1 Some Implications of a Variable EIS

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1.0.1 Abstract

The elasticity of intertemporal substitution (the EIS) measures the ease with which a consumer substitutes future for present consumption, other things equal. This important value is often assumed to be constant. Attanasio and Browning (1995) show how a variable EIS works well in a panel consumption study. The present paper examines some implications of a variable EIS for optimal growth, convergence, and poverty traps. Explicit direct utility functions that yield a variable EIS are exhibited. It is shown how the EIS may vary in such a way that the Diamond capital model has an infinity of steady state solutions.

JEL classification E21 O41

The Elasticity of Intertemporal Substitution The Elasticity of Intertemporal Substitution, henceforth the EIS, is a most important variable in macroeconomic theory. Put simply, it measures the willingness on the part of the consumer to substitute future consumption for present consumption. This willingness, naturally, will depend in part on the rate of return to saving, and also on the utility discount rate, and even risk. What the EIS measures, however, is the willingness to save given all these other influences. Without assuming an additively separable and stationary intertemporal utility function, the questions addressed in this paper lose their clarity at least. For this reason we follow the majority of the literature in assuming the form:

$$U\left[c_1, c_2, ..., c_t, ..\right] = \sum_{t=1}^{\infty} \delta^{t-1} U\left[c_t\right] \tag{1}$$

with $0 < \delta < 1$. Now let the consumer maximize (1) subject to the constraint:

$$\sum_{t=1}^{\infty} \pi_t c_t \leq I \tag{2}$$

where the π values are present values of consumptions in the various periods and I is the present value of infinite lifetime income. The short-run rate of return to saving in period t is:

$$\frac{\pi_t}{\pi_{t+1}} - 1 \tag{3}$$

So a fall in π_{t+1}, with everything else constant, represents an increase in the short rate of return at t.

With λ the Lagrange multiplier on the constraint (2), the maximization of (1) requires:

$$\delta^{t-1} \frac{dU\left[c_t\right]}{dc_t} - \lambda \pi_t = 0 \tag{4}$$

Then:

$$\frac{\frac{dU[c_{t+1}]}{dc}}{\frac{dU[c_t]}{dc}} = \frac{1}{\delta} \frac{\pi_{t+1}}{\pi_t} \tag{5}$$

Now with $\rho = \frac{c_{t+1}}{c_t}$, (5) can be written:

$$\ln \frac{dU\left[\rho c_t\right]}{dc} - \ln \frac{dU\left[c_t\right]}{dc} = \ln \pi_{t+1} - \ln \pi_t - \ln \delta \tag{6}$$

To see the effect on ρ of a fall in $\pi_{t+1}$with c_t and π_t constant, totally differentiate (6) to obtain:

$$\frac{c_t}{\frac{dU[\rho c_t]}{dc}} \frac{d^2U\left[\rho c_t\right]}{dc^2} \frac{d\rho}{d\pi_{t+1}} = \frac{1}{\pi_{t+1}} \tag{7}$$

Or,

$$\frac{d\rho}{d\pi_{t+1}} = -\frac{\sigma(c_t)}{\pi_{t+1}} \tag{8}$$

where:

$$\sigma(c) = -\frac{\frac{dU[c]}{dc}}{c\frac{d^2U[c]}{dc^2}} \tag{9}$$

is the EIS evaluated at c.

From (8) it will be seen that a rise in π_{t+1}, which is a fall in the rate of return to saving, lowers c_{t+1} relative to c_t. And this response is larger in absolute value the

larger is $\sigma(c)$. That justifies the identification of σ as the EIS. It is worth noting that σ is simply the inverse of the elasticity of marginal utility:

$$-\frac{c}{\frac{dU[c]}{dc}}\frac{d^2U[c]}{dc^2} \tag{10}$$

This reminds us usefully that it is the curvature of the marginal utility function that facilitates or inhibits intertemporal substitution.

