individual rationality of the government. Instead of merely assuming the government is a sort of *Deus ex machina* that intervenes at the right time by selecting the right equilibrium, we model explicitly what happens at stage (0). Specifically, at stage (0) the government chooses the sequence $\{c_t\}$, its new control variable, given that instrument independence has been granted to the central bank. It does so by backward induction, not only in the sense that the model has to be solved by dynamic programming, but also because it observes the way the central bank chooses its policy instrument at stage (iii) in the Markov Perfect Equilibrium we described above. In particular, (2.2) and (2.3) become the government's dynamic constraints for the problem:

$$\inf_{\{c_t\}} E_0 \left[\sum_{t=1}^{\infty} \beta^{t-1} L_t(\pi_t^b(c_t), y_t^b(c_t)) \right] \equiv v^g(y_0, \varepsilon_0) \quad s.t.(2.2),(2.3) \tag{3.1}$$

There are two policy regimes corresponding to this problem, which can be distinguished corresponding to government's treatment of expectations, just as in the original policy problem. In each regime, we state a Proposition (1 and 2 respectively) that relates optimality with time consistency and then we prove and discuss the implication of each Proposition.

PROPOSITION 1. *Optimal contracts require commitment.*

The first regime is labeled **'commitment'**: the government commits with respect to the other players (central bank and private sector) and hence (3.1) is solved under an additional constraint, $c_t^e = E_{t-1}c_t$, choosing the extra control c_t^e. The Bellman equation is:

(3.2)

$$v^g(y_{t-1}) = \inf_{\{c_t\},\{c_t^e\}} E_{t-1}\left[\frac{1}{2}\left[\left(\pi_t^b - \pi^*\right)^2 + \lambda\left(y_t^b - y^*\right)^2\right] + \beta v^g\left(y_t^b\right)\right], s.t.(2.2),(2.3), c_t^e = E_{t-1}c_t$$

Attaching the Lagrange multiplier φ_t to the rational expectations constraint, we obtain the first-order conditions with respect to c_t and c_t^e respectively:

$$\begin{aligned}(3.3)\qquad \left(\pi_t^b-\pi^*\right)+\alpha\lambda\left(y_t^b-y^*\right)+\alpha\beta\left(\gamma_1^g+\gamma_2^g y_t^b\right)-\varphi_t &= 0\\ -E_{t-1}\left[\left(\alpha^2\left(\lambda+\beta\gamma_2^b\right)\right)\left(\pi_t^b-\pi^*\right)+\alpha\lambda\left(y_t^b-y^*\right)+\alpha\beta\left(\gamma_1^g+\gamma_2^g y_t^b\right)\right]+\varphi_t &= 0\end{aligned}$$

Combining the two constraints we obtain:

$$\left(1-\alpha^2\left(\lambda+\beta\gamma_2^b\right)\right)\left[\left(\pi_t^b\right)^e-\pi^*\right]=0,$$

which further implies (as long as $\gamma_2^b\neq\frac{1-\lambda\alpha^2}{\beta\alpha^2}$, which is indeed the case as shown below):

$$\left(\pi_t^b\right)^e=\pi^*.$$

As shown before, the unique contracts consistent with this expected inflation are the optimal ones:

$$c_t^c=c_t^o.$$

Finally, it can be easily shown that the value functions for the central bank and the government are such that:

$$\left(\gamma_i^b\right)^c=\left(\gamma_i^g\right)^c=\gamma_i^c,\ i=1,2.$$

Therefore, the *optimal contracts c_t^o are a solution to government's choice only under commitment, c_t^c*. The value function for the government when choosing the contract under commitment $\left(\gamma_i^g\right)^c$ is the same as the value function it had when choosing policy directly under commitment γ_i^c; furthermore, this contract ensures that the value function of the central bank is also the same. It is in this sense that we view 'optimal delegation' solutions to time inconsistency as relying upon the presence of a *Deus ex machina*.

