Designing Domestic Institutions for International Monetary Policy Cooperation: A Utopia?[†]

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Abstract. In a wide variety of international macroeconomic models monetary policy cooperation is optimal, non-cooperative policies are inefficient, but optimal policies can be attained non-cooperatively by optimal design of domestic institutions/contracts. We show that given endogenous institutional design, inefficiencies of non-cooperation cannot and will not be eliminated. We model the delegation stage explicitly and show that *subgame perfect, credible contracts* (chosen by governments based on individual rationality) are non-zero, but are different from optimal contracts and hence lead to inefficient equilibria. Optimal contracts require cooperation at the delegation stage, which is inconsistent with the advocated non-cooperative nature of the solution. A general solution method for credible contracts and an example from international monetary policy cooperation are considered. Our results feature delegation as an equilibrium phenomenon, explain inefficiencies of existing delegation schemes and hint to a stronger role for supranational authorities in international policy coordination.

JEL codes: *F33, F42, D02, D62, E52, E58, E61.*

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1. Introduction

A large body of literature deals with optimal delegation of macroeconomic policy in an international context (see Persson and Tabellini (2000) for a comprehensive review). In this framework, optimal contracts or targeting regimes over some macroeconomic variable are viewed as panacea for solving inherent inefficiencies of non-cooperative (and discretionary) policymaking. Notably, much of the work concerning monetary policy institutions adopts this line of reasoning. The inefficiencies that optimal delegation is supposed to 'fix' in this case are problems due to non-cooperative policymaking in the presence of policy spillovers in a multi-country world (and/or 'credibility' problems like the inflation bias). A recurrent result (see e.g. the seminal contributions by Persson and Tabellini (1995, 1996, 2000)) is that the *cooperative optimum* (to be defined) can be achieved in a decentralised, non-cooperative manner by *delegating through optimal inflation contracts*. This is done by assuming that before the actual policy game takes place, there is an 'institutional design stage', where governments choose the appropriate delegation scheme for their central banks that implements the optimum.

This paper starts from the observation that these delegation schemes are *not subgame perfect, i.e. not credible*: indeed, they implicitly assume cooperation (or some form of coordination) at the delegation stage, which is hard to reconcile with the alleged 'purely non-cooperative implementation of the cooperative optimum'. We develop this argument analytically by explicitly modelling the institutional design stage and studying the (*credible, subgame perfect*) contracts that are consistent with governments' incentives and hence occur in equilibrium. Specifically, at the delegation stage governments choose the delegation parameters in a non-cooperative manner by backward induction, taking into account the reaction functions of the central banks at the policy stage. These *credible, subgame perfect contracts* turn out to be non-zero (hence delegation is always an equilibrium) whenever there is strategic complementarity or substitutability. However, they are always *different from the optimal contracts*, which would instead *require cooperation* of governments (or some form or coordination) at the delegation stage. But then, if binding agreements were possible, one wonders why would delegation be needed in the first place. These results are developed in Section 2 in a general linear-quadratic model.

In the international policy context, it has been long recognized (following Hamada (1976)) that cooperative policymaking[1] is Pareto optimal when sovereign policymaking has externalities on the other countries. Typically, externalities take the form of conflicts over shock stabilisation or over preferred levels of macroeconomic outcomes. The Pareto optimum is not enforceable for various reasons (individual incentives to deviate, suboptimality of cooperation when commitment with respect to the domestic private sector is impossible, uncertainty regarding models, loss functions, etc) - all these issues are extensively reviewed in Canzoneri and Henderson (1991) or Ghosh and Masson (1994). Given individual incentives to deviate from the optimal cooperative policies, the literature has moved towards

[1] We adopt the game-theoretical definition of cooperation as joint optimization by a group of players of their payoffs, implying a 'pregame' and the possibility of binding agreements. Coordination would by contrast mean choosing one particular equilibrium in the Nash Equilibrium set of the non-cooperative game (this might imply the presence of an external enforcing mechanism). Exchange of information is captured by the non-cooperative policymaking case.

identifying mechanisms that sustain the collusive outcome. We focus on the 'institutional design' approach pioneered by Persson and Tabellini (1995) and extended by Persson and Tabellini (1996, 2000) and Jensen (2000). This focus is reinforced in the international context by an observation of Rogoff (1985): in the presence of domestic credibility problems as the ones reviewed above cooperation itself might even be welfare-reducing[2]. But institutional design, or delegation to an independent monetary authority, could in principle act as a solution to correct inefficiencies coming from both discretion and non-cooperative policymaking.

The state of the art in the literature on optimal monetary policy delegation in an international context can be summarized as follows. Persson and Tabellini (1995, 1996, 2000) analyze performance contracts written by the governments before the game is played, at an 'institutional design' stage and show how these contracts can be designed such that the inefficiencies related to both discretionary and non-cooperative policymaking are eliminated[3]. The optimal linear contracts hence found are state-contingent, which is a non-desirable feature as it makes them difficult to implement (for example because they imply that the institution changes each time a shock occurs). However, Jensen (2000) addresses this issue by finding *state-independent* transfer functions that implement the cooperative outcome. These functions penalize quadratically inflation deviations from a certain level (chosen by the government) as well as inflation differentials between the two countries. He also provides interpretations of these contracts in terms of real-life institutions. A general criticism of this line of research is that welfare conclusions and prescriptions cannot be properly addressed in a model that lacks microfoundations (Obstfeld and Rogoff (1996)). However, recent research shows that the insights of optimal design of institutions carries over to more realistic setups in the new open-economy macroeconomics tradition. In a recent insightful contribution, Benigno and Benigno (2005) use a micro-founded, general equilibrium two-country model and show that targeting rules can be designed that implement optimal cooperative policies, and that optimal contracts exist that could make these targeting rules occur in a non-cooperative equilibrium[4].

Our Section 3 applies the general results of Section 2 to such a simple model of international monetary policy cooperation due to Persson and Tabellini (1996, 2000); it shows that, and explains why, credible subgame perfect contracts are different from optimal contracts. Section 4 concludes and points out some implications for the design of supranational institutions.

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[2]Canzoneri and Henderson (1991) interpret this insight as a particular case of a more general result: coalitions of only subsets of players are inefficient. See also Kohler (2002).

[3]Persson and Tabellini also look at non-linear discontinuous performance contracts with state-dependent parameters written directly over welfare functions that can implement the cooperative optimum. This is an application of a Folk Theorem in Delegation Games by Fershtman, Judd and Kalai (1991), where it is argued that in a two-player game the principals can obtain every Pareto optimal outcome as the unique subgame perfect Nash Equilibrium of the delegation game via such contracts written on target compensation form, as long as these contracts become common knowledge. However, the authors move to analyzing linear contracts, arguing that therse are highly non-realistic and difficult to implement.

[4]See also Benigno (2002) , for an earlier effort in this direction, using the framework of Corsetti and Pesenti (2001) .

2. Credible, subgame perfect contracts in a general linear-quadratic framework.

In this section we describe a general solution method for *credible contracts* (as a shorthand notation for *subgame perfect, non-cooperative contracts*) as opposed to *optimal contracts* (see e.g. Persson and Tabellini (2000) and references therein) in a two-country model with policy spillovers[5]. We start with a simplest setup in which there is no domestic credibility problem and prove formally three propositions. The first states that delegation always occurs in equilibrium when there is strategic complementarity or substitutability (in the sense that one country's equilibrium strategy is increasing/decreasing in the other country's strategy). The second states that optimal contracts -that implement the cooperative optimum in the non-cooperative game- require cooperation at the delegation stage, i.e. are a solution to government's delegation problem only under cooperation. The third shows that these cooperative contracts (optimal contracts) are always different from subgame perfect contracts - that are chosen by governments based on their individual rationality. We then describe the solution method for a more general setup in which we allow for domestic credibility problems.

2.1. A simple but general version of the argument. Following most of the literature on policy coordination and institutional design reviewed above, suppose that there are two countries, home and foreign, and distinguish the foreign country by an asterisk. In each country a policymaker (the government) minimizes a quadratic aggregate loss function[6] defined over deviations of some macroeconomic variables, which are related linearly to the policy instruments, i and i^* respectively. The loss functions are therefore $L(i, i^*)$ and $L^*(i, i^*)$, and their being quadratic implies that second-order derivatives are constant (we denote a derivative with respect to an argument by appending the corresponding argument as a subscript, i.e. L_{ii^*} is the cross-derivative of L). We assume that externalities (spillovers) are present in the sense that $L_{i^*} \neq 0, L^*_i \neq 0$, and their sign depends on the sign of these derivatives.