The EIS in Consumption Studies Many applied economists take the view that the value of σ is close to zero. See Hall (1988) and Mankiw, Rotenberg and Summers (1985). This reflects the failure of consumption studies to find a significant effect of the rate of interest on saving. See equation (8) above. Such estimates are seriously biassed if the consumer is capital-market quantity constrained, a feature ignored by the above computations. Or if, as in Deaton (1993), most consumers save only to replenish precautionary balances following negative shocks, the optimizing substitution-based theory does not apply. Recently the fine paper by Attanasio and Browning (1995) has shaken up the entire field. These authors show that representative consumer models give seriously biassed results when applied to aggregate consumption data. They use UK household expenditure data to model consumption at the individual level and obtain a greatly improved fit with the EIS varying with consumption. *The rich have a higher EIS than the poor*

I arrived at the same view concerning the EIS by a completely different route. I am interested in modelling *poverty traps*. The concept of something like a poverty trap is implicit in the Attanasio-Browning view of consumption. If the poor save little because they have a low EIS, and if they have a low EIS because they are poor, they will likely stay poor. This is a long term influence not examined in a short-run cross section. A leading problem with modelling the global poverty trap (as might apply to a whole nation) is that the most popular current growth models, in particular the Solow or Ramsey models promoted by Barro and Sala-i-Martin (1992) and (1995) all lead to asymptotic convergence for all like agents. Then we can look at a weaker notion of the poverty trap - the poor grow slowly because they are poor - or we can switch to a model that allows for a long-run low level steady state. I use the Diamond capital model for this purpose.

Consider another point. If the EIS varies systematically with consumption, is

there an over-arching utility function that produces that feature, and if there is such a function, what does it look like? Attanasio and Browning do not provide an answer to that question. They work with an indirect utility function that they log-linearize for econometric estimation, so we never get to see what the direct utility function looks like, or even whether such a function exists. I work with explicit direct utility functions.

Barro, Sala-i-Martin, and β-Convergence Suppose that all countries solve independent Ramsey model problems:[1]

$$\text{Max} \int_T^\infty U\left[AF\left\{k,1\right\} - \frac{dk}{dt}\right] e^{-\delta(t-T)}dt \tag{11}$$

where A is variable across countries and measures total factor productivity as it is affected by government policy, culture, corruption, etc. The Euler equation implies:

$$-\frac{d}{dt}\left\{U_1 e^{-\delta(\theta-t)}\right\} = U_1 AF_1\left\{k,1\right\} e^{-\delta(t-T)} \tag{12}$$

where subscripts denote partial differentiation. From (12):

$$-\frac{d}{dt}\left\{U_1\right\} e^{-\delta(\theta-t)} + \delta U_1 e^{-\delta(\theta-t)} = U_1 AF_1\left\{k,1\right\} e^{-\delta(t-T)} \tag{13}$$

Or:

$$-\frac{U_{11}c}{U_1}\frac{1}{c}\frac{dc}{dt} = AF_1\left\{k,1\right\} - \delta \tag{14}$$

Then (14) can be written:

$$\frac{1}{c}\frac{dc}{dt} = \sigma\left[AF_1\left\{k,1\right\} - \delta\right] \tag{15}$$

where σ is the EIS.

If the growth rate of consumption is monotonic with the growth rate of output, and because the higher is k the lower is F_1, then conditional β-convergence follows from (15) *provided that all units have equal values of* σ.

An Endogenous σ and Poverty Traps We have seen above that β-convergence can be guaranteed in the Barro/Sala-i-Martin model only if all the representative agents share the same value of the EIS σ. Could it be that β-convergence fails

[1] See Barro and Sala-i-Martin (1992).

because the poor have low σs? This possibility is no mere curiosum. Consider the condition of a poor unit (an individual or a family) or even a nation of such units. Imagine the said agent going to bed hungry each night and struggling to maintain a modicum of dignity in the way it lives. Even in this stark situation provision can be made for the future. If there is no expectation that income will be much better in the future, the only way to generate a small rise in consumption over time is to postpone current consumption and use the small resources released by that action to gain a return by means of which consumption later may be raised.

Is this intertemporal consumption substitution unattractive for the poor because they expect consumption to be higher later, and for this reason marginal utility will be lower later than it is today? That cannot be an explanation for the persistence of poverty, as it assumes that poverty will not persist. Might intertemporal consumption substitution be unattractive for the poor because they discount future utility more strongly than do the better off? That is taken here to mean an endogenous discount rate higher for the poor. Frank Ramsey called the discounting of utility "..a failure of the imagination". So are we to end by saying that the poor suffer from a special kind of feeble-wittedness which weakens particularly their imaginations concerning the future? That type of argument is all too familiar. It involves blaming the poor, however politely, for their poverty. They are too indolent, dim, drunk, whichever feature is at hand to lay on the poor the blame for their own condition.

