Economic Growth I: the 'classics'

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the development of growth theory

- Smith (1776), Malthus (1798), Ricardo (1817), Marx (1867)
  - growth falls in the presence of a fixed factor
- Ramsey (1928), Cass (1965) and Koopmans (1965)
  - growth with consumer optimisation (intertemporal substitution)
- Harrod (1939) and Domar (1946)
  - models with little factor substitution and an exogenous saving rate
- Solow (1956) and Swan (1956)
  - factor substitution, an exogenous saving rate, diminishing returns

Kaldor's stylised facts

- Per capita output grows over time and its growth rate does not tend to diminish;
- Physical capital per worker grows over time;
- The rate of return to capital is nearly constant;
- The ratio of physical capital to output is nearly constant;
- The shares of labour and physical capital in national income are nearly constant;
- The growth rate of output per worker differs substantially across countries.

international labour productivity

	1820	1870	1890	1913	1929	1938	1950	1960	1973	1987	1998
	UK=100			USA=100							
USA	83	96	99	100	100	100	100	100	100	100	100
Japan	31	18	20	18	22	23	15	20	45	60	68
Germany	62	48	53	50	42	46	34	52	73	91	106
France	80	54	53	48	48	54	42	51	74	99	102
Italy	58	39	35	37	35	40	38	46	78	96	100
UK	100	100	100	78	67	64	58	57	68	81	82
Canada	..	62	63	75	66	58	68	72	75	83	80

Source: Madison (1991) and OECD

Note: Labour Productivity is defined as GDP per man-hour

neo-classical production functions

- Consider a general production function

  (1.1) $Y = F(L, K)$

- This is a neo-classical production function if there are positive and diminishing returns to K and L; if there are constant returns to scale; and if it obeys the Inada conditions:

  $f(0) = 0;\ f'(0) = \infty;\ \lim_{k \to \infty} f'(k) = 0$

- with CRS, we have output per worker of

  (1.2) $Y / L = F(1, K / L)$

- If we write K/L as k and Y/L as y, then in *intensive form*: $y = f(k)$

  (1.3)

Cobb-Douglas production I

- One simple production function that provides a reasonable description of actual economies is the Cobb-Douglas:

  (1.4) $Y = AK^{\alpha}L^{1-\alpha}$

- where $A > 0$ is the level of technology and α is a constant with $0 < \alpha < 1$. The CD production function can be written in *intensive form* as

  (1.5) $y = Ak^{\alpha}$

- The marginal product can be found from the derivative:

$$MPK = \frac{\partial Y}{\partial K} = \alpha AK^{\alpha-1}L^{1-\alpha} = \alpha\frac{AK^{\alpha}L^{1-\alpha}}{K} = \alpha\frac{Y}{K} = \alpha APK$$

Cobb-Douglas production II

- If firms pay workers a wage of w, and pay r to rent a unit of capital for one period, profit-maximising firms should maximise:

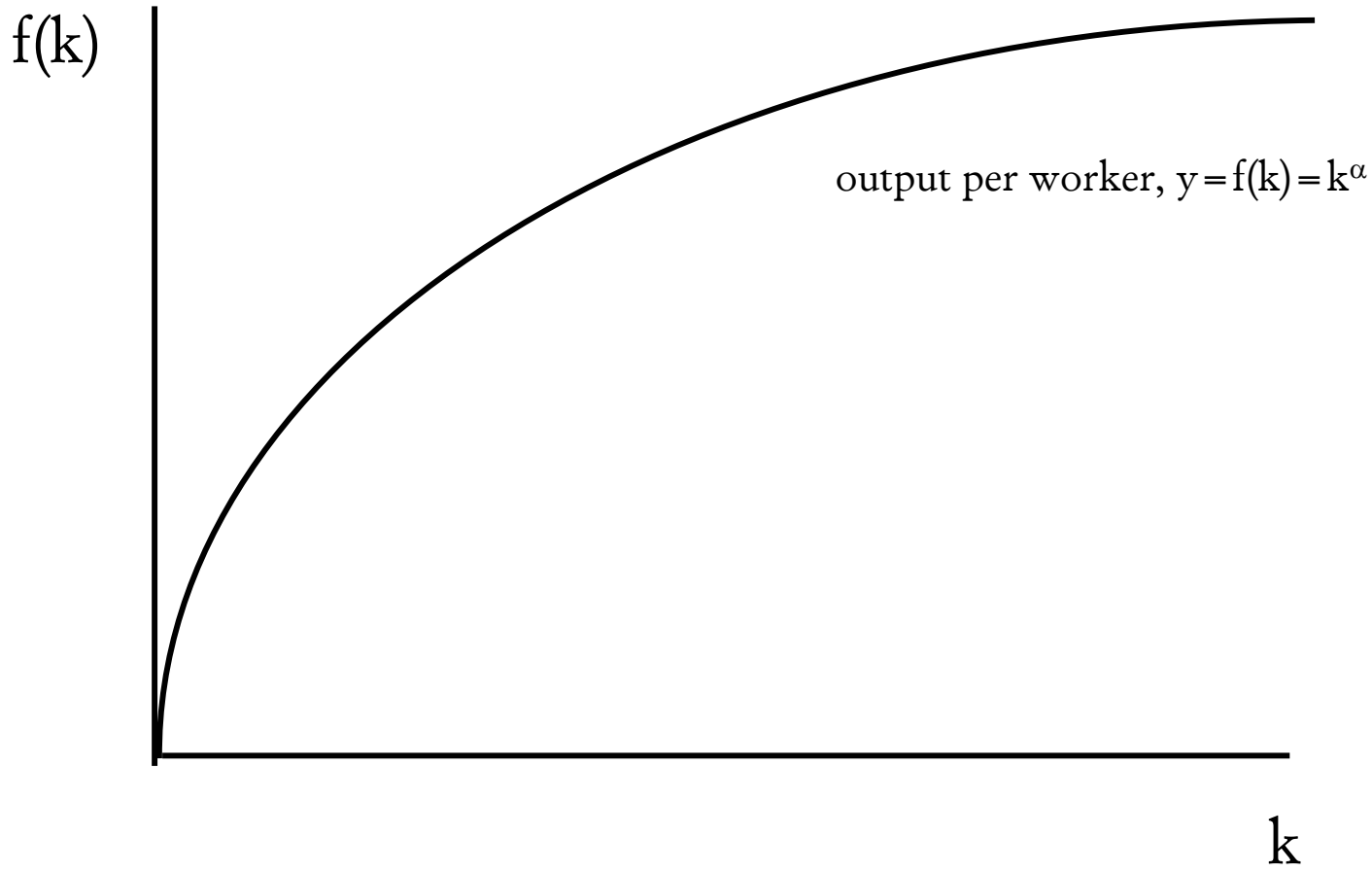
$$\max_{K,L} F(K,L) - rK - wL$$

- Under perfect competition firms are price-takers so they employ workers and rent capital until w and r are equal to the marginal products of labour and capital