The second regime we consider is a time-consistent one, without commitment; the reason to do so is that delegation-based solutions have been proposed as a substitute for a commitment technology. However, we have just shown that optimal delegation in fact *requires* commitment. It is therefore worthwhile to study the outcome of 'time-consistent' delegation, insofar as one is interested in studying an environment without commitment.

PROPOSITION 2. *Credible, time-consistent contracts are suboptimal and lead to implementation of the discretionary equilibrium.*

The Bellman equation associated with government's problem (3.1) is[12]:

(3.4)

$$v^g\left(y_{t-1}\right)=\inf_{\{c_t\}}E_{t-1}\left[\frac{1}{2}\left[\left(\pi_t^b-\pi^*\right)^2+\lambda\left(y_t^b-y^*\right)^2\right]+\beta v^g\left(y_t^b\right)\right],s.t.(2.2),(2.3)$$

The first order condition with respect to c_t is, treating the parameters of the value function in the central bank's problem γ_i^b as given and guessing that $v^g\left(y_t\right)$ is quadratic:

$$-\frac{1}{1+\alpha^2\left(\lambda+\beta\gamma_2^b\right)}\left(\pi_t^b-\pi^*\right)-\frac{\alpha\lambda}{1+\alpha^2\left(\lambda+\beta\gamma_2^b\right)}\left(y_t^b-y^*\right)+\beta\left(\gamma_1^g+\gamma_2^g y_t\right)\left[-\frac{\alpha\lambda}{1+\alpha^2\left(\lambda+\beta\gamma_2^b\right)}\right]=0$$

$$(3.5)\qquad \Rightarrow\left(\pi_t^b-\pi^*\right)+\alpha\lambda\left(y_t^b-y^*\right)+\alpha\beta\left(\gamma_1^g+\gamma_2^g y_t^b\right)=0$$

[12]Note that minimisation is done before uncertainty is resolved.

Taking expectations of (3.5) at $t-1$ we obtain (using the expression for y_t^b found in (2.3)):

$$\left(\pi_t^b\right)^e = \pi^* - \alpha\left(\beta\gamma_1^g - \lambda y^*\right) - \alpha\left(\lambda + \beta\gamma_2^g\right)\rho y_{t-1} \tag{3.6}$$

Combining this with (2.2) we find the expected part of the marginal contract, c_t^e :

$$c_t^e = \alpha\beta\left[\left(\gamma_1^g - \gamma_1^b\right) + \left(\gamma_2^g - \gamma_2^b\right)\rho y_{t-1}\right] \tag{3.7}$$

Substituting (3.7) back in the first order condition (3.5) we obtain (after some algebra):

$$c_t^* = \alpha\beta\left[\left(\gamma_1^g - \gamma_1^b\right) + \left(\gamma_2^g - \gamma_2^b\right)\rho y_{t-1} + \frac{\left(\gamma_2^g - \gamma_2^b\right)}{1+\alpha^2\left(\lambda + \beta\gamma_2^g\right)}\varepsilon_t\right] \tag{3.8}$$

Evaluating the expressions π_t^b, y_t^b from (2.3) at the c_t^* found in (3.8) as a solution to government's optimisation we obtain the equilibrium inflation and output conditional upon the time-consistent contract c_t^*, for a given value function of the government:

$$\begin{aligned} \pi_t^b\left(c_t^*\right) &= \pi^* - \alpha\left(\beta\gamma_1^g - \lambda y^*\right) - \frac{\alpha\left(\lambda + \beta\gamma_2^g\right)}{1+\alpha^2\left(\lambda + \beta\gamma_2^g\right)}\varepsilon_t - \alpha\rho\left(\beta\gamma_2^g + \lambda\right)y_{t-1} \\ y_t^b\left(c_t^*\right) &= \rho y_{t-1} + \frac{1}{1+\alpha^2\left(\lambda + \beta\gamma_2^g\right)}\varepsilon_t \end{aligned} \tag{3.9}$$

Heuristically, direct comparison of (3.9) with (1.15) shows that the equilibrium under the time-consistent contract would be the same as the discretionary equilibrium if the value function of the government were the same as the value function under discretion, $\gamma_i^g = \gamma_i^d$. Furthermore, the contract implementing this equilibrium would merely be $c_t^* = 0$, obtained (by direct inspection of (3.8)) when the value function of the central bank is identical to that of the government, $\gamma_i^b = \gamma_i^g = \gamma_i^d$. We now prove that this is indeed the case, and that this equilibrium is unique.