The Nash, non-cooperative equilibrium consists of strategies i and i^* that solve the following system of linear first order conditions (they are linear because the loss functions are quadratic):

$$L_i(i, i^*) = 0 \text{ and } L^*_{i^*}(i, i^*) = 0. \tag{2.1}$$

Solving this system one can find the (linear) reaction (or best response) functions of each policymaker as a function of the other policymaker's strategy, $i^N(i^*)$ and $i^{*N}(i)$. By implicit differentiation of (2.1) we obtain the slopes of these reaction functions as:

$$\frac{\partial i^N}{\partial i^*} = -\frac{L_{ii^*}}{L_{ii}}; \frac{\partial i^{*N}}{\partial i} = -\frac{L^*_{i^*i}}{L^*_{i^*i^*}}.$$

Note that strategic complementarity $\left(\partial i^N/\partial i^* > 0, \partial i^{*N}/\partial i > 0\right)$ or substitutability (< 0) will depend on the sign of the second derivatives L_{ii^*} and $L^*_{i^*i}$, as in

[5] Specific (parametric) examples of models most related to this one are i.a. Persson and Tabellini (1995, 2000). We provide an example in the next section.

[6] This loss function is usually quadratic and directly postulated, although possible to derive as a quadratic approximation of an aggregate welfare function describing society's preferences - see e.g. Woodford (2003) and Benigno and Benigno (2005) for the open-economy case.

Cooper and John (1988). Under usual concavity assumptions on L and L^*, *existence and uniqueness of equilibrium* require that these slopes, in the (i, i^*) space, be different or equivalently, that the Jacobian of the (2.1) system be non-singular. This condition is:

$$\Delta \equiv L_{ii} L^*_{i^*i^*} - L_{ii^*} L^*_{i^*i} \neq 0. \tag{2.2}$$

The cooperative equilibrium is obtained by joint minimization of an aggregate, global loss function $\Lambda(i, i^*) = L(i, i^*) + L^*(i, i^*)$ with respect to both instruments. The strategies that implement this cooperative equilibrium (denoted by a C superscript) i^C and i^{*C} will solve the linear system:

$$\begin{aligned} \Lambda_i(i, i^*) &= L_i(i, i^*) + L^*_i(i, i^*) = 0 \text{ and} \\ \Lambda_{i^*}(i, i^*) &= L_{i^*}(i, i^*) + L^*_{i^*}(i, i^*) = 0. \end{aligned} \tag{2.3}$$

The slopes of the optimal reaction functions can be found by implicit differentiation as before and compared to the Nash reaction functions. The condition for existence and uniqueness of the equilibrium is in this case: $\Lambda_{ii}\Lambda_{i^*i^*} - \Lambda_{ii^*}\Lambda_{i^*i} \neq 0$.The non-cooperative Nash equilibrium is inefficient in the presence of externalities since the terms L^*_i and L_{i^*} are non-zero.

Delegation to independent policy authorities is a prominent solution to solve these inefficiencies while preserving non-cooperative policymaking, i.a. in the pioneering work of Persson and Tabellini (1995)[7]. Consider that before the non-cooperative game is played, each government delegates policy to an independent policy authority by imposing a certain transfer function $T(i)$; assuming further that these contracts or transfer functions are linear (since the model is linear-quadratic), the delegated loss functions L^D become:

$$\begin{aligned} L^D &= L + ti \\ L^{*D} &= L^* + t^* i^*, \end{aligned} \tag{2.4}$$

where t and t^* are the marginal penalties/rewards. Policy authorities minimize these new loss functions in a non-cooperative manner, and the 'delegated' strategies $i^D(t, t^*), i^{*D}(t, t^*)$, contingent upon the contracts chosen by the governments previously, solve the first-order conditions:

$$L_i(i, i^*) + t = 0 \text{ and } L^*_{i^*}(i, i^*) + t^* = 0. \tag{2.5}$$

Under usual concavity assumptions on L and L^*, this linear system of equations has a unique solution if and only if the condition (2.2) is satisfied, as in the Nash equilibrium case. Under the same condition, the implicit function theorem can be applied to the system (2.5) in order to study the sensitivity of the equilibrium in terms of policy authorities' strategies to changes in the contracts (which are treated as parameters in the policy authorities' problem). These can be found as:

$$\begin{pmatrix} \frac{\partial i}{\partial t} & \frac{\partial i}{\partial t^*} \\ \frac{\partial i^*}{\partial t} & \frac{\partial i^*}{\partial t^*} \end{pmatrix} = -\begin{pmatrix} L_{ii} & L_{ii^*} \\ L^*_{i^*i} & L^*_{i^*i^*} \end{pmatrix}^{-1} = \Delta^{-1} \begin{pmatrix} -L^*_{i^*i^*} & L_{ii^*} \\ L^*_{i^*i} & -L_{ii} \end{pmatrix}, \tag{2.6}$$

where Δ has been defined in (2.2) above.

[7]Other solutions include repeated game mechanisms sustaining the equilibrium in (2.15) or (2.16) as a unique subgame perfect equilibrium in the repeated version of the game described above; see Canzoneri and Gray (1985) or Ghosh and Masson (1994) for an extensive treatment.

The **optimal contracts** that implement the desired cooperative equilibrium in the non-cooperative game can be easily found by comparing (2.5) with (2.3) as:

$$t^{O}=L_{i}^{*}\left(i^{C},i^{*C}\right);t^{*O}=L_{i^{*}}\left(i^{C},i^{*C}\right), \tag{2.7}$$

where the strategies are evaluated at their 'cooperative equilibrium' values found in (2.3). This is the result in Persson and Tabellini (1995,1996,2000). The argument of this paper is that these contracts are not a non-cooperative way of achieving cooperation through delegation. Intuitively, there is nothing to ensure that the precise marginal penalties implementing the optimum will in fact be chosen at the delegation stage: these contracts are not incentive-compatible.

We solve for the **subgame perfect, credible contracts** that governments will choose by backward induction based on their individual incentives, taking into account the reaction functions of the policy authorities i^{D},i^{*D}. These contracts are a solution to $\min_{t} L\left(i^{D}\left(t,t^{*}\right),i^{*D}\left(t,t^{*}\right)\right)$ and $\min_{t^{*}} L^{*}\left(i^{D}\left(t,t^{*}\right),i^{*D}\left(t,t^{*}\right)\right)$ respectively, where $i^{D}\left(t,t^{*}\right),i^{*D}\left(t,t^{*}\right)$ is the solution found in (2.5). First-order conditions of this problem constitute a system of linear equations in t and t^{*}:

$$\begin{aligned} L_{i}\left(i^{D},i^{*D}\right)\frac{\partial i^{D}}{\partial t}+L_{i^{*}}\left(i^{D},i^{*D}\right)\frac{\partial i^{*D}}{\partial t} &= 0 \\ L_{i}^{*}\left(i^{D},i^{*D}\right)\frac{\partial i^{D}}{\partial t^{*}}+L_{i^{*}}^{*}\left(i^{D},i^{*D}\right)\frac{\partial i^{*D}}{\partial t^{*}} &= 0 \end{aligned} \tag{2.8}$$

Using (2.5) and (2.6) into (2.8), subgame perfect contracts t^{P},t^{*P} are a solution to (the fixed point of):

$$\begin{aligned} t &= -\frac{L_{i^{*}i}^{*}}{L_{i^{*}i^{*}}^{*}}L_{i^{*}}\left(i^{D}\left(t,t^{*}\right),i^{*D}\left(t,t^{*}\right)\right) \\ t^{*} &= -\frac{L_{ii^{*}}}{L_{ii}}L_{i}^{*}\left(i^{D}\left(t,t^{*}\right),i^{*D}\left(t,t^{*}\right)\right). \end{aligned} \tag{2.9}$$

We are now in a position to state our main results.

PROPOSITION 1. *Delegation always occurs in equilibrium (subgame perfect contracts t^{P},t^{*P} are non-zero) if and only if there is strategic complementarity/substitutability.*

PROOF. The proof is by contradiction; suppose that $t^{P}=t^{*P}=0$, which implies $i^{D}\left(0,0\right)=i^{N},i^{*D}\left(0,0\right)=i^{N}$ and further from (2.9) that

$$\frac{L_{i^{*}i}^{*}}{L_{i^{*}i^{*}}^{*}}L_{i^{*}}\left(i^{N},i^{*N}\right)=0;\frac{L_{ii^{*}}}{L_{ii}}L_{i}^{*}\left(i^{N},i^{*N}\right)=0.$$

Since $L_{i^{*}}\left(i^{N},i^{*N}\right)$ and $L_{i}^{*}\left(i^{N},i^{*N}\right)$ are non-zero (otherwise the Nash equilibrium would be efficient, hence no need for delegation), this can hold if and only if $L_{i^{*}i}^{*}=L_{ii^{*}}=0$, i.e. if and only if there is no strategic complementarity or substitutability. □

Intuitively, incentives for delegation occur because each government/principal recognizes it can influence the other player's strategy by delegating. Whether the equilibrium contract requires a penalty or a reward depends on the sign of the cross-derivative (i.e. on whether there is complementarity or substitutability) and on whether there are positive or negative spillovers.