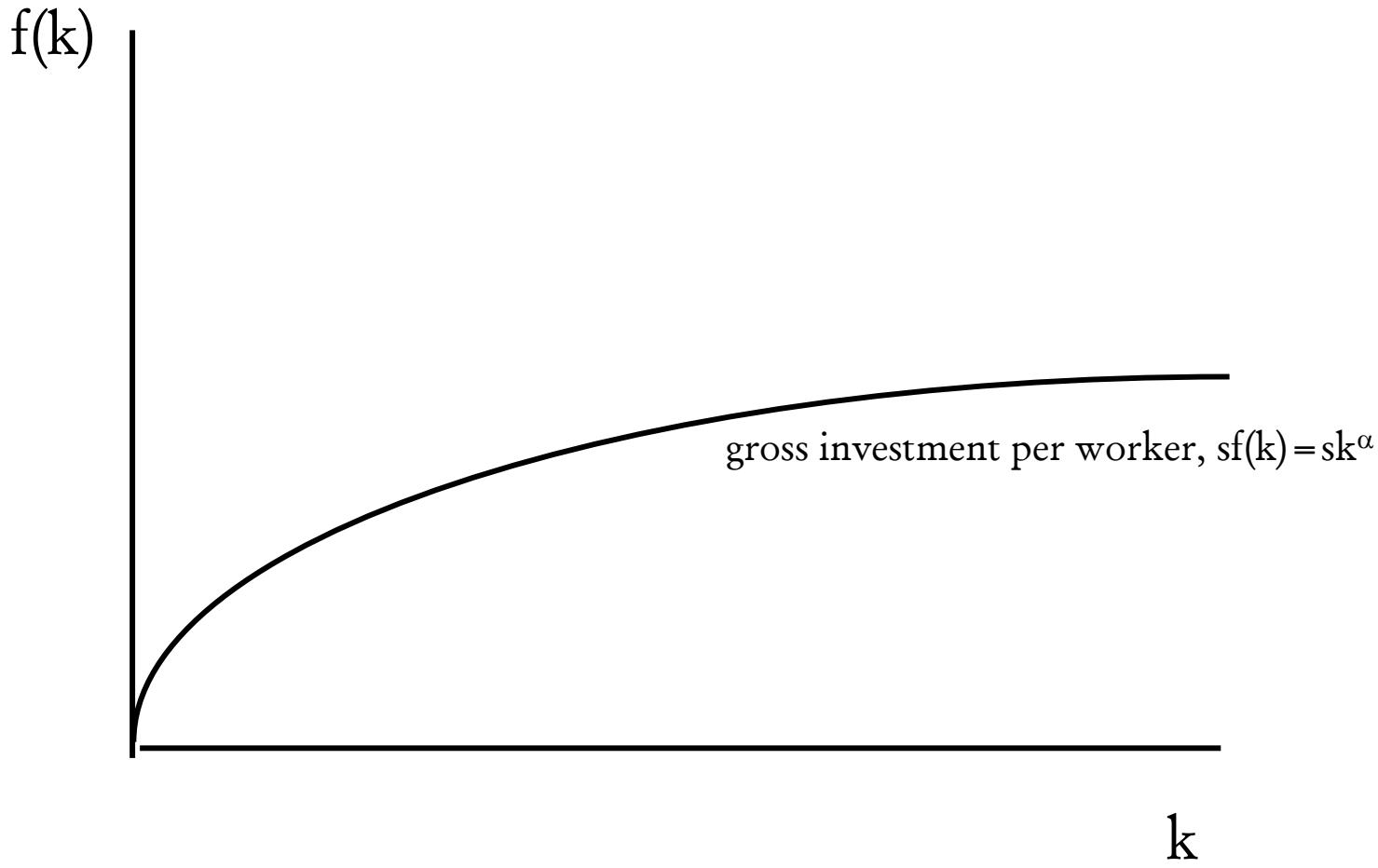
$$w = \frac{\partial F}{\partial L} = (1-\alpha)\frac{Y}{L};\ r = \frac{\partial F}{\partial K} = \alpha\frac{Y}{K}$$

- Notice that $wL + rK = Y$, that is, payments to inputs completely exhaust output so economic profits are zero.