An unwillingness to save when poor requires no kind of irrationality. While time discounting at variable rates involves problems of time-consistency, it is not obvious that a variable σ raises similar difficulties. One may dislike others' preferences but that does not convict them of irrationality. So should agents happen to have low values of σ at low levels of consumption, that is their business. Given the apparent importance of this question, it is natural to ask what the current growth literature has to say about it. The answer is simple: this issue is always assumed away. Constant elasticity utility functions are commonly employed, so that σ becomes a *constant* no matter where it is evaluated[2].

It is right to feel uncomfortable when economists make simplifying assumptions for analytical convenience. Yet this goes with the territory, everyone does it. The important question is whether too much rides on a particular simplification. Basing

[2]See Barro and Sala-i-Martin (1992), page 64.

the analysis of growth, convergence, and the explanation of poverty on the assumption that σ is a constant involves a huge restriction of the richness of the analysis, and may be fatally misleading.

A Variable σ One thing that may have deterred economic theorists from investigating the consequences of a variable level of σ is that such a feature should ideally emerge from a general overarching utility function. Then the different values of σ would simply express themselves when various budget constraints are presented to the agent. A tight budget constraint (poverty) would yield a low value of σ locally; a slack budget constraint (prosperity) would yield a high value of σ. All cases would represent the same agent with the same underlying preferences, but in different situations. This is directly contrary to an opinion of the author Scott Fitzgerald who wrote: "Let me tell you about the very rich. They are different from you and me". Ernest Hemingway responded acidly to this remark. "Yes, they have more money". Here the very rich are not different, they are just at another point of the utility function, with a far higher EIS.

These features are obtained when the utility function is chosen from a class of which the simplest case is:

$$U[c] = \int_0^c \exp\left\{\frac{1}{\alpha x}\right\} dx \tag{16}$$

where α is a positive constant and c is the level of consumption. We call this function a VEIS utility function, where VEIS stands for *variable elasticity of intertemporal substitution*. Then:

$$\frac{dU[c]}{dc} = \exp\left\{\frac{1}{\alpha c}\right\} \tag{17}$$

And:

$$\frac{d^2U[c]}{dc^2} = -\exp\left\{\frac{1}{\alpha c}\right\}\frac{1}{\alpha c^2} \tag{18}$$

$U[\bullet]$ is an increasing concave function. Now the elasticity of marginal utility may be computed as:

$$-\frac{\frac{d^2U[c]}{dc^2}}{\frac{dU[c]}{dc}}c = \frac{1}{\alpha c} \tag{19}$$

The elasticity of marginal utility decreases with consumption, and its inverse, the elasticity of intertemporal substitution, increases linearly with consumption at rate α. The poor have a lower EIS and β-convergence will not necessarily prevail.

The Oxford economists Godfrey Keller and John Quah pointed out to me that the formula for the EIS is the inverse of the coefficient of relative risk aversion. For this reason adding a linear term to a constant relative risk aversion function gives an increasing EIS function without recourse to the integral equation (16). This point is correct but not inconsistent with the above treatment.

For example, start with:

$$U[c] = \ln c + \gamma c \tag{20}$$

Then:

$$U_1[c] = \frac{1}{c} + \gamma \tag{21}$$

And:

$$U_2[c] = -\frac{1}{c^2} \tag{23}$$

The EIS is:

$$-\frac{U_1[c]}{U_2[c]\,c} = \frac{\frac{1}{c} + \gamma}{\frac{1}{c}} = 1 + \gamma c \tag{24}$$

From (24):

$$-\frac{U_2[c]}{U_1[c]} = \frac{1}{c}\frac{1}{1+\gamma c} \tag{25}$$

Integrating and substituting from the above model into (16) gives:

$$\ln U_1[c] = \ln\left[\frac{1}{c} + \gamma\right] = \int \frac{1}{c\,[1+\gamma c]} dc \tag{26}$$

The derivative of the left-hand side is:

$$\frac{1+\gamma c}{c}\frac{1}{(1+\gamma c)^2} = \frac{1}{c\,[1+\gamma c]} \tag{27}$$

Thius is the derivative of the right-hand side. The two approaches arrive at the same answer.