PROPOSITION 2. *Optimal contracts $t^{O};t^{*O}$ are equivalent to 'cooperative contracts' $t^{C};t^{*C}$ found by solving* $\min_{t,t^{*}}\left[L\left(i^{D}\left(t,t^{*}\right),i^{*D}\left(t,t^{*}\right)\right)+L^{*}\left(i^{D}\left(t,t^{*}\right),i^{*D}\left(t,t^{*}\right)\right)\right].$

PROOF. The first order conditions for cooperative contracts are:

$$L_i\left(i^D,i^{*D}\right)\frac{\partial i^D}{\partial t}+L_{i^*}\left(i^D,i^{*D}\right)\frac{\partial i^{*D}}{\partial t}+L_i^*\left(i^D,i^{*D}\right)\frac{\partial i^D}{\partial t}+L_{i^*}^*\left(i^D,i^{*D}\right)\frac{\partial i^{*D}}{\partial t} = 0$$
$$L_i\left(i^D,i^{*D}\right)\frac{\partial i^D}{\partial t^*}+L_{i^*}\left(i^D,i^{*D}\right)\frac{\partial i^{*D}}{\partial t^*}+L_i^*\left(i^D,i^{*D}\right)\frac{\partial i^D}{\partial t^*}+L_{i^*}^*\left(i^D,i^{*D}\right)\frac{\partial i^{*D}}{\partial t^*} = 0$$

From (2.5), we know $L_i\left(i^D,i^{*D}\right)=-t$ and $L_{i^*}^*\left(i^D,i^{*D}\right)=-t^*$, which substituted in the above yield:

$$\left[L_i^*\left(i^D,i^{*D}\right)-t\right]\frac{\partial i^D}{\partial t}+\left[L_{i^*}\left(i^D,i^{*D}\right)-t^*\right]\frac{\partial i^{*D}}{\partial t} = 0$$
$$\left[L_i^*\left(i^D,i^{*D}\right)-t\right]\frac{\partial i^D}{\partial t^*}+\left[L_{i^*}\left(i^D,i^{*D}\right)-t^*\right]\frac{\partial i^{*D}}{\partial t^*} = 0$$

Combining the last two we obtain:

$$\left[L_{i^*}\left(i^D,i^{*D}\right)-t^*\right] = \left[L_{i^*}\left(i^D,i^{*D}\right)-t^*\right]\frac{\frac{\partial i^{*D}}{\partial t^*}}{\frac{\partial i^D}{\partial t^*}}\frac{\frac{\partial i^D}{\partial t}}{\frac{\partial i^{*D}}{\partial t}},$$
$$\left[L_i^*\left(i^D,i^{*D}\right)-t\right] = -\left[L_{i^*}\left(i^D,i^{*D}\right)-t^*\right]\frac{\frac{\partial i^{*D}}{\partial t^*}}{\frac{\partial i^D}{\partial t^*}}.$$

We can substitute in the first equation the partial derivatives of i^D and i^{*D} using the result in (2.6) to obtain:

$$\left[L_{i^*}\left(i^D,i^{*D}\right)-t^*\right]=\left[L_{i^*}\left(i^D,i^{*D}\right)-t^*\right]\frac{L_{ii}L_{i^*i^*}^*}{L_{ii^*}L_{i^*i}^*}.$$

As long as the necessary and sufficient condition for existence and uniqueness of equilibrium (2.2) is satisfied ($L_{ii}L_{i^*i^*}^* \neq L_{ii^*}L_{i^*i}^*$), this implies that cooperative contracts t^C, t^{*C} obey:

$$t = L_i^*\left(i^D(t,t^*),i^{*D}(t,t^*)\right)$$
$$t^* = L_{i^*}\left(i^D(t,t^*),i^{*D}(t,t^*)\right)$$

Under our assumptions on L_i (notably, condition (2.2) holds) this is a linear system with a *unique* solution, t^C, t^{*C}. On the other hand, we know from (2.7) above that $t^O; t^{*O}$, is *a* solution, so we conclude that cooperative contracts and optimal contracts coincide $t^C=t^O; t^{*C}=t^{*O}$. □

Intuitively, Proposition 2 shows that optimal contracts can only be credible (subgame perfect) if governments cooperate at the delegation stage; however, the cooperation technology needed is equivalent to the one needed to implement optimal policies $i^C; i^{*C}$. If the possibility of binding agreements existed, there would be no need for delegation in the first place. In addition, Proposition 2 serves as an intermediary result for Proposition 3.

PROPOSITION 3. *Subgame perfect, credible contracts $t^P; t^{*P}$ are always different from cooperative contracts $t^C; t^{*C}$ and hence also from optimal contracts $t^O; t^{*O}$.*

PROOF. The proof is by contradiction. Suppose that $t^P; t^{*P}$ and $t^C; t^{*C}$ are identical: comparing the first order conditions in each case, (**??**) with (2.8), this can happen if and only if:

$$L_i^*\left(i^D, i^{*D}\right)\frac{\partial i^D}{\partial t} + L_{i^*}^*\left(i^D, i^{*D}\right)\frac{\partial i^{*D}}{\partial t} = 0 \tag{2.10}$$
$$L_i\left(i^D, i^{*D}\right)\frac{\partial i^D}{\partial t^*} + L_{i^*}\left(i^D, i^{*D}\right)\frac{\partial i^{*D}}{\partial t^*} = 0$$

We substitute the partial derivatives of i^D and i^{*D} using the result in (2.6) to obtain:

$$-L_{i^*i^*}^* L_i^*\left(i^D, i^{*D}\right) + L_{i^*i}^* L_{i^*}^*\left(i^D, i^{*D}\right) = 0 \tag{2.11}$$
$$L_{ii^*} L_i\left(i^D, i^{*D}\right) - L_{ii} L_{i^*}\left(i^D, i^{*D}\right) = 0$$

From (2.5) we know that $L_i\left(i^D, i^{*D}\right) = -t$ and $L_{i^*}^*\left(i^D, i^{*D}\right) = -t^*$, which substituted above give:

$$t = -\frac{L_{ii}}{L_{ii^*}} L_{i^*}\left(i^D, i^{*D}\right); t^* = -\frac{L_{i^*i^*}^*}{L_{i^*i}^*} L_i^*\left(i^D, i^{*D}\right)$$

However, we know that perfect contracts have to satisfy also (2.9). Equating the expressions for either t or t^* we obtain (using that there are externalities $L_{i^*}\left(i^D, i^{*D}\right) \neq 0, L_{i^*}\left(i^D, i^{*D}\right) \neq 0$ by assumption) :

$$\frac{L_{ii}}{L_{ii^*}} = \frac{L_{i^*i}^*}{L_{i^*i^*}^*},$$

which is a contradiction since violates the condition for existence and uniqueness of equilibrium (2.2). Therefore, subgame perfect contracts are always different from cooperative contracts, and hence also from optimal contracts (using Proposition 2). □

To summarize, we have introduced *subgame perfect, credible contracts* in a general linear-quadratic model with spillovers. We have shown that subgame perfect contracts are always non-zero (delegation is an equilibrium) whenever there are strategic complementarities or substitutabilities; however, they are always different from optimal contracts (which are instead equivalent to cooperative contracts, chosen by minimising an aggregate, 'global' loss function). The position of the equilibrium ocuring under credible contracts $i^D\left(t^P, t^{*P}\right), i^{*D}\left(t^P, t^{*P}\right)$ relative to the Nash equilibrium i^N, i^{*N} and the cooperative equilibrium i^C, i^{*C} depends on two factors, as can be seen by inspection of (2.9): the sign and magnitude of spillovers/externalities (first derivatives of loos functions) and the sign and magnitude of complementarity (substitutability) given by the second (cross-)derivatives.

