Abstract

"Creative destruction" refers to the way that economic advances make existing economic capital and ideas obsolescent: they are partially destroyed (in value terms). Existing models have only considered how additions to economic knowledge make existing knowledge less valuable. This paper recognises that both knowledge and capital obsolesce and considers the effect on both.

Knowledge, Physical Capital and Creative Destruction

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1. Introduction

One strand of the endogenous growth literature models technological change as determined by economically motivated agents doing "research", *e.g.* Romer [1990]. "Innovation" might be a better word than "research", since it lacks the highly academic overtones of the latter, but I shall continue to follow the practice of referring to "research". The result of research is either to increase the range of goods available or to make the existing goods better or cheaper to produce. In either case, given suitable technical conditions, output grows exponentially.

Aghion & Howitt [1992] observed that such processes could be referred to as "creative destruction", since the arrival of the new technologies reduced the value of the old. The phrase was drawn from Schumpeter [1942], who clearly saw this process occurring within a business cycle. It is possible, however, to ignore the business cycle part of the story and concentrate just on the trend in output, thus avoiding the extent to which business cycles are "real". So far as the trend in output is concerned, creative destruction has two effects upon growth: first, researchers realise that the value of their current research will be reduced by future research and are discouraged from doing research now; second, researchers do not take account of the fact that the rewards they receive from research are less than the societal gain and may do too much research. One corollary of this is that there may be paradoxical comparative dynamics. For example, consider two economies, identical in all respects except that research is easier in the second. The second economy is thus “better off”, but may have a lower growth rate: because potential researchers know that research is easier in their economy, they also know that the value of their discoveries will obsolesce more quickly and they refrain from research.

The rôle of capital in this process is small. Romer [1990] has durable intermediate goods, but they do not improve in quality: technological progress is achieved by variety. The durability of the intermediates is not an essential part of the model anyway and non-durables are used in similar models in Grossman & Helpman [1991] and Barro & Sala-i-Martin [1995]. Aghion & Howitt [1992] model technological progress by better intermediates rather than a greater variety, but their intermediates are non-durable.

If, however, we assume that intermediates are durable, we can consider more carefully what is being destroyed in creative destruction. Does the main burden of increased research activity fall on research activity or on investment in durable intermediate goods? Consider again the two economies identical except that research is cheaper in the second. We have already seen that the second economy might have lower growth. But there is now an additional consideration, because the growth rate will affect the speed at which physical capital obsolesces and hence the incentive to invest in physical capital. In some cases it is the destruction of the physical capital which deters research, not of the knowledge.

Note that the effect of physical capital on research is purely economic in this scenario. There are potentially two routes by which capital affects research-led growth. First, there may be a technical relationship, so that the existence of physical capital makes the process of research easier, for example King & Robson [1993]. The second is that the quantity of capital affects the incentives to do research. In this paper we consider only the second of these possibilities.

We structure of the model follows Aghion & Howitt [1992], in that research raises the productivity of an intermediate good in final output production. Unlike Aghion & Howitt we assume that the intermediate good is durable: physical capital. We also make some simplifying assumptions, namely that there is a continuum of production processes arriving at a constant rate, rather than a discrete set of such processes arriving stochastically. The result is a vintage capital model along the lines of Solow, Tobin, von Weizsäcker & Yaari [1966], but with technological progress endogenous.

The interest in the model lies in deepening the concept of "creative destruction" suggested by Schumpeter. Creative destruction means that the creation of new ideas or capital leads to the value of existing ideas and machines falling. Producers of ideas (researchers) and capital anticipate that this will happen in the future: the expectation of greater research effort or capital accumulation in the future discourages research effort or capital accumulation now, since the value is reduced by obsolescence. In extreme cases, this effect can be so strong that research is lower because of (correct) worries concerning the future.

We shall show that this concerns about obsolescence affect the research and the growth rate through both obsolescence in knowledge and capital, and that these two effects can move in opposite directions.

2. Assumptions

2.1. Production of the final good

Final output consists of one "representative" consumer good. Economic growth arises from more of this good being produced: there is no increase in the range of goods available. For simplicity, I shall assume that increases in productivity are achieved solely by giving workers a durable intermediate good which embodies better technology. We shall refer collectively to this durable as (physical) capital. The amount of capital used by a worker will be called "a machine" (this may consist of more or less than one machine when the word is used in the normal sense) and every worker must use one machine: the production technology has fixed coefficients[1].

When an intermediate good embodies knowledge which only became available at time t, the worker and intermediate good together will produce $y(t)$ units of output. Total economy output is just the integral of the output produced by intermediate goods of each vintage: if the density of machines in existence at t of vintage a is $f(t, a)$ and the output of each of these machines is $y(t-a)$, then

[1] In fact, if the ratio of the wage to the user cost of capital for each vintage is constant, then we only need assume that the technology is putty-clay, not clay-clay.

total final output is

$$Y(t) = \int f(t, a)\, y(t - a)\, da.$$

In the steady state the composition of the capital stock (*i.e.*, the values of $f(a)$) will not change over time and $Y(t)$ will be a constant multiple of $y(t)$. Thus the growth in output of the final good will be equal to the growth in the productivity $y(t)$. The value of research and intermediate good production will also be growing at this rate and hence the growth rate in the economy's total output will be the same as the growth rate in the productivity of the intermediate good.