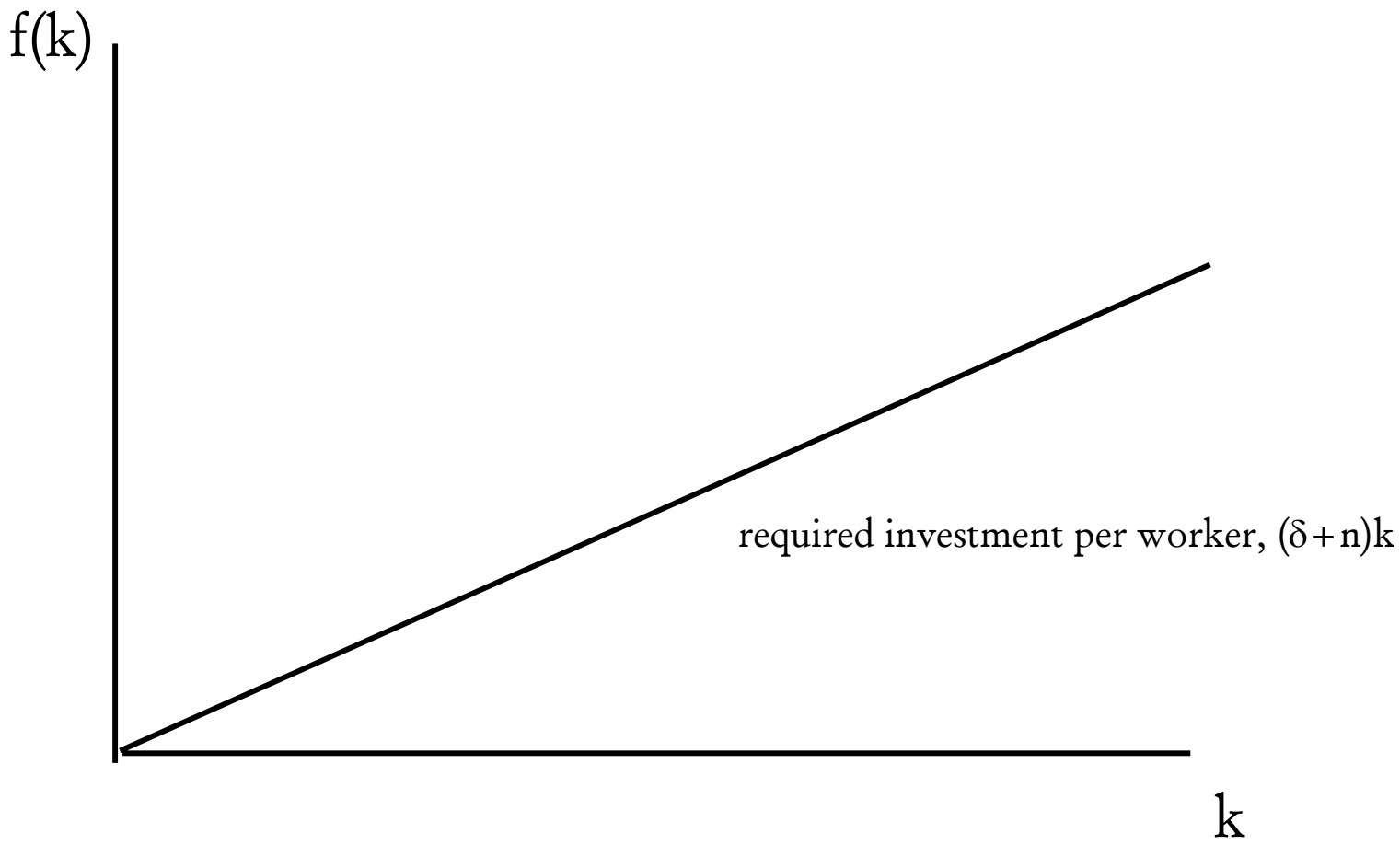
diminishing returns...