Optimal Growth with a VEIS function If the utility function is (16) the optimal growth condition can be written:

$$\frac{1}{c}\frac{dc}{dt} = \alpha c\left[AF_1\left\{k,1\right\} - \delta\right] \tag{28}$$

This can be summarized in the form:

- other things equal, the higher is k the lower is the rate of return to saving and the slower is the growth rate of consumption. This is the *Barro effect*.
- other things equal, the higher is c the higher is the growth rate of consumption. This is the *Anti-Barro effect*.
- other things equal, the higher is α, that is the more responsive is the EIS to the level of consumption, the higher is the growth rate of consumption. This is the *Ernest Hemingway effect*.

Notice that the steady-state level of capital per head that is implied by equation (20) is the same as that of the standard Ramsey model with a constant EIS utility function. If c converges asymptotically then so will the EIS if it depends uniquely on c. This is in accord with the carefully worded statement of Barro and Sala-i-Martin (1995, p.64):

> "Equation (2.8) shows that to find a steady state in which r and $\frac{\frac{dc}{dt}}{c}$ are constant, this elasticity must be constant asymptotically."

While the accuracy of these words can only be admired, these authors fail to examine the consequences of variation in the EIS for the transition dynamics that support their empirical studies. With technical progress, endogneous or otherwise, c will not be constant in steady state, and in that case the EIS must converge asymptotically as $c \to \infty$. Even then variation in the EIS is of great potential importance.

1.0.2 The Diamond Capital Model with a Variable σ

Diamond (1965) put capital into the overlapping generations model (OLG model) invented by Allais and Samuleson. A fine exposition and analysis of the model in its many ramifications can be found in De La Croix (2002). In the basic case the consumer lives for two periods. In the first period of her life she supplies 1 unit of labour inelastically and earns the wage rate corresponding to the marginal product

of the capital which the previous generation saved for its retirement. She may also save part of her wage and this becomes the capital saved until the next period. Here zero population growth is assumed. The consumer maximizes lifetime utility which is assumed to be additively separable. Thus the utility of a consumer born in period t is:

$$U\left(c_t^t\right) + \delta U\left(c_{t+1}^t\right) \tag{29}$$

A fundamental theorem for the Diamond Model says that k_t increases with k_{t-1}. Notwithstanding that result, multiple equilibria is a seeming possibility. Can the Diamond capital model offer the perfect theoretical realization of the poverty trap concept? Sadly the multiple equilibrium possibility has never attracted much theoretical interest. It is explained in every textbook that treats the model; yet this case is never developed. It has become a footnote point. Why is this? Probably a major explanation for the unpopularity of the multiple solution case is that it is seldom realised in combination with two highly appealing features:

- stability of steady state solutions of interest
- simple standard functional forms

It is most straightforward to produce multiple steady-states when one of these is the so-called *corner steady state*, see De la Croix and Michell (2002) p.28. This is the case in which the economy has a zero-capital no-activity equilibrium which is locally stable. Usually this case is judged to be uninteresting as there are no examples of zero capital economies in the world, or any such have disappeared and lost their populations. Perhaps the Empty Quarter of Saudi Arabia is a degenerate corner solution economy. If so the theory of its non-activity is not challenging, and in fact it cannot conceivably be seen as a particular solution to a general case embracing otherwise like economies.

To summarize, it is possible to obtain multiple stable steady-state solutions with simple functional forms, but these cases are a minority of all the cases concerned. If the production function is Cobb-Douglas and with a simple separable utility function there are no cases of multiple stable steady states. With a logarithmic utility function and the constant elasticity of substitution in production $\rho > 0$ there can be two positive steady-states, but it may be that only the corner degenerate outcome stable.

1.0.3 A Continuum of Steady states

We put a variable EIS into the Diamond model as follows. Assume:

$$-\frac{\frac{dU[c]}{dc}}{c\frac{d^2U[c]}{dc^2}} = \sigma(c) \tag{30}$$

where $\sigma(c)$ is an arbitrary positive increasing function of c. Then:

$$\frac{\frac{d^2U[c]}{dc^2}}{\frac{dU[c]}{dc}} = -\frac{1}{\sigma(c)\,c} \tag{31}$$

Integrating (31) gives:

$$\ln\frac{dU[c]}{dc} = -\int_0^c \frac{1}{\sigma(x)\,x}dx + \ln D \tag{32}$$

where D is a constant of integration.