To obtain the equilibrium, we need to find the parameters of the value function of the government γ_i^g; we apply the Envelope Theorem to (3.4) to get:

$$\gamma_1^g + \gamma_2^g y_{t-1} = E_{t-1}\left\{-\alpha\rho\left(\beta\gamma_2^g + \lambda\right)\left(\pi_t^b - \pi^*\right) + \rho\lambda\left(y_t^b - y^*\right) + \beta\rho\left(\gamma_1^g + \gamma_2^g y_t^b\right)\right\}$$

Substituting the values for inflation and output from (3.9) we obtain:

$$\gamma_1^g + \gamma_2^g y_{t-1} = \rho\left(\beta\gamma_1^g - \lambda y^*\right)\left[1+\alpha^2\left(\beta\gamma_2^g + \lambda\right)\right] + \rho^2\left(\beta\gamma_2^g + \lambda\right)\left[1+\alpha^2\left(\beta\gamma_2^g + \lambda\right)\right]y_{t-1} \tag{3.10}$$

Direct comparison of (3.10) with (1.16) shows that these equations are identical; therefore, solving for the value function of the government, we obtain:

$$\gamma_i^g = \gamma_i^d, \ i = 1, 2.$$

This result shows that indeed, the economic equilibrium in terms of inflation and output obtained under time-consistent delegation (3.9) is identical to the suboptimal, Markov perfect equilibrium obtained in the absence of delegation, (1.15):

$$\pi_t^b\left(c_t^*\right) = \pi_t^d, y_t^b\left(c_t^*\right) = y_t^d \tag{3.11}$$

What is the path of contracts that sustains this equilibrium? Trivially, we know that the outcome under delegation is identical to the outcome without delegation *if* c_t^* is identically zero. We now prove that this is also the **unique** time-consistent contract (i.e. the two equilibria are identical *only if* $c_t^* = 0$). As is clear from (3.8), in order to find the time-consistent contract we need to solve for the value function of the central bank under delegation, γ_i^b. To do so, we use the Envelope condition (2.4) taking into account that, as we have just shown, $\gamma_i^g = \gamma_i^d$, $\pi_t^b(c_t^*) = \pi_t^d$, $y_t^b(c_t^*) = y_t^d$ and the contracts satisfy (3.8). Substituting all these expressions into the Envelope condition (2.4), we obtain after some straightforward algebra:

$$\begin{aligned}\gamma_1^b + \gamma_2^b y_{t-1} &= \rho\left(\beta\gamma_1^d - \lambda y^*\right)\alpha^2\left(\beta\gamma_2^b + \lambda\right) + \rho\left(\beta\gamma_1^b - \lambda y^*\right) - \alpha^2\beta\rho\left(\beta\gamma_2^d + \lambda\right)\left(\gamma_1^d - \gamma_1^b\right) + \\ &+ \left[\rho^2\left(\beta\gamma_2^b + \lambda\right)\left[1 + \alpha^2\left(\beta\gamma_2^d + \lambda\right)\right] - \alpha^2\beta\rho^2\left(\beta\gamma_2^d + \lambda\right)\left(\gamma_2^d - \gamma_2^b\right)\right] y_{t-1}\end{aligned}$$

Identifying coefficients on y_{t-1} :

$$\gamma_2^b = \rho^2\left(\beta\gamma_2^b + \lambda\right)\left[1 + \alpha^2\left(\beta\gamma_2^d + \lambda\right)\right] - \alpha^2\beta\rho^2\left(\beta\gamma_2^d + \lambda\right)\left(\gamma_2^d - \gamma_2^b\right) \tag{3.12}$$