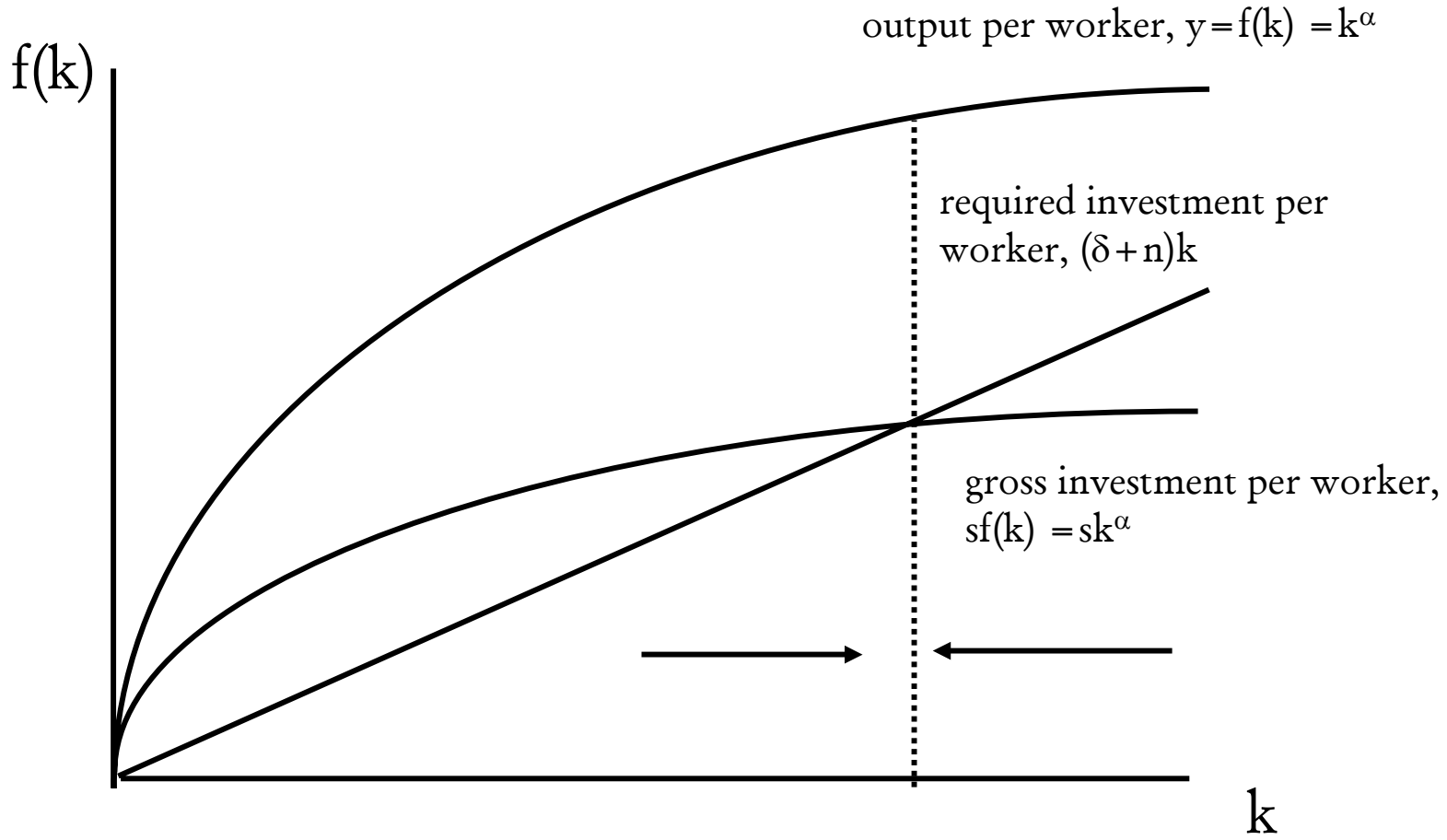
...saving a constant fraction of income...



...and a constant depreciation rate



...the Solow model



Solow model analysis

- The economy accumulates capital through saving, but the amount of capital per worker falls when capital depreciates physically or when the number of workers rises.
- Saving per worker is

  (1.6) $S/L = sY/L = sy = sk^{\alpha}$
- Depreciation per worker is a function of the capital stock

  (1.7) $(n+d)K/L = (n+d)k$
- In equilibrium, the capital stock will be constant when saving per worker equals depreciation per worker