Finally, we relate our results to a general argument regarding delegation in Persson and Tabellini (1995), which is based on the Folk Theorem for Delegation Games in Fershtman et al. (1991). This theorem provides conditions under which in a two-principal-two-agent game every Pareto optimal outcome of the principals' game can become the unique subgame perfect equilibrium of the delegation game; this is done if each principal delegates to an agent and both contracts are public information. However, contracts consistent with the condition of this theorem are of a form that does not seem easily mapped into real-world policy institutions; for instance in our example (for the home country):

$$
\text{(2.12)} \qquad \begin{aligned} T\left(i,i^{*}\right) &= \begin{cases} \Upsilon, \text{if } \left[L\left(i,i^{*}\right), L^{*}\left(i,i^{*}\right)\right] \leq \left(L^{C}, L^{*C}\right) \\ \Upsilon + c, c > 0 \text{ otherwise} \end{cases} \\ i &= \begin{cases} i^{C}, \text{iff } \left[T\left(i,i^{*}\right), T^{*}\left(i,i^{*}\right)\right] \leq \left(\Upsilon, \Upsilon\right) \\ i^{D}, \text{ otherwise} \end{cases} \end{aligned}
$$

The strategies in (2.12), together with mirroring strategies for the foreign country, implement the first best once contracts are public information[8], but require that each penalty be written over **both** *payoffs directly*. Although interesting theoretically, transferring this idea to linear contracts is dangerous. While this theorem shows that strategies of the form (2.12) implementing the cooperative optimum in a decentralised manner do *exist*, it says nothing about their implementability or their being chosen in equilibrium (obvious issues related to this are observability of payoffs and inconsistency of sovereign policymaking with making contracts depend on the other country's strategy). Moreover, if one is to think about governments choosing a delegation scheme and facing a decision problem that can be reduced to choosing a set of parameters of the transfer function T, then choice of the optimal contracts is by no means insured. In the 'linear contracts' example, each government has a choice parameter (t) and as we have shown, while it will always choose some contract, it will never choose the optimal contract t^{C}. Moreover, it is unclear why, if the government had the ability to commit to and actually implement the optimal contract, does it need to delegate policy rather than choose directly the optimal policy rule i^{C}.

2.2. Solution method for model with a credibility problem. We now briefly outline the solution method for a model in which there is a credibility problem. The main reason for doing so is that is has been recognized, since the work of Rogoff (1985) that policy cooperation may be suboptimal when a credibility problem exists. Therefore, optimal contracts are usually designed to correct for both distortions (one coming from non-cooperation, the other one from the lack of commitment). Suppose as before that in each country a policymaker (the government) minimizes an aggregate, quadratic loss function $L\left(\mathbf{X};\boldsymbol{\tau}\right)$ defined over deviations of some macroeconomic variables stacked in the vector $\mathbf{X}$ from some target (socially desirable) levels $\boldsymbol{\tau}$ ($\mathbf{X}$ would include e.g. inflation, the output gap, the exchange rate, etc). Suppose that in each country the policymaker has at its disposal one policy instrument (such as the interest rate or growth in a monetary aggregate for monetary policy) and denote this by i. Additionally, assume the model is stochastic, hence each variable will be hit by a stochastic shock and let the vector of such shocks be denoted by $\boldsymbol{\epsilon}$. Apart from the policymaker, in each country there is a private sector forming expectations over some relevant subset of variables of $\mathbf{X}$ and hence ultimately over the policy instruments conditional on some information available one period in advance (Ω_{-1}): $i^{e} = E\left[i \mid \Omega_{-1}\right], i^{e*} = E\left[i^{*} \mid \Omega_{-1}^{*}\right]$ .As the two countries are interdependent, we also assume that the instrument in one country influences at least one of the macroeconomic variables of the other, either directly or indirectly (e.g. through a variable such as the exchange rate). With these assumptions, the relevant macroeconomic variables can ultimately be expressed as a linear function of the instruments, expectations and shocks in both countries:

[8]For details see Persson and Tabellini (1995); for the game-theoretical argument see Fershtman et al (1991).

$$\mathbf{X} = \mathbf{X}\left(i, i^*, i^e, i^{e*}, \boldsymbol{\epsilon}, \boldsymbol{\epsilon}^*\right), \mathbf{X}^* = \mathbf{X}^*\left(i, i^*, i^e, i^{e*}, \boldsymbol{\epsilon}, \boldsymbol{\epsilon}^*\right) \tag{2.13}$$

Using (2.13), the loss functions can be expressed as functions of instruments, expectations, shocks and target levels:

$$L = L\left(i, i^*, i^e, i^{e*}, \boldsymbol{\epsilon}, \boldsymbol{\epsilon}^*; \boldsymbol{\tau}\right), L^* = L^*\left(i, i^*, i^e, i^{e*}, \boldsymbol{\epsilon}, \boldsymbol{\epsilon}^*; \boldsymbol{\tau}^*\right) \tag{2.14}$$

Strategic interactions in this model result from heterogeneity of targets ($\boldsymbol{\tau} \neq \boldsymbol{\tau}^*$) and from different preferences for the stabilisation of shocks, when there are spillovers. Assuming further L, L^* are differentiable, the policies have positive or negative externalities depending on whether $\frac{\partial L(.)}{\partial i^*}, \frac{\partial L^*(.)}{\partial i} \gtrless 0$. In addition, the presence of a private sector forming rational expectations of some variable(s) combined with a real distortion in the economy gives rise to domestic incentives to deviate from optimality (defined below) that are not related to cross-country spillovers[9]. Suppose that in choosing the policies, the policymaker faces the following timing in each period: (i) targets $\boldsymbol{\tau}, \boldsymbol{\tau}^*$ are revealed; (ii) expectations are formed, i^e, i^{e*} are determined; (iii) shocks $\boldsymbol{\epsilon}, \boldsymbol{\epsilon}^*$are realised; (iv) policy instruments i, i^* are chosen simultaneously; (v) macroeconomic variables $\mathbf{X}, \mathbf{X}^*$ are fully determined.

A policy regime whereby the two policymakers decide before stage (i) to cooperate (i.e. to minimise a joint loss function) and commit to an optimal rule with respect to the private sector will be Pareto optimal (see e.g. Persson and Tabellini 1995, 2000). We label the equilibrium occurring under this benchmark regime '**the cooperative and commitment equilibrium**' and denote it with superscript C as in the previous section. The policy instruments at this equilibrium obey:

$$(i, i^*)^C = \arg\min_{i,i^*} \left\{ \begin{array}{c} E\left[L\left(i, i^*, i^e, i^{*e}, .\right) + L^*\left(i, i^*, i^e, i^{*e}, .\right)\right] \text{ s.t.} \\ i^e = E\left[i \mid \Omega_{-1}\right], i^{*e} = E\left[i^* \mid \Omega^*_{-1}\right] \end{array} \right\} \tag{2.15}$$

The optimal policy rules $i^C\left(\boldsymbol{\epsilon}, \boldsymbol{\epsilon}^*\right), i^{*C}\left(\boldsymbol{\epsilon}, \boldsymbol{\epsilon}^*\right)$ can be found by solving for[10] the (linear, since loss functions are quadratic) first-order conditions, rewritten after eliminating the Lagrange multipliers of the rational expectations constraints:

$$\begin{aligned} \frac{\partial E\left[L\left(.\right) + L^*\left(.\right)\right]}{\partial i} + E\left[\frac{\partial E\left[L\left(.\right) + L^*\left(.\right)\right]}{\partial i^e} \mid \Omega_{-1}\right] &= 0 \\ \frac{\partial E\left[L\left(.\right) + L^*\left(.\right)\right]}{\partial i^*} + E\left[\frac{\partial E\left[L\left(.\right) + L^*\left(.\right)\right]}{\partial i^{*e}} \mid \Omega^*_{-1}\right] &= 0 \end{aligned} \tag{2.16}$$

Upon specifying functional forms for loss functions and for the models determining the macroeconomic variables, the policy rules are obtained by taking conditional expectations of the system (2.16), hence determining expected variables, and then substituting the latter in the original system (2.16). However, this equilibrium is unrealistic: since real-world policymaking is best described in a non-cooperative setup (i.e. binding agreements of any sort are not possible) then the appropriate equilibrium concept to use is **discretionary Nash equilibrium**. This equilibrium will obviously be inefficient, due to two reasons: ignoring the spillovers of policy to

[9]This is the case in the 'dynamic inconsistency' literature.