2.2. Labour Market Clearing

Since labour is homogeneous, labour market clearing simply requires that the numbers of workers in each sector sum to the total labour supply. We will write the total number of researchers as L, the total number of machine or intermediate good producers as M and the total number of workers producing the final good as N: when the total labour supply is Λ, then labour market clearing is just

$$\Lambda = L + M + N. \tag{2.1}$$

2.3. Technology & Knowledge

Economic knowledge is modelled as an endless and continuous list of production processes. The productivity of processes increases exponentially as one moves along the list. The more resources devoted to research, the faster these production processes are discovered, the greater the density of production processes being discovered and hence the faster the growth rate. We will impose a normalisation that the density of discovery is equal to the growth rate in productivity and thus relate the growth rate and the number of workers doing research as[2]

$$g = \upsilon\psi\left(L\right), \quad \psi\left(0\right) = 0, \quad \psi' > 0, \tag{2.2}$$

where ψ is invertible. We introduce υ as a simple means to consider changes in the productivity of research: if υ increases, productivity growth rises.

Researchers work independently because there are no benefits to working together. They do, however, benefit from external economies of scale, so their

[2] A more general specification would be

$$g = \Psi\left(L, M, N\right),$$

since there might be economies of scope between discovering knowledge and the other sectors of the economy.

productivity depends upon the total number of researchers. Discovery of productive processes is deterministic and, because researchers are identical, they each discover new knowledge at the same rate, which is thus

$$\frac{g}{L} = \frac{g}{\psi^{-1}\left(g/v\right)}. \tag{2.3}$$

Whether the rate at which an individual researcher discovers knowledge increases or decreases with g depends upon the second derivative of ψ:

$$\text{sign}\left[\frac{d\left\{g/L\right\}}{dg}\right] = \text{sign}\left[\psi''\left(L\right)\right]$$

Researchers set up companies which produce intermediate goods embodying the technology that the researcher has just discovered. These firms employ workers to produce intermediate goods and generate profits. The researcher receives the present value of the profit stream generated by the firm. These profits are "normal" profits, since the present value of the profits generated by profits must be just sufficient to prevent workers leaving the research sector.

2.4. The Interest Rate

We will assume that the interest rate, r, may be influenced by the growth rate according to the equation

$$r = \rho + \sigma g, \quad 0 \leq \sigma \leq 1, \quad \rho > 0, \tag{2.4}$$

where ρ and σ are constants. These processes could be generated by intertemporal utility maximisation by consumers, where the maximand is

$$U(t) = \int_0^\infty e^{-\rho s} \frac{C_t^{1-\sigma} - 1}{1 - \sigma} ds, \tag{2.5}$$

although other interpretations are also possible.

2.5. Expectations

There is no stochastic element in the model, so no probabilistic form of expectations is required. We analyse a steady state growth path with perfect foresight, so that the growth rate and depreciation are constant and fully known and acted upon by all agents.

3. Solving the Steady State

3.1. Machinery

Since physical depreciation is being ignored *a priori* for simplicity, all depreciation is a reduction in the value of capital, arising from economic events elsewhere in the economy. In a model with only a representative good, this is represented entirely by the increase in the wage paid to labour[3], which squeezes the gross profits earned by the owner of the machine.

Eventually the wage will rise to the point at which machines of a given vintage will be retired from production. We will refer to the age of the knowledge embodied in these machines as the retirement age, R. If machines always embodied the latest knowledge when they were produced, then the machines would also be R old when they were retired. We will, however, wish to allow the possibility that machines are produced which do not embody state-of-the-art knowledge. These machines will still be retired from production when the knowledge that they embody is R old, although the machines themselves will be younger.

[3]In an economy with many goods, capital could obsolesce for reasons other than wage increases, such as a fall in the demand for a good arising from cheaper substitutes. It is also possible that the value of some capital could rise, if technological advances made complements cheaper, but this would require horizontal differentiation.

We assume that final output is produced by perfectly competitive firms and that there are no economies of scale to either the firm or the industry as a whole. Thus the number and size of the firms is irrelevant so long as there are sufficiently many to maintain perfect competition. Because the industry is perfectly competitive, the output of a firm will be absorbed entirely by the inputs. The machine is paid for at the beginning of its life, the worker on a period by period basis. The machine will be retired at the point when the wage of machine operators, $w(t)$, just equals the output generated by the machine. From the definition of R, this output will be the output generated by a machine R units of time ago, which we will denote $y(t-R)$. Hence the wage is defined in terms of the retirement age

$$w(t) = y(t-R). \tag{3.1}$$

The amount that a firm is prepared to pay for the machine will be the present value of the flow of output over and above wages, gross profits. Gross profits will depend upon how much the wage lags behind the output of a machine. Let a denote how long the knowledge embodied in a machine has been available: a is the age of the knowledge (but not necessarily of the machine). A machine which embodies knowledge which is a old produces a constant flow of output $y(t-a)$

and the firm has to pay a wage $w(t)$, which is rising over time, so the gross profits generated by the machine is

$$\pi^m(t,a) = y(t-a) - w(t), \tag{3.2}$$

so long as $y(t-a) \geq w(t)$; otherwise the machine would be retired from production. When both the retirement age and the growth of productivity of machinery are constant, which they will be in the steady state, then $y\,(t-R) = y\,(t)\,e^{-gR}$, so, using equation (3.1), we can write the machine's output as

$$y\,(t-a) = w\,(t)\,e^{g(R-a)}. \tag{3.3}$$

The present value of the profit stream of a machine which embodies technology of age a is

$$V(t,a) = \int_0^{R-a} e^{-rs}\pi^m\,(t+s, a+s)\,ds, \tag{3.4}$$

which, after appropriate substitution is

$$V\,(t,a) = w\,(t)\int_0^{R-a} e^{(g(1-\sigma)-\rho)s}\left\{e^{g(R-a-s)} - 1\right\}ds \equiv w\,(t)\,\xi(R-a, g; \rho, \sigma). \tag{3.5}$$

We introduce the notation ξ because the solution to the integral is insufficiently tractable to allow an analytic solution to the model. We can, however, show that ξ is zero when $a = R$ (since that is the point when the machine is obsolete) or $g = 0$, in which case gross profits are zero because the wage does not lag behind machine productivity. Except in these cases, it is an increasing and convex function of both the retirement age and the growth rate and a decreasing function of ρ[4]. These results are intuitive: the longer a machine will be in operation, the more profit it will generate; the faster the growth rate, the more wages lag behind output; the higher the value of ρ, the higher the interest rate and the more profits are discounted.