This is a linear equation in γ_2^b and hence has a unique solution; furthermore, note that for $\gamma_2^b = \gamma_2^d$ (3.12) merely becomes identical to the equation determining γ_2^d (obtained by identifying the coefficient on y_{t-1} in (1.16)). Therefore, $\gamma_2^b = \gamma_2^d$ is the unique solution to (3.12). Moreover, using this result to obtain γ_1^b we obtain:

$$\gamma_1^b = \rho\left(\beta\gamma_1^b - \lambda y^*\right)\left[1 + \alpha^2\left(\beta\gamma_2^d + \lambda\right)\right],$$

which has as an unique solution $\gamma_1^b = \gamma_1^d$. Hence, we have just proved that the value function of the central bank when delegated with the time-consistent contract is identical (up to an irrelevant constant term) with the value function under discretion - and with the value function of the government when delegating in a time-consistent way:

$$\gamma_i^b = \gamma_i^d = \gamma_i^g,\ i = 1, 2.$$

Finally, using this result into (3.8) we obtain the time consistent contract as:

$$c_t^* = c_t^{*e} = 0\ \forall t.$$

Now, this is obviously different from the optimal contracts we solved for before in (2.6), and in an intuitive way. Our result is just an instance of the government's time inconsistency in the first place. The time consistent (discretionary, credible or Markov Perfect) contracts are different from the optimal contracts in the same way in which time consistent, discretionary policy was different from optimal policy (with commitment). If the government is subject to a time inconsistency problem, we show that this problem exists regardless of whether it chooses policy directly or the institution that chooses policy. The latter form is probably more subtle but, under rational expectations, leads to exactly the same outcome. The government has the same value function as when it chooses monetary policy directly, and now it effectively manipulates the value function of the central bank; based on its individual rationality, it chooses that contract that makes the value function of the central bank consistent with its preferred equilibrium: this penalty is zero.[13]

4. Time-inconsistent optimal delegation in a forward-looking New Keynesian model

For completion, we show that the same problem is binding in a forward-looking model of monetary policy based on Clarida, Gali and Gertler (1999) and Woodford (2003). In this model, gains from commitment occur in the absence of an inflation bias because the discretionary equilibrium implies suboptimal stabilization of

[13]It is intuitive by the same argument that in the case of delegation to an inflation-targeting central bank, the solution for the inflation target with which the government will delegate will not be the optimal one, obtained by Svensson (1997). It will instead be just π^*, the target that leads to the discretionary equilibrium.

supply ('cost-push') shocks, and optimal policy requires history-dependence - as detailed below. We show that the commitment equilibrium could in principle be achieved discretionarily by optimal delegation through an inflation contract (to the best of our knowledge, this is a novel result in the forward-looking New Keynesian framework). We then show that, as in the backward-looking model presented previously, this contract is not time consistent.

Following i.a. the aforementioned papers, suppose that inflation dynamics obeys the 'New Keynesian' Phillips curve (the natural rate has again been normalized to zero[14]):

$$\pi_t = \beta E_t \pi_{t+1} + \kappa y_t + \varepsilon_t \tag{4.1}$$

The **time-consistent** equilibrium is obtained by minimizing the discounted loss function under **discretion** (where we have assumed that there is no inflation bias):

$$L_t = \frac{1}{2}\left[\pi_t^2 + \lambda y_t^2\right] + E_t \sum_{i=t+1}^{\infty} \beta^{t+1-i}\left[\pi_i^2 + \lambda y_i^2\right]. \tag{4.2}$$

Under discretion, the central bank takes terms involving private sector expectations as given: namely, $\beta E_t \pi_{t+1}$ in (4.1) and the second term in (4.2) are treated parametrically. The solution is the targeting rule under discretion:

$$\pi_t^d + \frac{\lambda}{\kappa} y_t^d = 0 \tag{4.3}$$

Minimization of the loss function under **commitment** implies taking into account the influence on private sector expectations, and hence minimizing the whole intertemporal objective with respect to inflation and output, taking (4.1) as a dynamic constraint. The solution, which gives the *optimal policy from a timeless perspective* (see Woodford, 2003, Ch. 7), is obtained by attaching a (sequence of) Lagrange multipliers to the (sequence of) dynamic constraint(s) (4.1); upon elimination of the Lagrange multiplier the first-order condition can be written as an *optimal targeting rule*:

$$\pi_t^c + \frac{\lambda}{\kappa}\left(y_t^c - y_{t-1}^c\right) = 0, \ t \geq 1 \tag{4.4}$$