[10]Throughout we assume that certain properties of loss functions, policy sets, etc. are met so that the considered equilibria do exist and are unique, which is the case in most models considered in the literature.

the other countries' loss function (as in the previous section) and ignoring externalities on the own-country private sector (being more specific about the source of inefficiencies would require a parametric example which we postpone to the next section). We study again delegation as a possible solution to both these inefficiencies. Consider that at stage (0), before stage (i) above, each government delegates the policy to an independent policy authority by imposing a certain transfer function, or contract $T\left(\bar{\mathbf{X}}\right)$, where $\bar{\mathbf{X}}$ would be a subset of the relevant macroeconomic variables (e.g. only inflation for inflation contracts). Ultimately, this function can also be defined over instruments and shocks, hence delegation would mean assigning loss functions of the form (where superscript D stands for 'delegated'):

$$(2.17)\qquad \begin{aligned} L^{D}\left(.;T\right) &= L\left(.\right)+T\left(i,i^{*},\boldsymbol{\epsilon},\boldsymbol{\epsilon}^{*};\boldsymbol{\tau},\boldsymbol{\tau}^{*}\right) \\ L^{D*}\left(.;T^{*}\right) &= L^{*}\left(.\right)+T^{*}\left(i,i^{*},\boldsymbol{\epsilon},\boldsymbol{\epsilon}^{*};\boldsymbol{\tau},\boldsymbol{\tau}^{*}\right) \end{aligned}$$

The independent authorities face these loss functions when choosing their policy instruments simultaneously at stage (iv), in a non-cooperative and discretionary manner. The corresponding **discretionary Nash equilibrium** policy rules under **delegation** will be given by:

$$(2.18)\qquad \begin{aligned} i^{D} &= \arg\min_{i}\left[L\left(i,i^{*},i^{e},i^{*e},.\right)+T\left(i,i^{*},.\right)\right] \\ i^{D*} &= \arg\min_{i^{*}}\left[L^{*}\left(i,i^{*},i^{e},i^{*e},.\right)+T^{*}\left(i,i^{*},.\right)\right] \end{aligned}$$

The policy instruments can be solved starting from the first order conditions given below, taking expectations of these to pin down expected variables and substituting these back in the original system:

$$(2.19)\qquad \begin{aligned} \frac{\partial\left[L\left(i,i^{*},i^{e},i^{*e},.\right)+T\left(i,i^{*},.\right)\right]}{\partial i} &= 0 \\ \frac{\partial\left[L\left(i,i^{*},i^{e},i^{*e},.\right)+T^{*}\left(i,i^{*},.\right)\right]}{\partial i^{*}} &= 0 \end{aligned}$$

Consider first the case **without delegation**, i.e. $T\left(i,i^{*},.\right)=T^{*}\left(i,i^{*},.\right)=0$, leading to the pure Nash equilibrium choices, say i^{N},i^{*N}. The two sources of inefficiencies mentioned before are obvious by comparing the systems (2.16) with (2.19) evaluated at $T=T^{*}=0$; in the latter, two terms are absent that come from ignoring externalities on (i) the other policymaker and (ii) the private sector. Solutions to this inefficiency usually considered in the literature consist of governments choosing the functions $T\left(i,i^{*},.\right),T^{*}\left(i,i^{*},.\right)$ such that the solutions to the systems coincide. It is easily seen by direct comparison of (2.16) and (2.19) that **optimal contracts** (making the equilibrium under delegation identical with the Pareto optimum) should fulfil:

$$(2.20)\qquad \begin{aligned} \left(\frac{\partial T\left(i,i^{*},.\right)}{\partial i}\right)^{C} &= \frac{\partial E\left[L^{*}\left(.\right)\right]}{\partial i}+E\left[\frac{\partial E\left[L\left(.\right)+L^{*}\left(.\right)\right]}{\partial i^{e}}\mid\Omega_{-1}\right] \\ \left(\frac{\partial T^{*}\left(i,i^{*},.\right)}{\partial i^{*}}\right)^{C} &= \frac{\partial E\left[L\left(.\right)\right]}{\partial i^{*}}+E\left[\frac{\partial E\left[L\left(.\right)+L^{*}\left(.\right)\right]}{\partial i^{*e}}\mid\Omega_{-1}^{*}\right], \end{aligned}$$

where loss functions are evaluated at the cooperative and commitment optimum i^{C},i^{*C} found in (2.15). Specifying functional forms for the transfer functions usually results in a solvable system for the delegation parameters. To choose the most prominent example, linear inflation contracts, suppose T's are linear functions of the policy instruments and can be expressed as $T(i) = k + ti$, where k and t are the delegation parameters to be chosen, and $\partial T/\partial i = t$. The system (2.20) fully determines the 'optimal contracts' t and t^*, which we label by t^C and t^{C*}. In most cases, these marginal contracts are state-dependent (i.e. dependent on the realization of the shocks), which makes them hard to implement and undermines their credibility. Jensen (2000) addresses this problem by showing how the first best in (2.15) can nevertheless be implemented through state-independent delegation by choosing a quadratic form for the transfer function T.

Finally, when we model the delegation stage (0) as a separate stage of the game -whereby governments choose their 'strategies' (the parameters determining the transfer functions)- the timing is: (0) governments delegate policies to independent authorities by imposing the transfer functions $T(.)$, $T^*(.)$; (i)-(v): same as before. The solution method is based on backward induction: policy authorities choose their policy instruments independently and discretionarily taking delegation as given and governments choose delegation parameters taking into account the choice of policy instruments made previously by the delegated authorities. The policy rules governments face at stage (0) are $\left(i^D, i^{D*}\right)$ given by (2.18) and fulfill the first order conditions (2.19). For the sake of simplicity and for their widespread use, we restrict the functional form of the transfer functions to linear contracts $T(i) = k + ti, T^*(i^*) = k^* + t^*i^*$. The *subgame perfect, credible contracts* are determined by:

$$\begin{aligned} t^P(.) &= \arg\min_t \left\{EL\left[i^D(t,t^*,.), i^{D*}(t,t^*,.),.\right]\right\} \\ t^{*P}(.) &= \arg\min_{t^*} \left\{EL^*\left[i^D(t,t^*,.), i^{D*}(t,t^*,.),.\right]\right\} \end{aligned} \tag{2.21}$$

The other parameters k, k^* can be chosen such that the participation constraint of the policy authority is met. The first order conditions that credible contracts fulfil[11] are:

$$\begin{aligned} \frac{\partial E[L(.)]}{\partial i}\frac{\partial i^D(t,t^*)}{\partial t} + \frac{\partial E[L(.)]}{\partial i^*}\frac{\partial i^{D*}(t,t^*)}{\partial t} &= 0 \\ \frac{\partial E[L^*(.)]}{\partial i}\frac{\partial i^D(t,t^*)}{\partial t^*} + \frac{\partial E[L^*(.)]}{\partial i^*}\frac{\partial i^{D*}(t,t^*)}{\partial t^*} &= 0 \end{aligned} \tag{2.22}$$

The objects in (2.20) and (2.21) are different in most situations (i.e. when externalities are present, which is why one considers delegation in the first place). This implies that *optimal contracts* are not consistent with individual rationality of the governments, the cooperation problem being not solved but merely relocated to the delegation stage. To sustain optimal contracts as an equilibrium phenomenon, co-operation or some form of coordination of governments/principals is unequivocally necessary. Note again the difference with the Folk Theorem in delegation games: here, by delegating the principal modifies the reaction functions of the agent in a linear way (or else, if contracts non-linear) instead of 'forcing' the Nash equilibrium to overlap with the desired Pareto optimum. Even with this form of delegation,

[11] Note that this can be done more generally for a certain functional form of T as long as it is differentiable; the only modification would be that the number of parameters, and hence the number of first order conditions to solve, would increase (for example, for Jensen quadratic contracts there would be three parameters and three first order conditions).

the Nash equilibrium could be made identical to the Pareto optimum, but this is not compatible with individual incentives of governments. This is illustrated in the following example, where credible and optimal contracts are clearly different in an intuitive way.