A firm producing final output will be prepared to pay up to V for a machine. Notice that this is not the user cost of capital, which is a flow concept, but corresponds more closely to the list price of a machine or the prices implied by the price deflator for Gross Domestic Fixed Capital Formation in the national accounts[5].

[4]Note that gross profits are zero at $R-a$, so by Leibniz's rule we do not need to worry about changes in the upper limit to the integral. The integrand is clearly an increasing and convex function of R; these properties follow for g also because $1-\sigma$ and $R-a-s$ are both (weakly) positive, by assumption and definition respectively.

[5]A problem remains in that we have normalised the capital to be the amount of capital per worker: thus to obtain an estimate of V from the national accounts we would also need to take account of the number of workers expected to use the additional capital created, and the hours

3.2. Machine Production

To produce machines, firms only need workers and the knowledge in the patent. Given access to the relevant knowledge, the relationship between the number of workers and the number of machines produced is the same for all vintages, namely

$$f_p(t,a) = \eta\phi(M(t,a)), \quad \eta > 0.$$

The parameter η represents the level of productivity in this sector, $f_p(t,a)$ is the density of machines of vintage a in production at time t and $M(t,a)$ is the number of corresponding workers. The functional form of the production function is ϕ, which displays diminishing returns at some point. Instead the properties of ϕ, it will be more convenient to consider the average cost curve of the firm. We consider three possibilities, as shown in Figure 3.1.

Recall that each vintage of intermediate good is produced by one firm alone, which controls the legal rights to the particular knowledge which will be embodied in its intermediate goods. The firm does not allow other firms to use that knowledge and maximises profits based on its own output. Although the firm has

that they would spend operating it. If we could take these factors, together with changes in g and R into account, equation (3.5) implies that the price deflator of capital should move in a similar fashion to the wage.

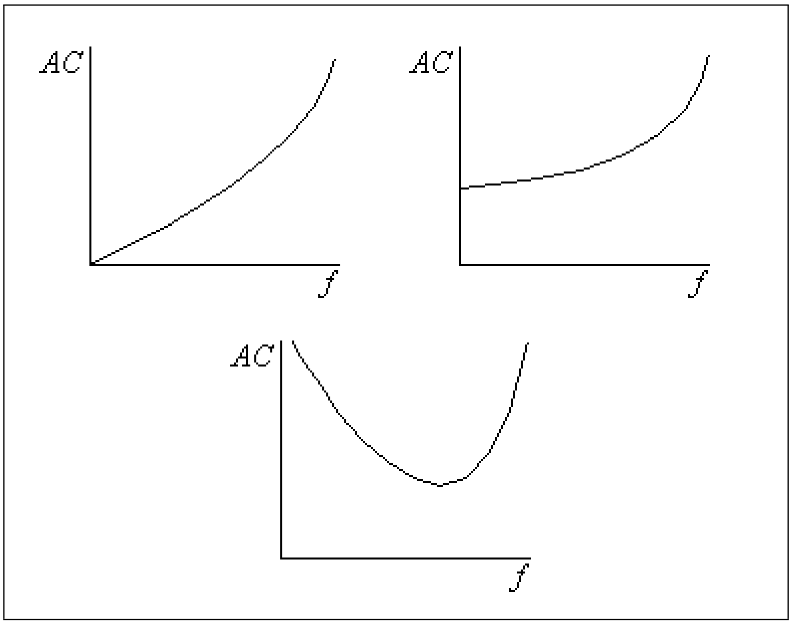

Figure 3.1: Average Cost Curves

a monopoly over its particular vintage, it has no market power, because all of the vintages are in competition with each other: no firm can charge a price higher than the value of its intermediate good because buyers can always substitute into other vintages at an appropriate price[6]. Thus the firm faces a perfectly elastic demand curve whose position is determined by the embodied knowledge of the intermediate good it is selling. Since the firm also takes the wage as given, out-

[6]An analogy might be the term structure of interest rates (in its (rational) expectations hypothesis form), where prices adjust to ensure that bonds of different terms are, in effect, substitutes.

put will be determined by price equals marginal cost, subject to price exceeding average cost. As other innovations occur, the price and hence profit will fall.

If the average cost curve is like the first example in the diagram, the firm will continue to produce until the price falls to zero, which will occur when the age of the knowledge it is using (a) is R old. In the other two cases, production will cease when the price is positive, implying that the age is less than R: we shall call the age of the knowledge at the moment when it stops being used Q. We shall not consider the first case in any detail, since the qualitative results, other than $Q = R$, are the same as for the other cases.

