Competition and Hold-ups*

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Abstract. In an environment in which both workers and firms undertake match specific investments, the presence of market competition for matches may solve the hold-up problems generated by the absence of complete contingent contracts. In particular, this paper shows that in a world in which workers' and firms' investments are separated by market competition and contracts specify a simple (non-contingent) wage payment, investments are constrained efficient. Indeed, workers and firms invest efficiently given the equilibrium matches in which they are involved.

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1. Introduction

A central concern for economists is the extent to which market systems are efficient. In the idealized Arrow-Debreu model of general competitive equilibrium, efficiency follows under mild conditions, notably the absence of externalities. But in recent years, economists have become interested in studying market situations less idealized than in the Arrow-Debreu set-up and in examining the pervasive inefficiencies that may exist. The subject of the present paper, the "hold-up problem", is one example of a situation that is thought to give rise to significant inefficiencies.

The hold-up problem applies when an agent making an investment is unable to receive all the benefits that accrue from the investment. The existence of the problem is generally traced to incomplete contracts: with complete contracts, the inefficiency induced by the failure to capture benefits will not be permitted to persist. In the standard set-up of the problem, investments are chosen before agents interact and contracts can be determined only when agents meet. Prior investments will be a sunk cost and negotiation over the division of surplus resulting from an agreement is likely to lead to a sharing of the surplus enhancement made possible by one agent's investment (Williamson 1985, Grout 1984, Grossman and Hart 1986, Hart and Moore 1988).

What happens if agent interaction is through the marketplace? In an Arrow-Debreu competitive model, complete markets, with price-taking in each market, are assumed; if an agent chooses investment *ex-ante*, every different level of investment may be thought of as providing the agent with a different good to bring to the market. If the agent wishes to choose a particular level of investment over some other, and the "buyer" he trades with also prefers to trade with the agent in question, rather than with an "identical" agent with another investment level, then total surplus to be divided must be maximized by the investment level chosen: investment will be efficiently chosen and there is no hold-up problem. In this situation, the existence of complete markets implies that agents know the price that they will receive or pay whatever the investment level chosen: complete markets imply complete contracts.

An unrealistic failure of the Arrow-Debreu set-up is that markets are assumed to exist for every conceivable level of investment, irrespective of whether or not trade occurs in such a market. But without trade, it is far-fetched to assume that agents will believe that they can trade in inactive markets and that a competitive price will be posted in such market.

The purpose of this paper is to provide an example of an environment where the trading pattern and terms of trade are determined explicitly by the interaction of sellers and buyers and, given the pattern of trade, investments are chosen efficiently. To ensure that there are no inefficiencies resulting from market power, a model of Bertrand competition is analyzed where some agents invest prior to trade; however, this does not rule out the dependence of the pattern of outcomes on the initial investment of any agent and the analysis concentrates on the case of a finite number of traders to ensure this possibility. Contracts are the result of competition in the marketplace and we are interested in the degree to which the hold-up problem is mitigated by contracts that result from Bertrand competition. In this regard, it should be said that we shall not permit Bertrand competition in contingent contracts; in our analysis, contracts take the form of an agreement to trade at a particular price. We are thus investigating the efficiency of contracts implied by a simple trading structure rather than attempting explicitly to devise contracts that help address the hold-up problem (Aghion, Dewatripont, and Rey 1994, Nöldeke and Schmidt 1995, Maskin and Tirole 1999, Segal and Whinston 1998).

With Bertrand competition, there is an asymmetry between sellers and buyers in a market. As a convention, we assume that sellers bid for the right to trade with buyers by naming a price that they wish to receive. There are two asymmetries: one side of the market (here the sellers) bids, and one side of the market (here, again, the sellers) will obtain a contract with a specified return. With this protocol, it is shown that the *ex ante* investments of sellers will be constrained efficient.