  (1.8) $\dot{k} = sk^{\alpha} - (n+d)k$

steady-state capital and output

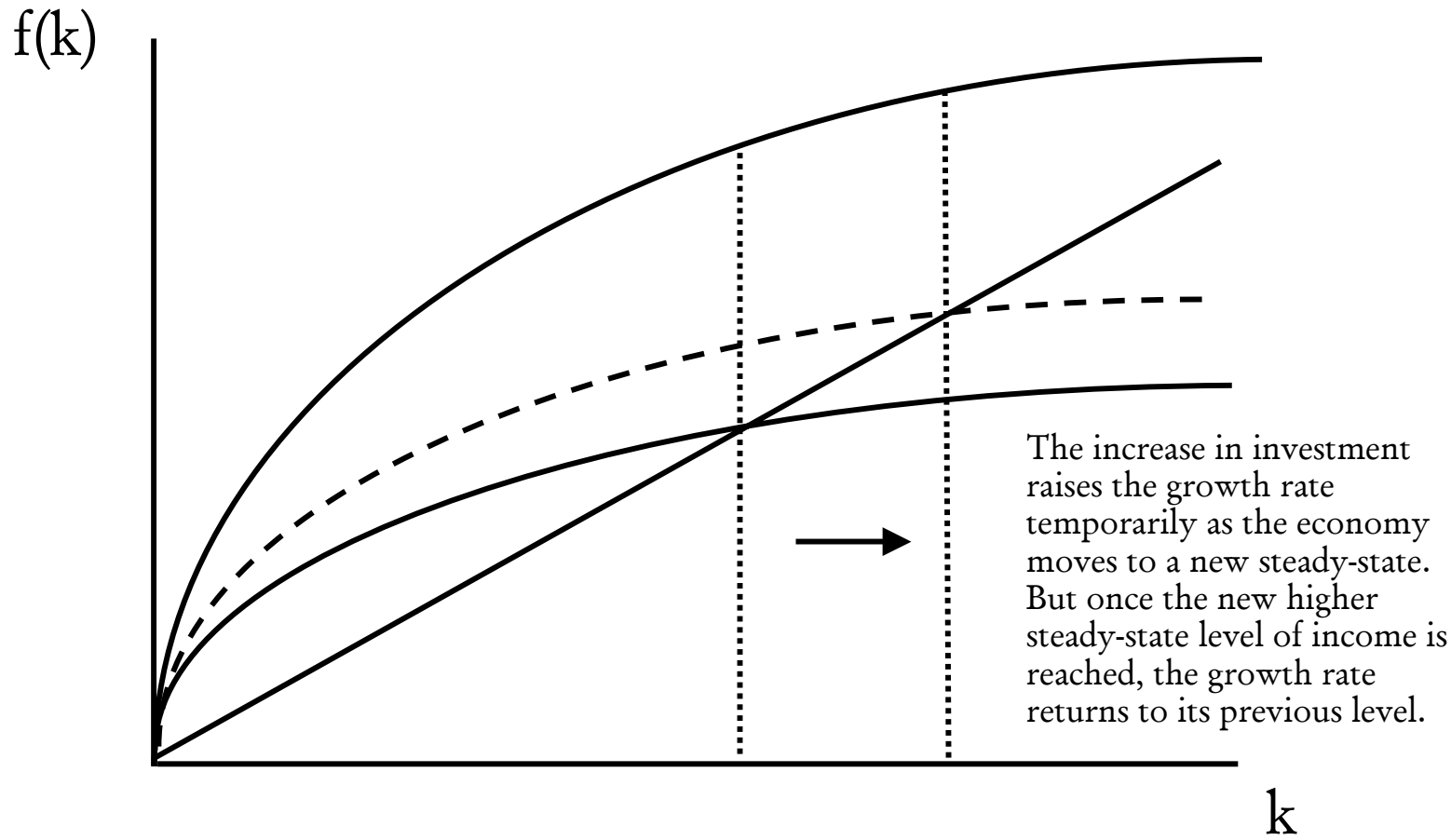
- Setting equation (1.8) to zero yields