Suppose that, at time t, a firm is selling intermediate goods incorporating knowledge whose obsolescence is a. Suppose that the firm pays a wage $w(t)$ to workers producing intermediate goods. When it produces f_p intermediate goods, its profits are

$$w(t)\,\pi(t,a) = V(t,a)\,f_p(t,a) - w(t)\,M(t,a) = w(t)\left\{\theta\xi f_p - M\right\}. \tag{3.6}$$

In the steady state, profits of a firm producing a vintage of a given age, prices and wages will all be growing at the same rate g, so henceforth we will simplify notation by ignoring the dependence of variables on t unless confusion may arise.

It follows from the properties of ϕ, that, so long as the firm is producing at all, its optimal output, $f_p^*(a)$, optimal labour input, $M^*(a)$, and resulting profit, $\pi^*(a)$, have the following properties:

$$\frac{\partial f_p^*(a)}{\partial \eta} > 0, \quad \frac{\partial f_p^*(a)}{\partial \xi} > 0;$$

$$\frac{\partial M^*(a)}{\partial \eta} > 0, \quad \frac{\partial M^*(a)}{\partial \xi} > 0.$$

$$\frac{\partial \pi^*}{\partial \xi} > 0; \quad \frac{\partial \pi^*}{\partial \eta} > 0. \tag{3.7}$$

This result is intuitive: the higher the ratio of the price of machinery to the wage, the more is produced. Since the firm ceases production only when profits are zero, we can formally define Q by

$$\pi^*\left(\xi\left(R-Q,...\right),\eta\right) = 0. \tag{3.8}$$

The present value of a firm's profits when the firm is just set up is

$$W(t) = \int_0^Q w(t+a)\,\pi^*(t+a,a)\,e^{-ra}da. \tag{3.9}$$

The remuneration from doing research is the value of each innovation multiplied by the rate at which researchers discover innovations, which we know from equation (2.3). For workers to be indifferent between research and producing intermediate goods, the remuneration from research must equal the wage. Substituting $\{g/\psi^{-1}(g)\} W(t) = w(t)$, we obtain the arbitrage condition

$$1 = \frac{g}{\psi^{-1}(g/\upsilon)} \int_0^Q e^{(g(1-\sigma)-\rho)a} \pi^* \left(\xi\left(R-a, g; \rho, \sigma\right), \eta\right) da. \tag{3.10}$$

R is the age at which knowledge embodied in the intermediate good stops being used, in contrast to Q, the age of knowledge when intermediate goods of that vintage are no longer produced.

3.3. The Indifference Condition

We simplify notation by rewriting equation (3.10) as

$$1 = \Theta\left(R, g; \rho, \sigma, \eta, \upsilon\right): \tag{3.11}$$

the complicated nature of ξ means that there is no useful analytical solution to the integral, even if we specify ϕ.

By Leibnitz's rule $\partial\Theta/\partial Q = 0$, so we can ignore the effect of Q for the moment and concentrate on the other properties of Θ:

$$\Theta_R > 0; \quad \Theta_g \begin{cases} \leq 0 & \text{iff } \psi'' \ll 0; \\ > 0 & \text{otherwise.} \end{cases}$$

$$\Theta_\rho < 0; \quad \Theta_\eta > 0; \quad \Theta_\upsilon > 0.$$

When $\Theta_g > 0$, for Θ to remain constant, then, as either g or R falls to zero, the other becomes infinite, so the locus of points solving $\Theta = 1$ is asymptotic to both axes: $\Theta = 1$ is a downward sloping curve. When $\Theta_g < 0$, then the curve $\Theta = 1$ slopes up. For economies with a lower value of ρ, the curve will be higher, since, for a given value of g, a higher interest rate can always be offset by an increase in R. For economies with higher values of η or υ, the curve will be lower.

The economic intuition for these results is remains clear. Higher growth rates mean that wages must lag further behind output, raising the value of machines and of innovation. Unless ψ is very concave, this will also mean higher remuneration for researchers, since the increase in the value of each innovation will more than offset the reduction (if any) in the number of innovations per researcher. But if the wage has fallen and the remuneration to researchers risen, workers would no longer be indifferent between occupations, so it is necessary for some off-setting factor to ensure that wages do not fall, namely that the retirement age fall.

If ρ and hence the interest rate is lower, the value of innovation is higher. When intermediate goods are non-durable, this happens because production is in the future and the resulting profits discounted less; when they are durable, not only are profits discounted less, but they also rise because machines are more valuable. If η is higher, intermediate good production is cheaper and profits and the value of innovation rise. If υ increases, then each researcher finds more innovations. In any of these cases, for a given growth rate, the retirement age must be lower to ensure that wages do not fall behind the remuneration to research.

3.4. Labour Market Clearing

The model is closed by the labour market clearing condition in equation (2.1). The number of researchers is known from equation (2.2). The total number of workers producing intermediate goods is the integral of the number of workers producing intermediate goods in each vintage, weighted by the density of vintages, which is

$$M = \int_0^Q gM^*(a)\, da. \tag{3.12}$$

We know that $M^*(a)$ is an increasing function of ξ and hence M is an increasing function of R, g and η and a decreasing function of ρ; by Leibnitz's rule, $\partial M/\partial Q = gM^*(Q) \geq 0$, where the equality only holds when the cost curve is always upward sloping.

To calculate the number of workers operating intermediate goods, we must remember that, for a given vintage, the number of intermediate goods in use may different from the number being made. The number of machines of a given vintage[7] currently in use is

$$f(a) = \int_0^a f_p(s)\, ds \tag{3.13}$$

and hence the total number of machines in use is

$$N = \int_0^R g \int_0^a f_p(s)\, ds\, da = \int_0^Q g \int_0^a f_p(s)\, ds\, da + (R - Q) \int_0^Q f_p(s)\, ds, \tag{3.14}$$

since $f_p(s) = 0$, when $s > Q$. It follows that N is an increasing function of R, Q, g and η and a decreasing function of ρ for the same reasons that M has these properties, since the number of machines produced of a given vintage increases

[7] When intermediate goods are non-durable the results are similar: see the previous section for the precise formula.

with the number of workers.