Additionally, we require that $\pi_0^c + \frac{\lambda}{\kappa} x_0^c = 0$.Optimal policy implies inertia because in this model current inflation depends on future expected inflation (see Woodford, 2003 for an extensive discussion)[15]. Substitution of the relevant optimality condition ((4.3) or (4.4)) into the Phillips curve (4.1) would give the equilibrium outcomes in terms of inflation and output in each case and would illustrate the -by now- well-known stabilization bias (suboptimal response to shocks) present in the discretionary equilibrium (see the cited papers for an extensive discussion).

Using the same logic as previously, the commitment equilibrium could in principle be achieved under discretion by delegating monetary policy via an inflation

[14]This boils down to assuming that there are no technology, preference or government spending shocks.

[15]Woodford argues that policy that is optimal from a timeless perspective is not subject to a time inconsistency problem: by definition, the policymaler would have wished to commit itself to this policy at a date very far in the past. However, as Jensen (2002) points out, this policy still requires a commitment technology. Consistent with our overall focus on environments without commitment, we study institutional design mechanism that could act as a substitute for this commitment technology - and lead to implementation of the optimal policy discretionarily.

contract to a central bank. To the best of our knowledge, this result is novel[16]. The loss function of the central bank becomes:

$$L_t = \frac{1}{2} E_t \sum_{i=t}^{\infty} \beta^{t-i} \left[\pi_i^2 + \lambda y_i^2 + c_i \pi_i \right], \tag{4.5}$$

The first-order condition of the central bank acting under discretion is:

$$\pi_t^b + \frac{\lambda}{\kappa} y_t^b + c_t = 0 \tag{4.6}$$

which by direct comparison with (4.3) and (4.4) gives the following Proposition.

Proposition 3. *In the forward-looking New Keynesian model outlined above, the* ***optimal contract*** *leading to the implementation of the (timeless) commitment optimum is:*

$$c_t^o = -\frac{\lambda}{\kappa} y_{t-1}, \ t \geq 1.$$

Additionally, we require $c_0^o = 0$.

By having the marginal penalty depend on past values of output (gap), this contract captures the general idea that optimal policy should feature inertia, or history dependence (see Woodford, 2000, 2003 and Jensen, 2002). The optimal contract 'works' intuitively by inducing history dependence: if output gap last period was negative (for example because of a cost-push shock), it requires an inflation penalty today. However, a result that mirrors Proposition 2 also holds in this case.

Proposition 4. *In the forward-looking model, credible, time-consistent contracts are also suboptimal.*

The **time consistent contract** is obtained, just as before, by backward induction, solving:

$$\min_{\{c_t\}} \frac{1}{2} E_t \sum_{i=t}^{\infty} \beta^{t-i} \left[\pi_i^2 (c_i) + \lambda y_i^2 (c_i) \right],$$

taking as given expectations, just as in the original problem under discretion, and having as constraints the Phillips curve (4.1) and the optimal rule of the central bank under delegation (4.6). Using the two constraints we solve for inflation of output as functions of the contract:

$$\begin{aligned} y_t^b (c_t) &= -\frac{\kappa}{\lambda + \kappa^2} c_t - \frac{\kappa}{\lambda + \kappa^2} (\beta E_t \pi_{t+1} + \varepsilon_t) \\ \pi_t^b (c_t) &= -\frac{\kappa^2}{\lambda + \kappa^2} c_t + \frac{\lambda}{\lambda + \kappa^2} (\beta E_t \pi_{t+1} + \varepsilon_t). \end{aligned}$$