In a steady state solution to the Diamond model we must have:

$$\ln\frac{dU[c_1]}{dc} - \ln\frac{dU[c_2]}{dc} = \ln R + \ln\delta \tag{33}$$

where c_1 and c_2 are consumption in respectively the first and second period of a life, R is the gross rate of return to saving, and δ is the discount factor. From (32) and (33):

$$\int_{c_1}^{c_2} \frac{1}{\sigma(x)\,x}dx = \ln R + \ln\delta \tag{34}$$

Now in steady state c_1, c_2 and R all depend upon capital per head k. If over some range of values of k every value gives a steady state, which is the type of case that will be constructed, then (34) will be an identity in k. Let the per capita production function be:

$$6\sqrt{k} \tag{35}$$

Then:

$$c_1 = 3\sqrt{k} - k \tag{36}$$

And:

$$c_2 = k + 3\sqrt{k} \tag{37}$$

Finally:

$$R = 1 + 3k^{-0.5} \tag{38}$$

Differentiating (33) totally with respect to k gives:

$$\frac{1 + 1.5k^{-0.5}}{\sigma(c_2)\left[k + 3\sqrt{k}\right]} - \frac{1.5k^{-0.5} - 1}{\sigma(c_1)\left[3\sqrt{k} - k\right]} = -\frac{1.5k^{-1.5}}{1 + 3k^{-0.5}} \tag{39}$$

By multiplying both sides of (39) by $\left[k + 3\sqrt{k}\right]\left[3\sqrt{k} - k\right]$ we obtain an expression that facilitates computation.

$$\frac{\left[k^{0.5} + 1.5\right]\left[3\sqrt{k} - k\right]}{\sigma(c_2)} - \frac{\left[1.5 - k^{0.5}\right]\left[k + 3\sqrt{k}\right]}{\sigma(c_1)} = -\frac{1.5k^{-1.5}}{1 + 3k^{-0.5}} \tag{40}$$

Or,

$$\sigma(c_2) = \frac{1.5k^{-.5} - 1}{k + 3\sqrt{k}} \frac{\left(9k - k^2\right)\sigma(c_1)}{(1.5k^{-.5} - 1)\left(k + 3\sqrt{k}\right) - 1.5\left(3 - \sqrt{k}\right)\sigma(c_1)} \tag{41}$$

Given a value of k, and if $\sigma(c_1)$ is known for the c_1 value implied by that k from (36), $\sigma(c_2)$ is defined by (41). This indicates that a continuous function $\sigma(.)$ might be constructed such that when the EIS increases with c according to $\sigma(.)$ all values of k on a connected interval are steady-state equilibrium levels.

1.0.4 Computing an h(c) Function for a Continuum of Solutions

The equation (41) seems to hold out the possibility of computing a function $\sigma(c)$ that will imply a utility function for which the Diamond model will have a continuum of steady-state equilibria. But will that approach work, and how should the said function be computed? The details are important. And even if we find the required function will it be of a form that will justify the intuition promoted in this paper, according to which an EIS increasing with consumption is what gives an uncountable multiplicity of solutions?

Here is how we proceed. Choose a particular value for k, selected to give a good result. Here we take $k = 0.1$. Then from (36) and (37) we have:

$$c_1 = 0.84868 \text{ and } c_2 = 1.04868 \tag{42}$$

Now given a value for $\sigma(0.84868)$ we can use (41) to compute $\sigma(1.04868)$. The computation is complex but essentially trivial. The choice of an appropriate value for $\sigma(0.84868)$ is important, as it is not just a normalization, and a poorly chosen value can lead to an $\sigma(c)$ function that lacks the basic features required. It may decrease with c for instance, and can even take negative values.

Take $\sigma(0.84868) = 0.2$. Then $\sigma(1.04868)$ follows from (41) as $\sigma(1.04868) = 0.2521728$. Notice that $\sigma(c)$ has increased with c, at least for that pairwise comparison, which is what we want.