We can now rewrite the labour market clearing in terms of the variables of interest by substituting equations (2.2), (3.12) and (3.14) into (2.1) to get

$$0 = \psi^{-1}(g/v) + M(R, g, Q; \rho, \eta) + N(R, g, Q; \rho, \eta) - \Lambda \equiv \Sigma(R, g, Q; \rho, \eta, v, \Lambda) \tag{3.15}$$

where, from the properties of ψ, M and N,

$$\Sigma_R > 0; \quad \Sigma_g > 0; \quad \Sigma_Q > 0$$

$$\Sigma_\rho < 0; \quad \Sigma_\eta > 0; \quad \Sigma_v < 0.$$

The intuition for these results are as follows: if the retirement age is higher then the demand for workers is higher, because more machines are in operation and more are being produced (because they are in production for longer and more are produced at any given age). If the growth rate is higher, then there must be more researchers and the demand for operators and producers is higher for the same reasons as for R. If the interest rate is lower, machines are less valuable and so less are produced. If η is higher, then machines are cheaper and more are produced and operated. If υ is lower, more workers are needed to find the same number of innovations.

3.5. Retirement of Knowledge

Finally we turn to the determination of Q. Each intermediate good firm ceases production when the price is just equal to the minimum average cost, which we shall write as

$$\xi\left(R-Q,g;\rho\right)=\chi\left(\eta\right);\quad \chi'<0.$$

The functional form of χ follows from that of the production function, ϕ. The derivative follows because the minimum of the average cost curve must be lower if costs fall. Hence we can deduce the following properties[8] of Q:

$$\frac{\partial Q}{\partial R}=1>0;\quad \frac{\partial Q}{\partial g}=\frac{\xi_g}{\xi_R}>0;\quad \frac{\partial Q}{\partial \rho}=\frac{\xi_\rho}{\xi_R}<0;\quad \frac{\partial Q}{\partial \eta}=\frac{\chi'}{\xi_R}. \tag{3.16}$$

3.6. Summary of the model

There are three endogenous variables solved by the indifference condition, labour market clearing and the condition determining the point at which knowledge is

[8] Assuming g and R are both strictly positive.

no longer used:

$$\{R, g, Q\} : \begin{cases} 1 &= \Theta\left(R, g, Q; \mathbf{p}\right) \\ 0 &= \Sigma\left(R, g, Q; \mathbf{p}\right) \\ 0 &= \xi\left(R - Q, g; \rho\right) - \chi\left(\eta\right). \end{cases}$$

where we introduce the notation $\mathbf{p}$, which is a vector of all the parameters of interest.

4. Equilibrium and Comparative Dynamics

We shall analyse the equilibrium in two stages: we start by considering the growth rate and the retirement age of capital alone by eliminating Q. Once this is done we consider the behaviour of Q.

4.1. Retirement of machines and growth

We begin by differentiating the solution equations to obtain

$$0 = \Theta_R dR + \Theta_g dg + \mathbf{\Theta}_p d\mathbf{p}, \tag{4.1}$$

$$0 = \Sigma_R dR + \Sigma_g dg + \Sigma_Q dQ + \mathbf{\Sigma}_p d\mathbf{p}, \tag{4.2}$$

$$0 \quad = \quad \xi_R \left(dR - dQ\right) + \xi_g dg + \left(\boldsymbol{\xi}_p - \boldsymbol{\chi}_p\right) d\mathbf{p}. \tag{4.3}$$

From the third equation we can obtain dQ, and thus eliminate dQ from the labour market clearing equation, which we rewrite as

$$0 = \Sigma_R^Q dR + \Sigma_g^Q dg + \boldsymbol{\Sigma}_p^Q d\mathbf{p}. \tag{4.4}$$

It is easy to show that the partial derivatives are qualitatively the same, whether we have solved for Q or not:

$$\operatorname{sign} \Sigma_i^Q = \operatorname{sign} \Sigma_i \quad i = \{R, g, \mathbf{p}\}$$

so $\Sigma^Q = 0$ will also be a downward sloping curve $\{g, R\}$ space and we the analysis for g and R will be similar regardless of Q.

Since the curves $\Sigma = 0$ and $\Theta = 1$ can both slope down, there may be no equilibrium or many equilibria, possibilities which we do not discuss further. If there are equilibria, however, we can say that they will fall into one of three categories, as shown in Figure 4.1.

First, when ψ is very concave, $\Theta = 1$ will slope up and thus cut $\Sigma = 0$ from