Substituting in the loss function, the solution to this problem is:

$$\pi_t^b + \frac{\lambda}{\kappa} y_t^b = 0,$$

[16] Jensen (2002) and Walsh (2003) have recently studied how alternative intitutional design schemes (nominal income growth targeting and output gap growth targeting, respectively) can improve upon the discretionary equilibrium. However, none of these papers focused on either exact implementation of the first-best or on inflation contracts.

which implies again:

$$\begin{aligned}\pi_t^b(c_t) &= \pi_t^d; y_t^b(c_t) = y_t^d, \\ c_t^* &= 0.\end{aligned}$$

As before, the time consistent contract ensures implementation of the suboptimal, discretionary equilibrium, which does not feature inertia and in which there is suboptimal stabilization of supply shocks. Therefore, the time inconsistency of optimal delegation occurs *regardless of whether the structural model is backward- or forward-looking*, and of whether there are endogenous state variables or not. The only condition is that a time inconsistency problem exist in the original policy problem.

5. Do reappointment costs sustain optimal delegation?

While sometimes the literature acknowledges the possibility of a failure of *enforcement* of optimal delegation (e.g. Walsh 1995 footnote 5), the counter-argument, albeit implicit, is that constitutions are always binding. But that again is a solution by assumption, let aside that it might not be true in practice[17]. Another way to interpret this argument is that changing institutions is costly, as Jensen (1997) does when modelling a separate enforcement stage in the game. We now introduce such costs in our model and study whether they can alleviate the time inconsistency of delegation.

Suppose the government faces a cost in changing its delegation parameters, c_t. We will model this in two ways: first, consider that the government dislikes deviations of the contract it chooses from the optimal contract c_t^o. This means that in a first period it was able to implement this contract, and now, when it has the opportunity to renege on it, it faces a cost. We model this as a quadratic cost, not necessarily in monetary terms, but easily interpretable e.g. as a loss of reputation for financial markets. Then, at stage (0), the government faces the following loss function, again taking as constraints the choice of the central banker given delegation:

$$L_t = \frac{1}{2}\left[\left(\pi_t^b - \pi^*\right)^2 + \lambda\left(y_t^b - y^*\right)^2\right] + \frac{\delta}{2}\left(c_t - c_t^o\right)^2 \tag{5.1}$$

For simplicity, we focus on the case without output persistence of the backward-looking model[18] whereby $\rho = 0$ and equation (1.1) becomes an usual Lucas supply curve. As there is no intertemporal constraint the dynamic problem boils down to minimising period-by period the loss function in (1.3), hence the value function becomes a constant and the coefficients γ_1 and γ_2 become identically zero in all regimes. In this case the optimal contract c_t^o is constant, $c^o = \lambda\alpha y^*$. Substituting this and the expressions for equilibrium in the central bank problem and minimising with respect to c_t, we obtain the following time-consistent contract:

$$c^* = \frac{\delta\left(1 + \alpha^2\lambda\right)}{1 + \delta\left(1 + \alpha^2\lambda\right)} c^o < c^o$$

---

[17]Moreover, McCallum (1995) argues that institutions need not bind, giving the most prominent example: the gold standard in the US. Although this has been abandoned *de facto* a long time ago, it has not yet been abandoned *de jure*. Moreover, New Zealand's 'inflation contract' has also been changed more than once. See the discussion in Jensen 1997.

[18]This case is studied in more detail in an Appendix available upon request or in a working paper version on the author's webpage.

For the forward-looking model of Section 4, the optimality condition is:

$$c_t^* = \frac{\delta\left(\kappa^2 + \lambda\right)}{\kappa^2 + \delta\left(\kappa^2 + \lambda\right)} c_t^o < c_t^o$$

In either case, the time consistent contract is always smaller than the optimal contract. The intuition is straightforward: as δ, the 'reappointment aversion' parameter, goes to zero, c^* is simply 0, the equilibrium contract without any reappointment costs. As the aversion to institutional change (or the difficulty to implement this) increases, the equilibrium contract approaches the optimal contract. It is trivial to see that $\lim_{\delta\to\infty} c^* = c^o$. 'Infinite' aversion to institutional change might sustain the optimal contract as subgame perfect.

