Macroeconomic Theory I: Growth Theory

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macroeconomic theory course

- These lectures introduce macroeconomic models that have microfoundations. This provides a neoclassical benchmark, with optimising individuals and competitive markets. Of course, incomplete markets and imperfect competition are important phenomena in macroeconomics, but their effects are perhaps best understood as deviations from a well-understood benchmarks.
- In this lecture series we will examine:
  - Growth Theory
  - Investment
  - Competitive Equilibrium (Real) Business Cycles
  - New Keynesian Economics

the Ramsey model

- Ramsey (1928) analysed optimal economic growth under certainty, by deriving the intertemporal conditions that are satisfied on the optimal consumption path that would be chosen by a central planner.
- Intertemporal optimisation is usually analysed by use of a Hamiltonian function. This function maximises the present value of utility over an infinite horizon with respect to a *state* variable, a *control* variable, and a *co-state* variable (the shadow price of an extra unit of the *state* variable in terms of utility). This typically involves two first-order conditions and a transversality condition (to ensure that the *state* variable asymptotically approaches zero).

the OLG model

- Another approach to intertemporal optimisation is not to assume that economic agents live forever, but to assume that they live in overlapping generations, as pioneered by Allais (1947), Samuelson (1958) and Diamond (1965).
  - These models imply that at any one time individuals of different generations are alive and trading with one another, and that future generations may be neglected by current generations.
  - The simplest possible such model, with just two generations alive at any one time, is used extensively in life-cycle consumption models.
- Chapter 2 of Romer (1996) provides a very good introduction to both infinite-horizon and overlapping generations models.

a primer on growth theory

- In the Solow model, growth is exogenous since it is driven by a rate of technical progress that is assumed to be constant.
- In the 1980s, economists became interested in models where growth was endogenous, that is, was explained from within the system.
- To do this, it is necessary to explicitly solve the consumer optimisation problem of the economy - the Solow model omits this by assuming a constant saving rate (although importantly it does allow factor substitution).
- In practice, variables such as saving, human capital formation, and R&D should be endogenous.

the development of growth theory

- Smith (1776), Malthus (1798), Ricardo (1817), Marx (1867)
  - growth falls in the presence of a fixed factor
- Ramsey (1928), Cass (1965) and Koopmans (1965)
  - growth with consumer optimisation (intertemporal substitution)
- Harrod (1939) and Domar (1946)
  - models with little factor substitution and exogenous saving rate
- Solow (1956) and Swan (1956)
  - factor substitution, an exogenous saving rate, diminishing returns
- Arrow (1962) and Sheshinski (1967)
  - growth as an unintended consequence of learning by doing

human capital models

- One-Sector Models
  - with exogenous saving, diminishing returns: the Solow model
  - with exogenous saving and constant returns: the AK model
  - with consumer optimisation
- Two-Sector Models
  - with exogenous saving: similar results to Solow model
  - with consumer optimisation: the Rebelo model
  - with consumer optimisation and no physical capital in education: the Lucas-Uzawa model

human capital models

- What if production is not just a function of labour and capital, but also depends upon human capital?
- Workers can be given the incentive to spend time learning new skills if those skills will receive higher rewards in the workplace.
- Therefore, we can have a perfectly competitive production sector with human capital as an input into production.
- The simplest way to do this is to treat human capital as just another form of capital (i.e. for it to be produced using the same production function as physical capital and output): a one-sector model.
- The harder way to do this is to treat human capital as being produced by a different production function, presumably this production function will itself be relatively intensive in human capital: a two sector model.

a one-sector model (endogenous saving)

- Assume a standard Cobb-Douglas production function with human capital input of H (i.e. the workforce L times the average quality of the workforce, h):

(1) $Y = AK^{\alpha}H^{1-\alpha} = C + I_K + I_H$

- Changes in capital stocks are given by:

(2) $\dot{K} = I_K - \delta K \qquad \dot{H} = I_H - \delta H$

- The equilibrium growth rate of C, Y, K and H can be shown to be

(3) $\gamma^{*} = (1/\theta).[A\alpha^{\alpha}.(1-\alpha)^{(1-\alpha)} - \delta - \rho]$

- Where 1/θ is the inter-temporal elasticity of substitution of utility and θ>0 (when θ is low, households care little about consumption smoothing) and ρ is the rate of time preference.
- It should be apparent that there are constant returns to broad capital in this model, and it consequently behaves like the AK model. All variables grow at the rate of equation (3). Indeed, where nonnegative gross investment is allowed, the model has no transitional dynamics either since if K and H are unbalanced, they adjust discretely to their equilibrium values.

a two-sector model (exogenous saving)

- Once again, consider the human-capital augmented model:

  (4) $Y = K^{\alpha}(hL)^{1-\alpha}$

- Where h is human capital per person. This evolves according to:

  (5) $\dot{h} = (1-u)h$

- Where (1-u) is time spent learning and u is time spent working . Re-writing this shows that an increase in time spent learning raises the growth rate of human capital.