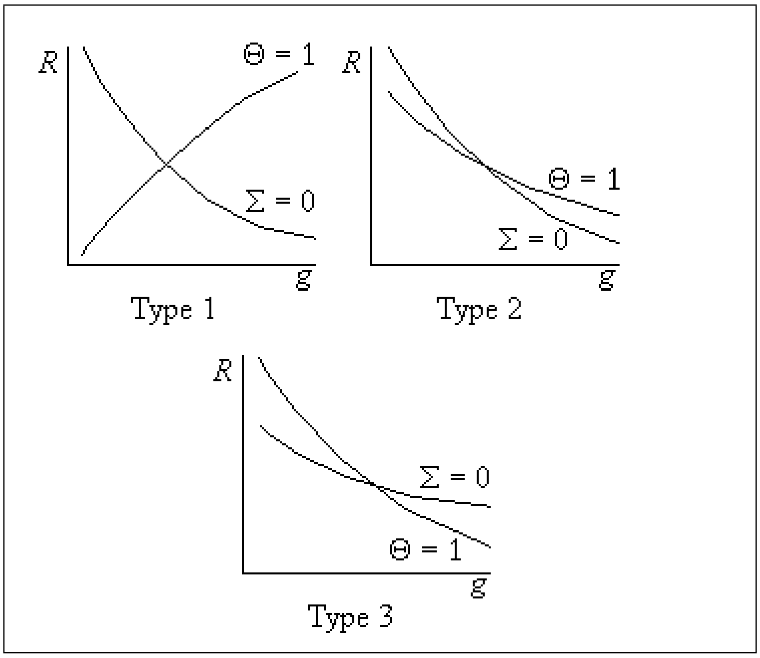

Figure 4.1: Equilibrium Types

below. We shall call this a Type 1 equilibrium.

Second, both curves may slope down but $\Theta = 1$ will still cut $\Sigma = 0$ from below. This can occur when ψ is concave or not too convex, so that $\Theta = 1$ is relatively shallow; more precise statements cannot be made without specifying the equations. We shall call this case a Type 2 equilibrium.

Third, both curves slope down but $\Theta = 1$ cuts $\Sigma = 0$ from above. This will occur when $\Theta = 1$ is relatively steep, probably when ψ is not too concave or is convex. This will be a Type 3 equilibrium.

The comparative static properties of the model will depend upon the type of equilibrium. The derivatives of the two endogenous variables with respect to the parameters can be obtained by solving equations (4.1) and (4.4):

$$\frac{dg}{d\mathbf{p}} = \frac{\boldsymbol{\Sigma}_p/\Sigma_R - \boldsymbol{\Theta}_p/\Theta_R}{\Theta_g/\Theta_R - \Sigma_g/\Sigma_R} \tag{4.5}$$

$$\frac{dR}{d\mathbf{p}} = \frac{(\Sigma_g/\Sigma_R)\,(\boldsymbol{\Theta}_p/\Theta_R) - (\Theta_g/\Theta_R)\,(\boldsymbol{\Sigma}_p/\Sigma_R)}{\Theta_g/\Theta_R - \Sigma_g/\Sigma_R} \tag{4.6}$$

To discuss the properties of these derivatives for every parameter would be tedious, so we only consider Λ and η in detail and summarise for the other parameters.

Consider two economies, A and B, where B has a larger labour force. Indifference between occupations is unaffected by the size of the labour force, so both will have the same $\Theta = 1$ curve. The labour market clearing curve for economy B, however, will be further from the origin, since more workers are available for either research or investment in machinery, which will lead to higher values of g or R respectively. The three possibilities are shown in Figure 4.2, with the labour market clearing conditions labelled "A" and "B" as appropriate. When there is

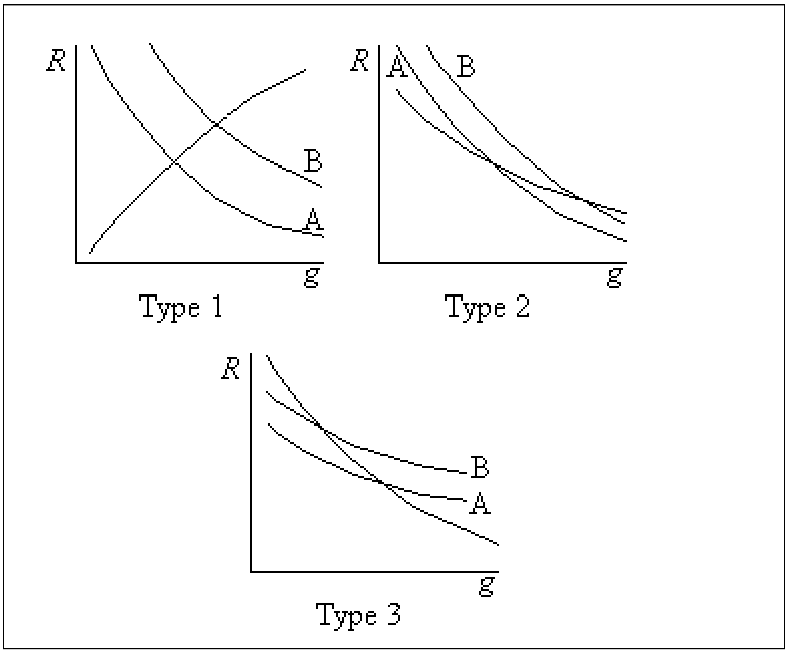

Figure 4.2: Economies with different labour supplies

a Type 1 equilibrium, the growth rate and retirement age will both be higher in B; when there is a Type 2 equilibrium, the growth rate will be higher and the retirement age lower; when there is a Type 3 equilibrium, the growth rate will be lower and the retirement age higher.

The intuition behind these results is fairly straight-forward. More workers are available in B, so there is the potential for greater research effort. But potential researchers will need to be compensated for two costs. First, if ψ is concave, there are negative externalities in the research sector, so the greater the number of researchers the fewer innovations discovered by each individual. Second, if the growth rate is higher in the future, then potential researchers may anticipate that newer vintages will displace older ones more quickly. This is "creative destruction" of machinery, since the increased speed of innovation leads to machinery being scrapped more quickly (corresponding to a lower retirement age). Creative destruction of machinery does not directly concern researchers; their remuneration per innovation is the value of the patent. The patent price does, however, depend upon the value of machinery amongst other things, so researchers are indirectly affected by the creative destruction of machinery. Thus the researchers in economy B must be compensated for the externalities and their expectations concerning the effect of creative destruction on the value of each patent.