3. An example: credible vs. optimal inflation contracts in international monetary policy

We use a parameterized version of the model in the previous section for an illustrative example. The model is an adapted version of Persson and Tabellini (1996, 2000) and consists of directly postulated reduced forms[12]. The world consists as before of two countries, each one being specialized in producing a consumer good, which is an imperfect substitute for the other country's good. This generates the main spillover of policy through the real exchange rate. Each country has a monetary policy instrument, which it uses for short-run stabilisation (it being able to do so is insured, for instance, by some nominal rigidity). The policy is also subject to a credibility problem generated by a real distortion making the natural rate of output (employment) suboptimally low. The model parameters are symmetric for simplicity but the shocks hitting the economy are arbitrarily correlated. All variables are in log-differences, a star denotes a foreign-country variable (for brevity just the home country's model is presented) and time subscripts have been suppressed:

$$y = \gamma(p - p^e) - \varepsilon \tag{3.1}$$

$$p = m \tag{3.2}$$

$$z \equiv s + p^* - p \tag{3.3}$$

$$z = \delta(y - y^*) \tag{3.4}$$

$$\pi = p + \beta z \tag{3.5}$$

Deviations of output growth y from the natural rate (normalised to zero) are defined in (3.1) by an usual expectations-augmented Phillips curve, where inflation surprises in producer price inflation p matter. For simplicity, in (3.2) we suppose the growth rate of money is the same as that of producer inflation, and therefore we abstract from velocity shocks[13]. Real exchange rate appreciation z is defined in (3.3) as nominal depreciation plus the differential of producer inflation. (3.4) relates the relative prices z of the two goods to their relative demand, hence defining an inverse demand equation, where $\delta > 0$ is the inverse relative demand elasticity of outside goods. A higher supply of foreign goods reduces z (real appreciation) by inducing a relative excess demand for home goods. Finally, consumer price index inflation π is producer inflation plus inflation induced by the consumption of foreign goods, where β is the share of the latter in the domestic consumption basket. Observe that the only source of uncertainty in the economy is given by adverse supply shocks $(\varepsilon, \varepsilon^*)$ with zero mean, different variances and arbitrary covariance $(\sigma_\varepsilon^2 \lesseqgtr \sigma_{\varepsilon^*}^2, \sigma_{\varepsilon\varepsilon^*} \lesseqgtr 0)$. The private sector forms rational expectations of

[12]Although it can be derived from microfoundations (see Rogoff (1985) or Canzoneri and Henderson (1991))

[13]In fact, in contrast to Persson and Tabellini, we abstract from all shocks other than supply shocks for simplicity. Having one shock that creates incentives to deviate from copperative policy is sufficient for our point.

producer inflation (and hence money growth), as the expectation of the latter over the distribution of shocks, conditional upon the information set Ω_{-1} of previous realizations of macroeconomic variables and model parameters:

$$p^e = m^e = E\left[p \mid \Omega_{-1}\right] = E\left[m \mid \Omega_{-1}\right] \tag{3.6}$$

Social welfare in each country is defined over variability of output and inflation from some socially desirable levels. For simplicity, normalize the socially optimal inflation to zero and suppose the desirable output θ is greater than the natural rate due to some real distortion (monopolistic competition, for instance). Then the policymakers' task is to minimise the expected value of the following conventional period loss function[14], using as instruments the money growth rates m:

$$L\left(.\right) = \frac{1}{2}\left\{\pi^2 + \lambda\left(y-\theta\right)^2\right\} \tag{3.7}$$

We assume that $\theta > 0$ giving rise to the domestic inflation bias described in the previous section. The timing is as in the previous section: just substitute $\mathbf{X}$ with (y, π, p, z), i with m, $\boldsymbol{\epsilon}$ with ε and $\boldsymbol{\tau}$ with $(0,\theta)$.

3.1. The cooperative optimum, non-cooperative inefficiency and optimal contracts. For the sake of brevity, we shall apply directly the solution method described in the last section, without getting into computational details. The **cooperative and commitment equilibrium** (as in (2.15) and (2.16)) is attained in this example for the *optimal state-contingent policy rules*:

$$\begin{aligned} m^C\left(\varepsilon, \varepsilon^*\right) &= \frac{b}{1+b\gamma}\varepsilon + \frac{d\left(1+2a-b\gamma\right)}{\left(1+b\gamma\right)\left(\left(1+2a\right)^2+b\gamma\right)}\left(\varepsilon-\varepsilon^*\right), \\ m^{*C}\left(\varepsilon, \varepsilon^*\right) &= \frac{b}{1+b\gamma}\varepsilon^* - \frac{d\left(1+2a-b\gamma\right)}{\left(1+b\gamma\right)\left(\left(1+2a\right)^2+b\gamma\right)}\left(\varepsilon-\varepsilon^*\right), \end{aligned} \tag{3.8}$$

where we used the change of notation $b = \lambda\gamma, a = \beta\delta\gamma$ and $d = \beta\delta$. At the optimum, the policymaker stabilizes domestic supply shocks (due to their influence on output and inflation). She also stabilizes relative shocks $\varepsilon - \varepsilon^*$ due to their indirect impact on welfare through real exchange rate appreciation/depreciation[15]. The responses are optimal due to the cooperative features of the equilibrium. Note also the absence of the inflation bias due to commitment (expected policies are zero). Each policymaker internalizes the effects of its instruments on both the other country's welfare and its domestic private sector.

The **non-cooperative and discretionary equilibrium** is also solved by the corresponding method described in the previous section ((2.19)). Suppose delegation to an independent central bank has taken place before the stage game is played,

[14] When the game is repeated over time, the policymakers minimize the expected present discounted value of this loss function. Since the stage game is always identical, this is equivalent to period-by-period minimisation.

[15] See e.g. Persson and Tabellini (1995, 1996, 2000) or Bilbiie (2000) for details and the solution of slightly more general models.

at stage (0). For the moment we assume this takes the form of linear inflation contracts of the type considered by Persson and Tabellini[16]: each government imposes a transfer function on its central bank of the form $T = k + t\pi, T^* = k^* + t^*\pi^*$. The marginal penalties t and t^* are allowed to be state-contingent. Given the linear nature of the model and linearity in stochastic shocks we model this by assuming that each marginal penalty is additively separable in a state-independent and a state-dependent component, namely:

$$
\begin{aligned}
(3.9) \qquad t &= \bar{t} + \tilde{t}\left(\varepsilon, \varepsilon^*\right) \text{ where } E\left[\bar{t}\right] = \bar{t} \text{ and } E\left[\tilde{t}\right] = 0 \\
t^* &= \bar{t}^* + \tilde{t}^*\left(\varepsilon, \varepsilon^*\right) \text{ where } E\left[\bar{t}^*\right] = \bar{t}^* \text{ and } E\left[\tilde{t}^*\right] = 0
\end{aligned}
$$

Given these linear penalties, each central bank will minimise its loss function (3.7) modified as in the system (2.17), e.g. $L(.) = \frac{1}{2}\left\{\pi^2 + \lambda\left(y - \theta\right)^2\right\} + T(.)$. The Nash discretionary equilibrium policy instruments given delegation are found as in the system (2.18) in the previous section as (where the same change of notation as before was used and additionally $A = (1+a)^2 + b\gamma, B = a(1+a)$):

$$
\begin{aligned}
(3.10) \quad m^D\left(\theta, \varepsilon, \varepsilon^*; t, t^*\right) &= -\bar{t} + \frac{b}{1+a}\theta - \frac{(1+a)A}{A^2 - B^2}\tilde{t} - \frac{(1+a)B}{A^2 - B^2}\tilde{t}^* + \\
&\quad \frac{b}{A-B}\varepsilon + \frac{d\left(1+a^2\right)}{A^2 - B^2}\left(\varepsilon - \varepsilon^*\right) \\
m^{*D}\left(\theta, \varepsilon, \varepsilon^*; t, t^*\right) &= -\bar{t}^* + \frac{b}{1+a}\theta - \frac{(1+a)B}{A^2 - B^2}\tilde{t} - \frac{(1+a)A}{A^2 - B^2}\tilde{t}^* + \\
&\quad \frac{b}{A-B}\varepsilon^* - \frac{d\left(1+a^2\right)}{A^2 - B^2}\left(\varepsilon - \varepsilon^*\right)
\end{aligned}
$$

The purely non-cooperative discretionary equilibrium *without delegation* ($\bar{t} = \tilde{t} = \bar{t}^* = \tilde{t}^* = 0$) features two inefficiencies, as expected. First, a familiar inflation bias (the θ term) is present in each country (and is the same in both due to the assumption on homogeneity of targets) due to discretionarity. Secondly, the responses to both domestic supply shocks and relative shocks are different from the optimal ones due to not internalizing of policy externalities when acting non-cooperatively. The exact nature of the distortions will depend on the shocks and the values of parameters but as a general rule the policies would have a contractionary bias when a favorable shock hits (positive externalities) and would be too expansionary, at the other country's cost, when an adverse shock is realised . Following for instance Persson and Tabellini (1996) we shall call this a *stabilisation bias*[17].

Following the analysis in the general case we can easily find the marginal penalties that implement the cooperative and commitment optimum when policy m, m^* is chosen in a non-cooperative and discretionary manner. *Mutatis mutandis*, system

[16] In Bilbiie (2000) we show equivalence of linear inflation contracts and inflation *targets* in this framework. We shall focus only on contracts for the sake of exposition but the results applpy equally to delegating with a non-zero inflation target.

[17] More details on interpretation of incentives in this equilibrium can again be found in Persson and Tabellini (1996), Bilbiie (2000) or Jensen (2000).