(1.9) $$k^* = \left(\frac{s}{n + d} \right)^{1/(1-\alpha)}$$

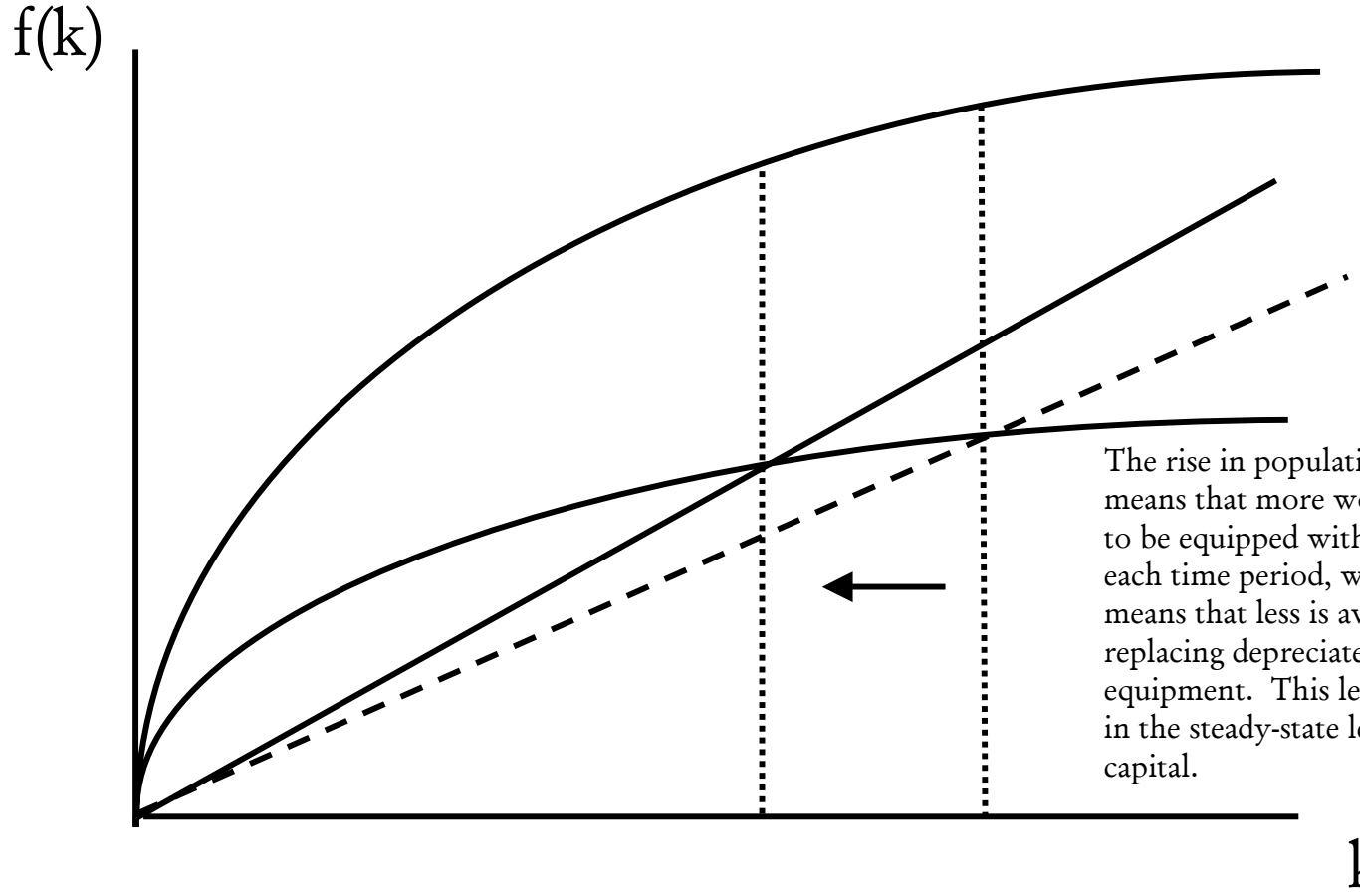
- Substituting this into the production function reveals the steady-state level of output per worker:

(1.10) $$y^* = \left(\frac{s}{n + d} \right)^{\alpha/(1-\alpha)}$$

higher saving

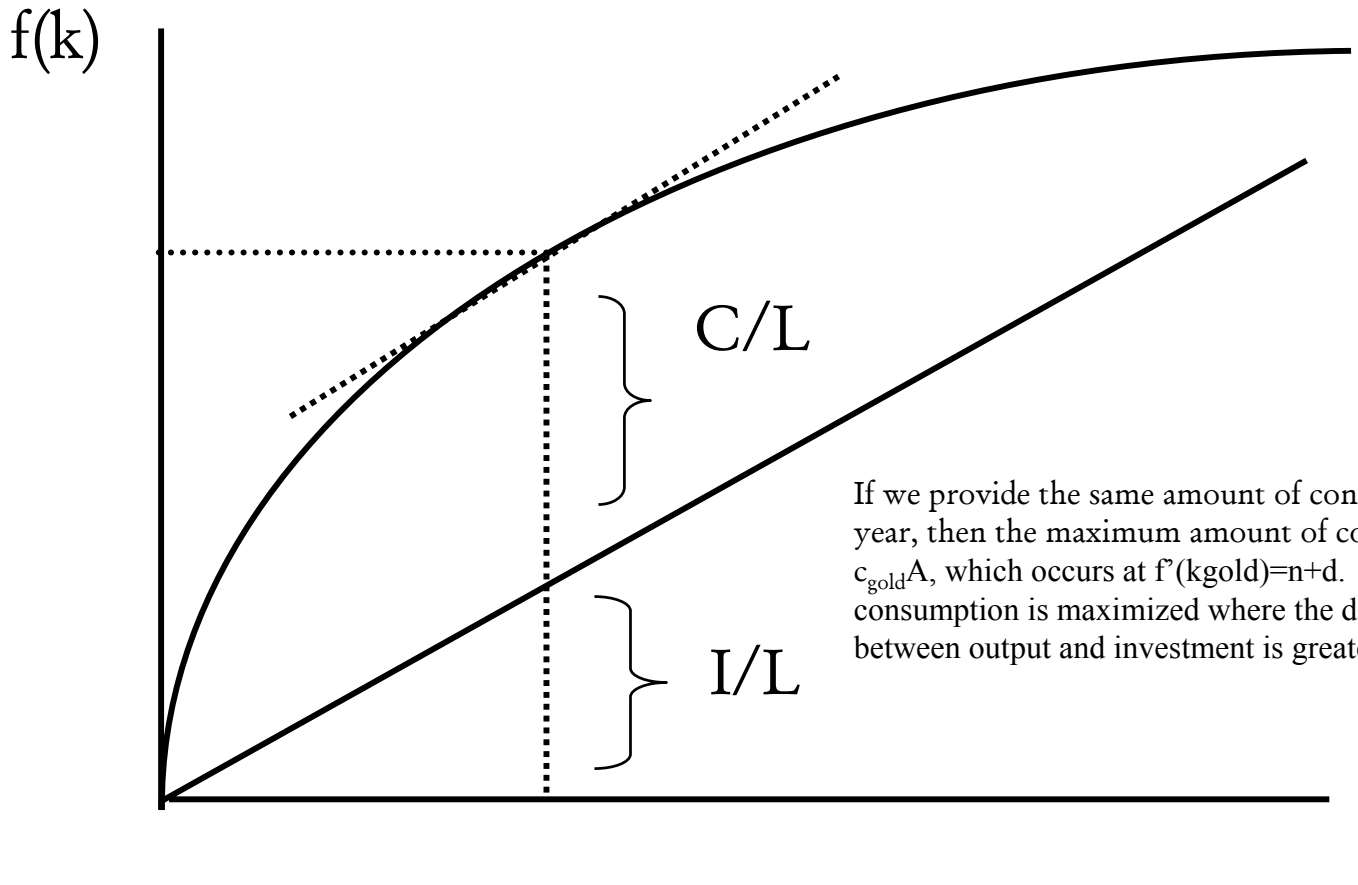


faster population growth



The rise in population growth means that more workers need to be equipped with capital each time period, which means that less is available for replacing depreciated equipment. This leads to a fall in the steady-state level of capital.

the Golden Rule



If we provide the same amount of consumption every year, then the maximum amount of consumption is $c_{gold}A$, which occurs at f'(kgold)=n+d. Aggregate consumption is maximized where the difference between output and investment is greatest.

summary

- The study of economic growth has a long history, with major contributions from Adam Smith, David Ricardo and even Karl Marx.
- These early theories failed fully to take into account the important roles played by factor accumulation and substitution and technical progress.
- To recap, the Solow model model has two main predictions:
  - For countries with the same steady-state, poor countries should grow faster than rich ones.
  - An increase in investment raises the growth rate temporarily as the economy moves to a new steady-state. But once the new higher steady-state level of income is reached, the growth rate returns to its previous level – there is a levels effect but not a growth effect.

looking ahead

- The Solow model is an advance on earlier models since it allows for factor substitution and accumulation.
- However, saving and technical progress are still exogenous.
- In lecture two we will examine the role of exogenous technical progress in the Solow model.
- In lecture three we will examine endogenous technical progress.

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