Ceteris Paribus, if the growth rate were expected to be higher in the future, researchers would expect the value of each patent to be higher, since wages would lag further behind machine prices making machinery production, and hence innovation, more profitable relative to producing or operating machinery. This effect will often suffice to compensate researchers for any fall in the value of each patent due to creative destruction and any negative externalities and, anticipating this, more workers will choose to do research. This is a Type 2 equilibrium.

Sometimes, however, the negative externalities and difference in the patent price cannot be compensated by expectations of a higher growth rate, in which case more workers will choose to do research in economy B only if they expect there to be less creative destruction (a higher retirement age). This is a Type 1 equilibrium.

Finally, it may be the case that there is no growth rate higher than that in economy A which will compensate potential researchers in economy B for the patent price. Anticipating this, workers in B will avoid research and the growth rate will be lower. But this will also mean that the ratio of machine prices to wages will also be lower unless there is less creative destruction, so the retirement age will have to be higher.

We now assume that economies A and B are the same in all respects except that economy B has a more productive machine producing sector, that is a higher value of η. This means that both the curve $\Theta = 1$ and the curve $\Sigma = 0$ for economy B lie below those of economy A. For a Type 1 equilibrium the retirement age will be lower in B than in A, but the growth rate may either rise or fall. For both a Type 2 and a Type 3 equilibrium, all that we can say is that it is impossible for both the retirement age and the growth rate to be higher in B than in A; the retirement age may be higher and the growth rate lower, the retirement age may be lower and the growth rate higher or both may be higher.

The reason for these ambiguous results is that higher productivity in the machine will encourage more production and reduce the retirement age. This will happen even if there is no increase in the growth rate, but it will still reduce the patent price through creative destruction. On the other hand, higher productivity will increase the profitability of machine producing firms and mean that the value of a patent will be higher, encouraging more innovation. Again, this is true even if there is no difference in the growth rate. Thus firms will expect to produce machines more quickly but for a shorter period of time. Thus the direct productivity effect on patents is ambiguous. On top of this effect, however, there is the two effects mentioned in the previous section: any changes in the growth rate will also affect the value of patents and the number of innovations produced by each researcher.

In Type 2 and Type 3 equilibria, we can exclude the possibility that both the retirement age and the growth rate will be higher. This would mean that researchers benefited from higher η, R and g, all of which would make workers prefer to do research and thus violate the indifference condition. In Type 1 equilibria, we can also exclude the possibility that the retirement age be higher and the growth rate lower; because of the large externalities, a lower growth rate would make researchers better off and also violate the indifference condition.

We now summarise the comparative static properties without further discussion:

Machines	Type 1	Type 2	Type 3
$\eta \uparrow$	$g\ ?\quad R \downarrow$	?	?
$\upsilon \uparrow$	$g \uparrow\ \ R\ ?$	$g \uparrow\ \ R \downarrow$	$g \downarrow\ \ R \uparrow$
$\rho \downarrow$	$g\ ?\ \ R \downarrow$	?	?
$\Lambda \uparrow$	$g \uparrow\ \ R \uparrow$	$g \uparrow\ \ R \downarrow$	$g \downarrow\ \ R \uparrow$

(4.7)

A question mark in isolation means that at least one of the variables must fall. This completes the first stage of the analysis of the comparative dynamics. We now turn to the creative destruction of the patents, that is the retirement age of knowledge, Q.

4.2. Retirement of knowledge

From equation (4.3), we can obtain the change in Q relative to that of R:

$$dQ = dR + \frac{\xi_g}{\xi_R} dg + \frac{\boldsymbol{\xi}_p - \boldsymbol{\chi}_p}{\xi_R} d\mathbf{p} \tag{4.8}$$

From table (4.7) we know that dg and dR will usually have opposite signs. Thus there is scope for dQ to have a different sign from that of dR, in other words, creative destruction of knowledge does not need to mimic creative destruction of capital. If dg has the same sign as dR, then creative destruction of the two forms of capital will move together

The last term in equation (4.8) is zero when the parameter of interest is υ or Λ, so we will start by ignoring that term; thus

$$\frac{dQ}{dR} = 1 + \left(\frac{\xi_g}{\xi_R} \frac{dg}{dR} \right)$$

If dg and dR have the same sign, then the sign of dQ/dR will be positive. Thus if a higher value of η leads to a fall in both g and R, then Q will fall also. But in most cases g and R will move in contrary directions, so it is possible that despite creative destruction of machinery (lower R), there may be no creative destruction of knowledge and that patents will increase in value. This will happen when the term in brackets is large (and negative). We cannot, however, evaluate the bracket without precise knowledge of the functional form, even by a Taylor expansion around $g = 0$: the limit of ξ_g/ξ_R as g tends to zero is infinite[9] and the limit of dg/dR must be zero as g tends to zero if the indifference condition is to be satisfied.