(2.20) in this case translates to:

$$
\begin{aligned}
t^{C}(\theta,\varepsilon,\varepsilon^{*}) &= \frac{1}{1+a}\left\{b\theta - a\pi^{*C}(\varepsilon,\varepsilon^{*})\right\} \\
&= \frac{b}{1+a}\theta - \frac{ab}{(1+a)(1+b\gamma)}\varepsilon^{*} - \frac{2a^{2}b}{(1+b\gamma)\left[(1+2a)^{2}+b\gamma\right]}(\varepsilon-\varepsilon^{*}) \qquad (3.11)\\
t^{*C}(\theta,\varepsilon,\varepsilon^{*}) &= \frac{1}{1+a}\left\{b\theta - a\pi^{C}(\varepsilon,\varepsilon^{*})\right\} \\
&= \frac{b}{1+a}\theta - \frac{ab}{(1+a)(1+b\gamma)}\varepsilon + \frac{2a^{2}b}{(1+b\gamma)\left[(1+2a)^{2}+b\gamma\right]}(\varepsilon-\varepsilon^{*})
\end{aligned}
$$

The marginal penalties are intuitive. The first terms are the familiar ones correcting for the domestic inflation bias in each country. The other terms correct for suboptimal stabilisation of shocks. The penalty is weaker if the foreign country suffers an adverse supply shock ($\varepsilon^{*} > 0$) or a less sever supply shock as compared to the home country ($\varepsilon - \varepsilon^{*}$). In these two cases foreign inflation is positive and the real exchange rate appreciates at home. In this case the home policy is too contractionary and a reward (or lower penalty, depending on the θ term) for additional inflation is needed to correct for that.

While the marginal penalties' being state-contingent is intuitive, the design of 'state-dependent institutions' in practice is not (for instance, they would require changes in institutions for every realization of shocks)[18]. If only state-independent contracts are feasible, then only the domestic incentives are corrected for, leaving suboptimal shock stabilisation unaltered. This problem is solved by Jensen (2000) by proposing a delegation scheme based on quadratic contracts with targets of the form: $T(.) = \frac{1}{2}\left\{\alpha\left(\pi-\pi^{B}\right)^{2} + \mu\left(\pi-\pi^{*}\right)^{2}\right\}$, where α, π^{B}, μ are decision variables of the government when delegating. In our example, following the same solution method the optimum is implemented for: $\left(\alpha = \frac{a}{1+2a}, \pi^{B} = -\frac{b}{a}\theta, \mu = -\frac{a}{1+a}\right)$. For details and an intuitive interpretation of this see Jensen (2000).

3.2. Credible contracts: the linear inflation contracts case. Although linear contracts *à la* Persson and Tabellini implement the optimum (albeit with state-contingent parameters), and the quadratic contracts with targets *à la* Jensen also solve the problem of state-contingency, they are both subject to the problem we identified in the previous section. We shall now see an example of this at work[19].

To see what contracts government will choose (and hence implement) based only on their individual rationality and their perception of rationality of the agents to which they delegate (central banks) we follow the solution method outlined in the general case. By backward induction, at the delegation stage (0), governments face the policy rules contingent on contracts that we solved for previously in (3.10). They then minimise the expected values of the social losses given by (3.7) (and its foreign counterpart), where inflation and the output gap are evaluated at the delegated Nash Equilibrium. We treat the state-independent part of the contract as control

[18]For a critique of state-dependent delegation see for instance Jensen (2000).

[19]Jensen recognises this problem himself in the last paragraph of the mentioned paper '[...] incentives causing policymakers to deviate from cooperative policies would also cause governments to deviate from cooperative institutions', but he focuses on identification of optimal institutions and not their implementability.

variables of the government. Hence, e.g. the 'home' government will only choose $\bar{t}$, and for finding the equilibrium state-contingent part of the contract $\tilde{t}(\varepsilon, \varepsilon^*)$ we use (3.9). Substituting $m^D(.; t, t^*), m^{*D}(.; t, t^*)$ found in (3.10) into $E[L(.)], E[L^*(.)]$ and minimising the latter two with respect to $\bar{t}$ and $\bar{t}^*$ respectively yields the two first order conditions:

$$p^D(\bar{t}, \tilde{t}, \bar{t}^*, \tilde{t}^*; \theta, \varepsilon, \varepsilon^*) = 0 \tag{3.12}$$
$$p^{*D}(\bar{t}, \tilde{t}, \bar{t}^*, \tilde{t}^*; \theta, \varepsilon, \varepsilon^*) = 0$$

Substituting the delegated Nash equilibrium money growth rates from (3.10), we get two equations in four unknowns $\bar{t}, \bar{t}^*, \tilde{t}(\varepsilon, \varepsilon^*), \tilde{t}^*(\varepsilon, \varepsilon^*)$. Using state independence of the first two and zero-mean of the last two one gets a solution for credible contracts as (where a and b are defined as before):

$$t^P(\varepsilon, \varepsilon^*) = \frac{b}{1+a}\theta + \frac{b}{1+a}\varepsilon^* + \frac{b}{1+2a}(\varepsilon - \varepsilon^*) \tag{3.13}$$
$$t^{*P}(\varepsilon, \varepsilon^*) = \frac{b}{1+a}\theta + \frac{b}{1+a}\varepsilon - \frac{b}{1+2a}(\varepsilon - \varepsilon^*)$$

We are now ready to compare these non-cooperative *credible contracts* with the *optimal contracts* implementing the first best, focusing on the home country. A first thing to note is that, maybe surprisingly so, the state-independent term leading to elimination of the systematic inflation bias is the same in $t^C(\varepsilon, \varepsilon^*)$ and $t^P(\varepsilon, \varepsilon^*)$. Another way to read this is that if only state-independent delegation were possible, the two contracts would coincide, although they would then be both suboptimal in that they would not affect stabilisation of shocks.

The inefficiency of credible contracts comes from suboptimal responses to shocks (second and third term). Consider again the case where the foreign country is hit by an adverse supply shock $\varepsilon^* > 0$ and this is less severe than in the home country (or equivalently, there is a larger favorable shock), i.e. $\varepsilon - \varepsilon^* > 0$. In equilibrium π^* is greater than zero and there is a contractionary bias of the home country's monetary policy. In the optimal contract t^C, both coefficients on shock stabilisation are *negative*: the optimal penalty in the home country decreases to correct for the deflationary bias. On the contrary, in the credible contract t^P both coefficients are *positive:* the penalty is increasing in ε and $\varepsilon - \varepsilon^*$, aggravating the contractionary bias of home policy. Equivalently, note from (3.12) that the credible contract is always imposed so that producer inflation is zero, whereas in the considered case the optimal response would be a positive producer inflation. To achieve the zero inflation in the perfect equilibrium an increasing marginal penalty is needed. In the converse case, where the foreign country faces a favorable supply shock $\varepsilon^* < 0$ relatively smaller than in the home country ($\varepsilon - \varepsilon^* < 0$) there is an expansionary bias of home monetary policy. This negative spillover would be eliminated through optimal delegation: the penalty becomes larger (see (3.11)) to reduce inflationary incentives. As the spillovers are ignored in the perfect equilibrium, the penalty will be smaller, tailored to achieving a zero inflation consistent with the first order condition (3.12). But the optimal response should in fact target deflation.

The two contracts are different even when shocks are perfectly correlated, in the symmetric case whereby $\varepsilon = \varepsilon^*$. The optimal marginal penalty is $t^C = \frac{b}{1+a}\theta - \frac{ab}{(1+a)(1+b\gamma)}\varepsilon$, whereas the credible one is $t^P = \frac{b}{1+a}\theta + \frac{b}{1+a}\varepsilon$. An adverse common supply shock generating a contractionary bias is optimally corrected by a decrease in the penalty for additional inflation. Not recognizing the positive externality that would result from both countries inflating more, at the delegation stage governments increase the penalty, therefore aggravating the deflationary bias.

The different contracts can be directly compared by substituting them in the best response functions (3.10)[20]. The linear optimal contracts t^C, t^{*C} implement the first best optimal money growth rates m^C, m^{*C}, and so do the Jensen quadratic contracts and the contracts consistent with the Folk Theorem in delegation games presented in (2.12). Without delegation, the Nash equilibrium policies would feature an inflation bias and suboptimal shock stabilisation. Delegation with credible contracts would mean elimination of the inflation bias but still suboptimal shock stabilisation, so they will not lead to implementation of m^C, m^{*C}. By substituting t^P, t^{*P} in the best response functions one gets $m^D\left(.; t^P, t^{*P}\right) = \frac{d}{1+2a}\left(\varepsilon - \varepsilon^*\right), m^{*D}\left(.; t^P, t^{*P}\right) = -\frac{d}{1+2a}\left(\varepsilon - \varepsilon^*\right)$. Whether these will be higher or lower than m^C, m^{*C} depends on the nature of the shocks and the parameters.