For higher values of k we can compute $\sigma(c_2)$ in a similar manner, but to do that we need to know the level of $\sigma(c_1)$ on the open interval $(0.84868, 1.04868)$. So to complete the computation we have to seed the equation (33) with an initial specification of $\sigma(c)$ on the open interval. The shape of the seeding function will influence the shape of $\sigma(c)$ for $c > 1.04868$, so ideally we should select that function with great skill to obtain a beautiful result. I have some ideas about how to do that, but they must await development of this analysis. For the time being I just take $\sigma(c)$ to be the increasing linear function:

$$\sigma(c) = 0.2 + 0.260864(c - 0.84868) \tag{43}$$

for $0.84868 \leq c \leq 1.04868$.

We can solve for the value of k that gives $c_1 = 1.04868$. This value is approximately $k = 0.1632$. We can use this number to push forward the computation of $\sigma(c)$. Figure 1 shows the function $\sigma(c)$ that results when these calculations are carried out by the program **Mathematica**. The value of c for which σ has been computed is on the horizontal axis, the corresponding value of σ is on the vertical axis. The functional relation is approximately linear, although this surely reflects the small range over which the function has been computed. What matters is that σ increases with c. That justifies the intuitive interpretation of the role of a variable EIS presented above.

These computations are reassuring. In particular we have $\sigma(c)$ increasing with c. That would vindicate the presumption that a flexible EIS may give multiple steady-

state solutions in the Diamond capital model. Of course we do not need a continuum of solutions. But where we can obtain that result it is then clear that multiple isolated solutions is a real possibility.

1.0.5 Concluding Remarks

There is an important question that is not answered either by the Attanasio and Browning paper, or by my own calculations above. The former authors claim rightly that theirs is a long-run study. This is because their panel covers a long run of years (1970-1986). Nonetheless it remains to be determined how precisely the dependence of σ on c operates. Consider the following point. US real per household consumption, was 62.3% higher in 1995 than it was in 1970. As is well known, this comparison is potentially misleading for several reasons. Typical household composition changed greatly over the period. Labour market participation, especially by women altered radically. Finally the shape of the US income distribution was different in the two years. If we ignore all these points, and in particular the last one concerning income distribution, it would follow that a household that consumed in 1995 the median consumption for 1970 would be consuming only 62% of 1995 median real household consumption. That would not be extreme poverty, but it would amount to a severely restricted standard of living. If σ depends only and invariantly on the absolute level of real consumption the implication would be that a household with 62% of 1995 median real household consumption should have an EIS value equal to that of a median household in 1970. In this case long-run growth inevitably raises the levels of σ in the population. Then there would be an upward trend in the average and the typical levels of σ, and with that would come a growing sensitivity of consumption to interest rates. I know of no evidence for such a trend, which is not to say that it is not a fact.

This discussion may seem familiar. It is similar to the old classic debates concerning the consumption function, and particularly the issue of why the long-run propensity to consume seems to be higher than the short-run propensity to consume. If for instance the rich save more than the poor, does that imply that eventually most households will save at a high level as today's poor become tomorrow's well off? The *relative income hypothesis,* Duesenberry (1949) says that this does not happen because saving depends upon relative not absolute income. Then a family that occupies the same rank position in the income distribution will be richer as time goes

forward, but will not save at the same rate as a household that enjoyed its absolute income in the past.

Could the EIS vary with relative consumption rather than absolute consumption? Unlike the relative income hypothesis, the models that have been examined in this paper are strongly micro-founded. That inhibits the insertion of a dependence of individual utility functions upon the consumptions of others. In any case just asking the question shows how radically consumption theory is affected when we allow one little value σ to vary rather than assuming it constant.

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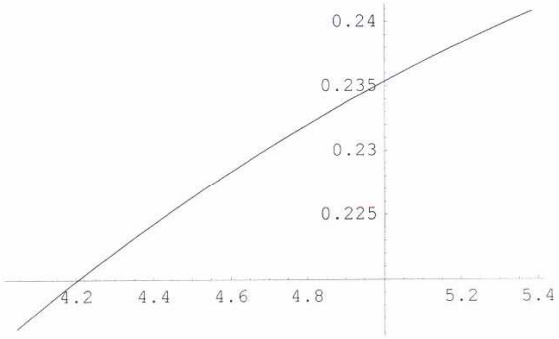

- Graphics -