The value of dg/dR can be found by dividing equation (4.5) by (4.6). When we are considering different values of Λ, the resulting relationship between Q and R is

$$\frac{dQ}{dR} = 1 - \frac{\xi_g}{\xi_R}\frac{\Theta_R}{\Theta_g}.$$

The advantage of considering differences in Λ is that only the labour market clearing curve is affected and we only need consider the slope of the indifference

[9]Since

$$\left.\frac{\partial \xi}{\partial R}\right]_{g=0} = \left.\frac{g e^{g(R-a)}\left\{1 - e^{-r(R-a)}\right\}}{r}\right]_{g=0} = 0$$

and

$$\left.\frac{\partial \xi}{\partial g}\right]_{g=0} = \frac{\rho\,(R-a) + e^{-\rho(R-a)} - 1}{\rho^2} \geq 0$$

condition. Hence

$$\frac{dQ}{dR} \lesseqgtr 0 \Leftrightarrow \frac{\Theta_g}{\Theta_R} \lesseqgtr \frac{\xi_g}{\xi_R}, \quad \text{when} \quad \Theta_g > 0.$$

$$\frac{dQ}{dR} \lesseqgtr 0 \Leftrightarrow \frac{\Theta_g}{\Theta_R} \gtreqless \frac{\xi_g}{\xi_R}, \quad \text{when} \quad \Theta_g < 0.$$

To see what this means, consider a Type 2 equilibria, where an economy with more labour has a higher growth rate and a lower retirement age for machinery. The higher value of g and the lower value of R must just offset each other to ensure that $\Theta = 1$. If the slope of that curve is very shallow (*i.e.*, Θ_g/Θ_R is small in magnitude), then any difference in the growth rate will be offset by quite a small difference in the retirement age. Now when ξ_g/ξ_R is large, then, for a given value of Q, any change in g will need a large contrary change in R for the knowledge retirement condition to be satisfied, but, as just outlined, the change in the retirement age will be small. A fall in Q will be needed to ensure that knowledge is retired.

We now consider the economic intuition behind these results. Recall that economy B has a larger labour force than economy A. In the Type 1 equilibrium, the negative externalities in the research sector mean that higher growth is only possible if workers anticipate less depreciation. There is no *prima facie* reason why they should require less depreciation of both machinery and knowledge, but the theory indicates that this is the case: Θ_g/Θ_R, which is negative, cannot be greater than ξ_g/ξ_R, which is positive.

In the Type 2 equilibrium, a higher growth rate means that each researcher is making more discoveries and to keep workers indifferent between research and other occupations this is compensated by greater creative destruction. This will always be achieved by greater creative destruction of machinery, but will only sometimes be achieved by additional creative destruction of knowledge. The latter will be unnecessary when the creative destruction of machinery has a relatively large effect on the patent price compared to the effect on the knowledge retirement price.

Finally, when workers are discouraged from research by their anticipation that the value of patents will be quickly undermined by future research, there will be less creative destruction of machinery, which will only sometimes need to be accompanied by less creative destruction of knowledge.

The effects of higher values of Λ are summarised in Figure 4.3. It is assumed that there is some measure of convexity of ψ which can be plotted on the horizontal axis: at the origin ψ is concave and as one moves to the right it becomes less concave and eventually convex and then increasingly convex. Under these circumstances, the slope of the indifference curve is initially negative and continually increases as the convexity of ψ increases, eventually becoming positive. The slope of the labour market clearing condition is always positive (although it need not decrease with the convexity of ψ as shown here). The comparative dynamics depend upon the type of equilibrium and the value ξ_g/ξ_R, which is plotted on the vertical axis.

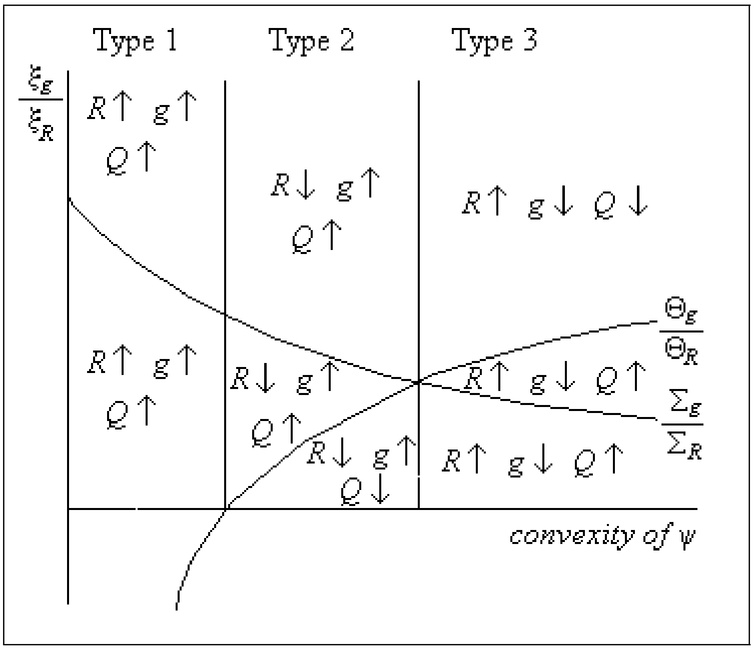

Figure 4.3: Comparative Dynamics: Higher value of Λ

Similar results can be obtained for the other parameters.

5. Local Stability

Faced with this range of comparative static properties it is reasonable to ask if any of them can be excluded for any reason. Here we consider local stability, when expectations are that the economy will remain in the steady state. From a position of equilibrium, we consider the forces acting on the economy when it moves a little way from equilibrium, but expectations do not change

First of all, consider the condition that workers be indifferent between research and either producing or operating intermediate goods. In the steady state this is determined by

$$\Theta\left(R^{*}, g^{*}, r\right) = 1.$$

We know that the effect on Θ of a small increase in R is positive. The effect on Θ of a small increase in $g(t)$ is negligible. The reason that g enters as an argument of Θ is that, in the steady state, it determines the rate at which wages increase and hence the profitability of intermediate good and, in turn, the profitability of producing intermediate good. Out of the steady state, the rate at which wages increase will depend upon the rate at which productivity was increasing when