Hence, two governments seeking to delegate policy in a manner consistent with their individual rationality (or with their mandate, that is maximising social welfare) would fail to achieve a first best equilibrium. In order to delegate with the scheme that would insure that optimal policies are followed they would need to cooperate at the delegation stage. Alternatively, a supranational institution able to coordinate the two governments on the 'right' institutions would do the same job. However, this is far from the non-cooperative setup one wishes to describe in the first place. If compromises are to be made in terms of allowing for the possibility of binding agreements at the level of governments, it is hard to understand why then wouldn't governments cooperate directly without any need for delegating policies.

4. Concluding comments

Various inefficiencies associated with policy making, whether at a domestic or international level, can allegedly be solved by delegation of policy to independent monetary authorities. In a prominent example, monetary policy, delegation schemes have been viewed as panacea for both domestic credibility problems and inefficiencies coming from cross-country spillovers. Given policy externalities, a policy regime where governments cooperate (and commit with respect to the private sectors) is unequivocally Pareto optimal, but there are strong incentives to deviate from it. Some simple and intuitive delegation schemes easily mappable into real-life institutions have been found to 'fix' both these incentives: *i.a.* the linear inflation contracts proposed by Persson and Tabellini (1995, 1996, 2000), quadratic contracts with targets of Jensen (2000), or targeting rules in a new open-economy model as in Benigno and Benigno (2005). In these cases, each government delegates to an independent policy authority by imposing to the latter a certain transfer function. The governments can then in theory design the contract to ensure that the delegated authorities choose the policy instruments that implement the desired equilibrium.

The argument of this paper is that this implementation mechanism hides an implicit assumption about governments being actually able to sign binding agreements in order to coordinate on exactly those delegation parameters that 'do the job'. But if this were the case, it is hard to see why governments need to delegate

[20]In Bilbiie (2000) we provide a more detailed welfare comparison of equilibria.

instead of committing themselves to the optimal policy rules. The way out from this dilemma is, in our view, an explicit modelling of the delegation decision of the governments. We do this by supposing that each government chooses the (subgame perfect, credible) contracts based on its individual rationality, taking into account the agents' choices at a future stage[21]. First, we provide a general solution method and define the new equilibrium concept in a non-parametric model of policymaking with spillovers. We first show that delegation always occurs in equilibrium when there is strategic complementarity or substitutability. However, we then show that optimal contracts occur in equilibrium only under cooperation at the delegation stage (the same sort of cooperation needed to implement optimal policies): optimal contracts are equivalent to 'cooperative contracts' found by minimizing the global loss function. We then arrive at our main point: subgame perfect, credible contracts are always different from cooperative contracts, and hence also from the optimal ones[22]. We extend the analysis to a model featuring a domestic credibility problem and present an example from international monetary policy cooperation, where it turns out that the contracts governments would actually choose are different in an intuitive way from the optimal ones for any correlation of shocks[23].

Our analysis raises a normative question: what could then insure implementation of the 'right' institutions, preserving the non-cooperative assumption about policymaking? One possible solution consists of creating of a supranational institution that is able to 'coordinate' governments on the optimal contracts at the delegation stage. An alternative would be strengthening the role of some existing supranational institutions (such as the I.M.F.): this reinforces arguments made by Canzoneri and Henderson (1991) in a different setup. There, such an institution helped governments choose, i.e. coordinate on, a best equilibrium among a multiplicity of feasible equilibria. The equilibrium (and hence coordination by the international principal), however, is in terms of policies directly, which seems hard to map into real-life practice. In our context, the supranational institution would help design the appropriate incentive schemes of countries' policy authorities and monitor their implementation over time. While this might seem akin to centralization or cooperation on policies directly (which would be a solution by assumption), we think it is indeed more realistic to assume that national governments agree to coordinate on some institutional features than to systematically pursue cooperative policies that are not consistent with their incentives. By the presence of such an international institution, sovereignty of policymaking is preserved. Further research is needed in order to analyse the incentives and the design of such supranational

[21]It is important to note that once we consider transfer functions we are no longer in the conditions of the Folk Theorem in Delegation Games of Fershtman et al, which dealt with 'take-it-or-leave-it' target compensation functions by which the equilibrium in the agents' game can be made identical to a Pareto optimum. Once we make the transfer functions linear (e.g.), the equilibrium in the agents' game will depend on the delegation in no simple way as this will change their reaction functions. In order to pin down the equilibrium one has to pin down some values for the delegation parameters.

[22]Note that our results are different from McCallum's (1995) critique concerning closed-economy monetary institutions or more specifically enforceability of inflation contracts. Rather than studying enforceability (and sustainability over time) of optimal contracts, we solve for incentive-compatible, credible contracts in an open-economy framework.

[23]The two are identical only if shocks are absent and/or only state-independent institutions are feasible, but the latter case features again inefficiency.

institutions, and mechanisms by which they could implement and monitor globally optimal policy regimes.

References

[1] Benigno, P. 2002, 'A Simple Approach to International Monetary Policy Coordination,' *Journal of International Economics*,' 57(1), 177-196.

[2] Benigno, G. and Benigno, P. 2005, 'Designing Targeting Rules for International Monetary Policy Cooperation', Forthcoming in *Journal of Monetary Economics*.

[3] Bilbiie, F.O., 2000, 'Inflation contracts, targets and strategic incentives for delegation in international monetary policy games', *Working Paper* ECO 2001/16, European University Institute.

[4] Canzoneri, M. and Gray, J.A., 1985. 'Monetary Policy Games and the Consequences of Non-cooperative behaviour', *International Economic Review,* vol. 26, pp. 547-64

[5] Canzoneri, M. and Henderson, D., 1991. '*Monetary Policy in Interdependent Economies: A Game-Theoretic Approach*', MIT Press, Cambridge MA

[6] Cooper, Richard, 1969. 'Macroeconomic policy adjustment in interdependent economies', *Quarterly Journal of Economics*, 83, pp. 1-24.

[7] Cooper, Russell and A. John, 1988, 'Coordinating Coordination Failures in Keynesian Models,' *The Quarterly Journal of Economics*, 103(3), 441-464.

[8] Corsetti, G. and Pesenti, P., 2001. "Welfare And Macroeconomic Interdependence," *The Quarterly Journal of Economics*, 116(2), 421-445.

[9] Dolado, J.; Griffiths, M. and Padilla, J., 1994. 'Delegation in international monetary policy games', *European Economic Review*, 38, 1057-69

[10] Fershtman, C.; Judd, K. and Kalai, E. 1991. 'Observable contracts: strategic delegation and cooperation' *International Economic Review*, 32, 551-9.

[11] Ghosh, A.; Masson, P., 1994. '*Economic cooperation in an uncertain world*', Blackwells, Oxford.

[12] Hamada, K., 1976. 'A strategic analysis of monetary interdependence", *Journal of Political Economy* 84, 677-700.

[13] Jensen, H., 2000. 'Optimal monetary policy cooperation through state-independent contracts with targets', *European Economic Review* 44, 517-539.

[14] Kohler, M., 2002. 'Coalition formation in international monetary policy games,' *Journal of International Economics*, 56(2), 371-385.

[15] Laskar, D., 1989. 'Conservative central bankers in a two-country world', *European Economic Review*, 33, pp. 1575-95

[16] McCallum, B., 1995, 'Two fallacies concerning central-bank independence', *American Economic Review Papers and Proceedings*, 85, 207-211

[17] Obstfeld, M. and Rogoff, K., 1996, *Foundations of International Macroeconomics*, MIT Press.

[18] Persson, T. and Tabellini, G., 1996. 'Monetary cohabitation in Europe', *American Economic Review Papers and Proceedings*, 111-17

[19] Persson, T. and Tabellini, G., 1995. 'Double-edged incentives: Institutions and policy coordination', in Grossman, G. and Rogoff, K. (eds) *Handbook of International Economics*, Vol. III, North-Holland

[20] Persson, T. and Tabellini, G., 2000. '*Political economics: explaining economic policy*', MIT Press

[21] Rogoff, K., 1985. 'International monetary policy co-ordination may be counterproductive', *Journal of International Economics* 18, 199-217

[22] Svensson, L.E.O., 1997. 'Optimal inflation targets, conservative central bankers and linear inflation contracts', *American Economic Review*, 87-1, pp. 99-115

[23] Walsh, C., 1995. 'Optimal Contracts for Central Bankers', *American Economic Review* 85(1), pp.150-67

[24] Woodford, M., 2003, Woodford, M., 2003, '*Interest and Prices*' Princeton University Press.