  (6) $\dot{h}/h = (1-u)$

- This models works just like the Solow model where we call A human capital and let g=(1-u). Therefore, in this simple two-sector model, a policy that leads to a permanent increase in the time spent learning leads to a permanent rise in the growth of output per worker.

the Lucas-Uzawa model

- In the full Lucas-Uzawa model, the proportion of time spent learning is endogenous. Consider the following output and human capital accumulation equations:

  $$Y = C + \dot{K} + \delta K = AK^{\alpha}.(uH)^{1-\alpha} \qquad \dot{H} + \delta H = B.(1-u)H$$

- To find the steady-state we look for a solution where u, K/H and C/K are constant. In which case, the common growth rate in steady-state of C, K, H, and Y is

  (7) $\gamma^{*} = (1/\theta)(B - \delta - \rho)$

- If we define the following

  (8) $\varphi \equiv \left[\rho + \delta.(1-\theta)\right] / B\theta$

- Then the steady-state proportion of time devoted to not learning is

  (9) $u^{*} = \varphi + (\theta - 1)/\theta$

dynamics of Lucas-Uzawa

- In general, the growth rate of consumption in the model is
  (10) $\gamma_C = (1/\theta).[\alpha A.u^{(1-\alpha)}(\omega)^{-(1-\alpha)} - \delta - \rho]$
- Where ω=K/H. Notice that (10) is inversely related to (K/H). Hence, the growth rate tends to rise with the amount of the imbalance between human and physical capital if human capital is abundant relative to physical capital ($\omega<\omega^*$) but falls if human capital is relatively scarce ($\omega>\omega^*$).
- The model therefore predicts that the economy recovers faster from a war that destroys physical capital than from an epidemic that destroys human capital (the one-sector model predicts equally fast recoveries from either).
- This is the result of the assumption that education is relatively intensive in human capital. If $\omega>\omega^*$, the marginal product of human capital in the goods sector is high and so are wages. But the education sector is relatively intensive in human capital and therefore has a high cost of operation and low output.

ideas-based growth models

- Linear Knowledge Production Function
  - Romer/Grossman-Helpman/Aghion-Howitt
- Non-linear Knowledge Production Function
  - Jones/Kortum/Segerstrom
- Linear Increasing Variety
  - Young/Peretto/Aghion-Howitt/Dinopolous-Thompson
- Non-Linear Increasing Variety
  - Jones

the Romer model

- One of the most influential new growth models is that of Romer (1990) which stresses the importance of profit-seeking research in the growth process.
- There are three sectors in the full Romer model:
  - A competitive production sector;
  - A monopolistic intermediate-goods sector that produces particular capital goods using designs purchased from the research sector;
  - A monopolistic research sector where inventors race to invent and then receive a patent from the government.
- The aggregate production function exhibits increasing returns since there are constant returns to L and K but ideas, A, are also an input.
- Increasing returns require imperfect competition. Firms in the intermediate-goods sector are monopolists so capital goods sell for more than their marginal cost.
- However, the profits of intermediate-goods firms are extracted by inventors and compensate them for the time spent in inventing.
- There are no economic rents in the model; all rents compensate some factor input.

the basic Romer model

- The aggregate production function takes the familiar form:
  (11) $Y = K^{\alpha}(AL_Y)^{1-\alpha}$
- There are constant returns to labour and capital but the presence of ideas (A) leads to increasing returns overall. Capital accumulates at the rate:
  (12) $\dot{K} = s_K Y - dK$
- And the workforce grows at a constant rate:
  (13) $\dot{L}/L = n$
- Labour is used to either produce new ideas or to produce output
  (14) $L_A + L_Y = L$

ideas-based growth

- Views of the knowledge production function:

(15) $$\frac{\dot{A}}{A} = \delta L_A$$ R/GH/AH

(16) $$\dot{A} = \delta L_A A^{\phi}$$ J/K/S

(17) $$\dot{A} = \delta L_A^{\lambda} A^{\phi}$$

- δ is the productivity of each researcher; L_A is the number of researchers
- ϕ is the returns to the stock of ideas
  - ϕ>0 increasing returns to ideas 'standing on shoulders'
  - ϕ<0 decreasing returns to ideas 'over-fishing'
- λ is the degree of congestion in current research
  - λ<1 'stepping on toes'
  - λ>1 'network externality'

early endogenous growth

Romer/Grossman-Helpman/Aghion-Howitt

(18) $Y = A^{\sigma} L_Y$ There are constant returns to rivalrous inputs and increasing returns to labour and ideas together, $\sigma > 0$.

New ideas are produced using research labour and the existing stock of knowledge:

(19) $\dfrac{\dot{A}}{A} = \delta L_A$ where $L_A = sL$ and $L_Y = (1-s)L$ with $0 < s < 1$.

The growth rate of output per worker is therefore given by:

(20) $g_Y \equiv \dfrac{\dot{Y}}{Y} - \dfrac{\dot{L}}{L} = \sigma \delta s L$

semi-endogenous growth

Jones/Kortum/Segerstrom

(21) $\dot{A} = \delta L_A A^{\phi}$

Where $\phi>0$ increasing returns to ideas and $\phi<0$ decreasing returns to ideas. Note that the R/GH/AH models assume that $\phi=1$. Imposing $\phi<1$, when L grows at rate $n>0$ there is a balanced growth path where:

(22) $g_A = \dfrac{n}{1-\phi}$

and therefore

(23) $g_Y = \sigma g_A = \dfrac{\sigma n}{1-\phi}$

In the case where $\phi=1$, there is no balanced growth path and the presence of population growth leads to explosive growth. Along the balanced growth path (with $\phi<1$), the level of output per worker ($y \equiv Y/L$) is given by:

(24) $y^*(t) = (1-s)\left(\dfrac{\delta(1-\phi)}{n} sL(t)\right)^{\sigma/1-\phi}$

growth with increasing variety

Young/Peretto/Aghion-Howitt/Dinopoulos-Thompson

Suppose that consumption is a constant-elasticity of substitution (CES) aggregate of a variety of goods:

(25) $$C = \left[\int_0^B Y_i^{1/\theta} d_i \right]^\theta$$

where B is the number of different varieties of goods, Y_i is the consumption of variety i and θ>1 is related to the elasticity of substitution between goods. The total number of varieties evolves over time according to:

(26) $$B = L^\beta$$

in Y/P/AH/DT models, β=1 so that the variety of goods is proportional to the population. Under a range of assumptions about Y_i, per capita output is given by:

(27) $$g_c = \theta g_B + \sigma g_A = \theta \beta n + \sigma g_A$$

with the R/GH/AH knowledge production function, the growth rate of A depends on research effect per variety, L_A/B:

(28) $$g_A = \delta s L / B = \delta s L^{1-\beta}$$

substituting this into equation (27) yields:

(29) $$g_c = \theta \beta n + \sigma \delta s L^{1-\beta}$$

the Jones (1999) model

The Y/P/AH/DT models assumed that β=1. The intuition for this is that as the population grows, the number of varieties grows in proportion so that the number of researchers per variety stays constant. Therefore, there is no growth effect of scale.