A second way to model institutional inertia is to assume that the government 'smooths' the contract, i.e. it dislikes deviations of this period's contract from last period's contract (it values institutional stability directly). In this case, the period loss function is:

$$L_t = \frac{1}{2}\left[\left(\pi_t^b - \pi^*\right)^2 + \lambda\left(y_t^b - y^*\right)^2\right] + \frac{\mu}{2}\left(c_t - c_{t-1}\right)^2 \tag{5.2}$$

By the same method as above, the solution will be a homogenous difference equation:

$$c_t^{**} = \psi c_{t-1}^{**}.$$

where $\psi = 1 - \left[1 + \mu\left(1 + \alpha^2\lambda\right)\right]^{-1}$ for the backward model and $\psi = 1 - \left[\kappa^2 + \mu\left(\kappa^2 + \lambda\right)\right]^{-1}$ for the forward-looking model. As the coefficient ψ is less than one in both models, the solution is stable and the solution is:

$$c_t^{**} = \psi^t c_0 \tag{5.3}$$

This converges asymptotically to zero, so the contract will in the limit be the Markov perfect one we solved for before, i.e. $c_t^{**} = 0$. Even if governments implements in the first period the optimal contract $c_0 = c^o$, it will follow its incentives and start to decrease the penalty to its preferred zero level, the speed with which it does so being dependent on how much it values institutional stability, i.e. on μ.The results for the persistence case would be different but the main intuition should remain. Reappointment costs do not solve the problem, they merely postpone it.

6. Conclusions

Kydland and Prescott's insight quoted in the introduction applies as much to delegation (when viewed as a control problem consisting of choosing the institution to which policy is delegated) as it does to choosing policy directly. Delegation does not solve the time inconsistency problem, and is subject to a time inconsistency problem itself: *optimal* delegation is not time consistent, while *time consistent* delegation is suboptimal. Nothing ensures that the government chooses the optimal institution, unless it acts as a *Deus ex machina* does in an antique tragedy; but if it had this ability it is hard to understand why it does not use it when choosing policy. We articulated this argument in two models of monetary policy, and showed that it is independent of whether the structural model is backward-looking (and features endogenous state variables) or forward-looking.

Normatively, our results imply that the search for solutions to the time inconsistency problem should continue. This search should be directed towards 'deep' ways to sustain commitments, since optimal delegation requires precisely the same commitment technology that is required by optimal policy. These could include solutions based on reputational mechanisms (as e.g. in Stokey (1989)) and informational imperfections (Backus and Driffill (1985) and Canzoneri (1985)). Another possible solution is suggested by the very nature of the problem, hinted to by the sentence cited at the outset of this paper. Since time inconsistency occurs because of *rational expectations*, mechanisms based on bounded rationality by private agents could help sustain optimal policy as a time-consistent outcome (see e.g. Ireland (2000) for an effort in this direction, building on early work by Taylor (1982)).

Positively, our results question explanations of the recent success in fighting inflation based on 'better institutions' having solved the time inconsistency problem. While the strengthening of central bank independence in developed countries may have well led to less political pressures, and hence to less inflation, this is independent of time inconsistency issues; incidentally, central banks that are not 'inflation targeters' (most obviously, the Fed) have been successful in fighting inflation. Moreover, many authors (e.g. Sims and Zha (2005)) also argue that the variance of shocks hitting the economy has decreased more generally. Therefore, to an explanation reading 'the delegation of monetary policy helped solve the time inconsistency problem, by reducing average inflation (by eliminating the inflation bias) and inducing better stabilization of shocks', one can oppose an explanation that does not involve monetary policy delegation at all. In this latter view, better inflation performance is a combination of (i) central banks having achieved independence (thereby being isolated from political pressures), (ii) having learned that there is no exploitable long-run inflation-output trade-off and (iii) having faced less adverse shocks. The results of Sargent, Williams and Zha (2005) give support to the last two features of this second interpretation in explaining US inflation dynamics in the post-war period.

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