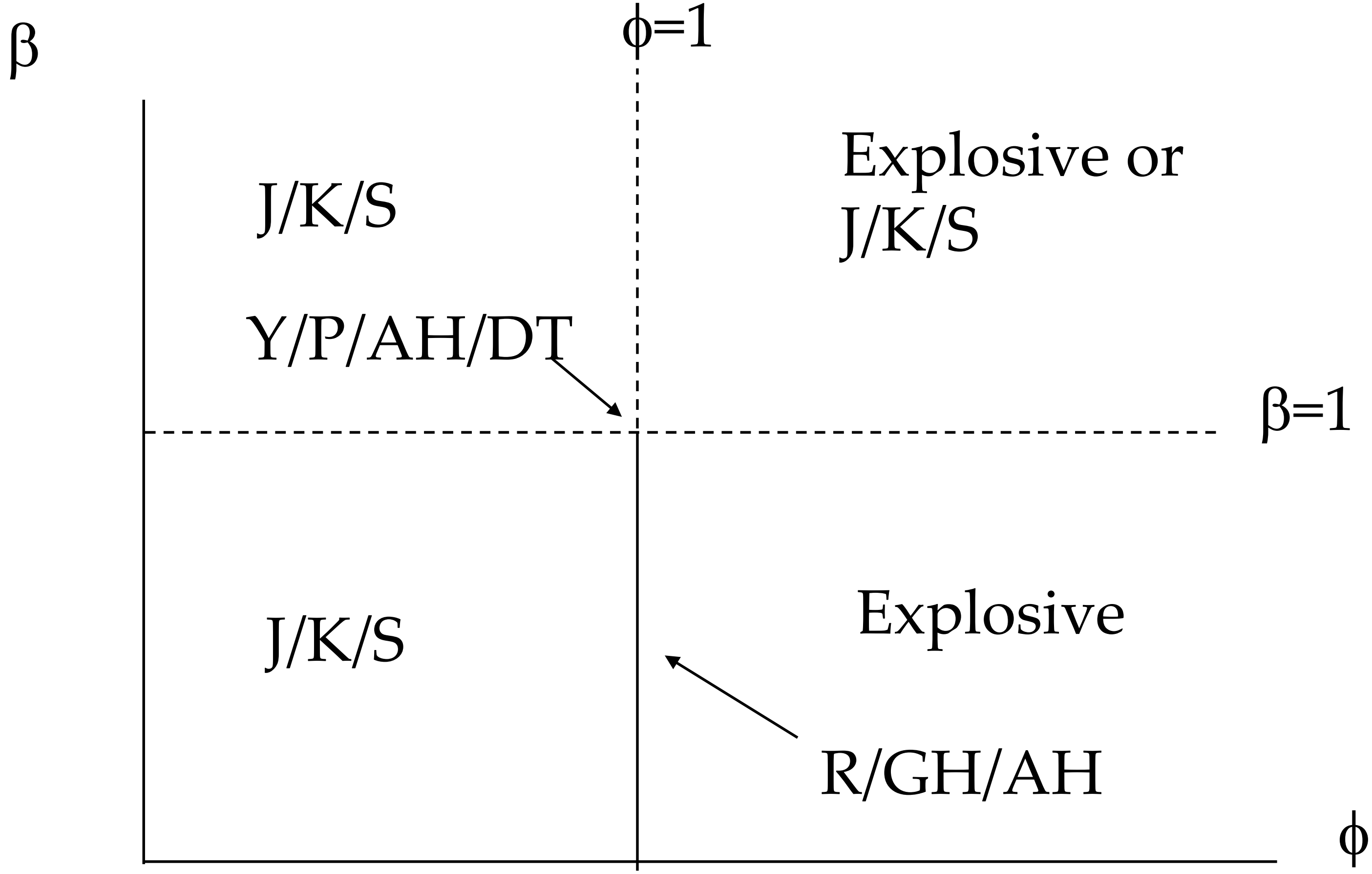
If β<1, there is a growth effect of scale. If β>1, there is a negative growth effect of scale.

In addition, we can also look at the effect of the J/K/S knowledge production function, $\dot{A} = \delta L_A A^{\phi}$, on the growth rate of output in (29):

(30) $$g_c = \theta\beta n + \sigma\delta s \frac{L^{1-\beta}}{A^{1-\phi}}$$

This general models encompasses each of the three cases discussed earlier.

the Jones model



optimal growth

- In human capital models, the investment rate chosen by the representative agent is typically Pareto-optimal.
- In ideas-based models, the investment rate chosen by the social planner is rather different from that chosen by the market. This is due to four distortions:
    - Current research affects the productivity of future research, but this is not rewarded – *standing on shoulders*.
    - Current research may also duplicate existing research and hence lower the productivity of research – *stepping on toes*.
    - Inventors cannot appropriate the entire consumer surplus due to their inventions – *surplus appropriability*.
    - Inventions may reduce the profitability of previous inventions – *creative destruction*.

intuition and growth models

- To generate permanent growth in the absence of population growth (a growth effect of scale), a model must contain a fundamental linearity in a differential equation.
- In the AK model this occurs in the production function and in the Romer model this occurs in the technology equation.
- In the Young-Peretto model this occurs in the growth rate of varieties equation.
- The Lucas model of human capital also contains such a linearity if u is assumed exogenous or when there are constant returns to H and K.

you can download the pdf files from:

http://www.nuff.ox.ac.uk/Users/Cameron/lmh/