In essence, a seller will bid just enough to win the right to trade with a buyer and, if he were to have previously enhanced the value of a trade by extra investment, he would have been able to win the right with the same bid, as viewed by the buyer, and so receive all the benefits of the extra investment. When buyers make *ex-post* investments — as in our setting — the terms of trade, defined by what sellers receive, are fixed and buyers receive any enhancement to surplus resulting from their investments: buyers' investments will also be constrained efficient. Thus in a world of sequential investments, separated by competitive bidding, the residual rights to the surplus of a trade can switch from the seller to the buyer and both sets of agents will make constrained efficient investments.

The constraints imposed on efficiency are given by the matches that are observed in equilibrium. Indeed, given the very few assumptions we make on the matching process, it is possible that a number of coordination problems arise. Bertrand competition, on the other hand, is enough to deliver weakly efficient equilibrium matches. However, it is still possible that a seller might undertake a high investment with the sole purpose of changing the buyer with whom he will be matched. This may lead to inefficient equilibrium matches. Notice that, in spite of this problem, sellers and buyers will invest efficiently given the equilibrium matches in which they will be involved. In such an environment, hold-up problems are solved and the only inefficiencies left are due to coordination problems. These coordination problems are not the focus of the present paper. It should be said, however, that they cannot be solved by a bilateral contractual arrangement or by market competition (Hart 1979, Cooper and John 1988).

The structure of the paper is as follows. After a discussion of related literature in the next section, Section 3 lays down the basic structure of the model. The firms' investment choices are analyzed in Section 4 while Bertrand competitive equilibria are characterized in Section 5. Finally, Section 6 investigates the workers' ex-ante investment choices and the efficiency properties of the equilibria we construct. Section 7 provides concluding remarks.

2. Related Literature

The literature on the hold-up problem has mainly analyzed the bilateral relationship of two parties that may undertake match specific investments in isolation (Williamson 1985, Grout 1984, Grossman and Hart 1986, Hart and Moore 1988). In other words, these papers identify the inefficiencies that the absence of complete contingent contracts may induce in the absence of any competition for the parties to the match. This literature identifies the institutional (Grossman and Hart 1986, Hart and Moore 1990, Aghion and Tirole 1997) or contractual (Aghion, Dewatripont, and Rey 1994, Nöldeke and Schmidt 1995, Maskin and Tirole 1999, Segal and Whinston 1998) devices that might reduce and possibly eliminate these inefficiencies. We differ from this literature in that we do not alter either the institutional or contractual setting in which the hold-problem arises but rather analyze how competition among different sides of the market may eliminate the inefficiencies associated with such a problem.

The literature on bilateral matching, on the other hand, concentrates on the inefficiencies that arise because of frictions present in the matching process. These inefficiencies may lead to market power (Diamond 1971, Diamond 1982), unemployment (Mortensen and Pissarides 1994) and a class structure (Burdett and Coles 1997, Eeckhout 1999). A recent development of this literature shows how efficiency can be restored in a matching environment thanks to free entry into the market (Roberts 1996, Moen 1997) or Bertrand competition (Felli and Harris 1996). We differ from this literature in that we abstract from any friction in the matching process and focus on the presence of match specific investments before and after the matching process.

A small literature considers investments in a matching environment. Some of the papers focus on general investment that may be transferred across matches and identify the structure of contracts or competition that may lead to efficiency (MacLeod and Malcomson 1993, Acemoglu and Shimer 1998, Holmström 1999) or the inefficiencies due to the presence of an exogenous probability that the match will dissolve (Acemoglu 1997).

Three recent papers consider, instead, specific investments in a matching environment as we do (Cole, Mailath, and Postlewaite 1998, de Meza and Lockwood 1998, Felli and Roberts 1999).

Cole, Mailath, and Postlewaite (1998) focus on *ex-ante* match specific investment and analyze efficiency when matches and the allocation of the shares of surplus are in the core of the assignment game. They demonstrate the existence of an equilibrium allocation that induces efficient investments as well as allocations that yield inefficiencies. A critical assumption for their efficiency result is a condition of 'overlapping'. This is essentially an assumption on the specificity of the investments chosen by both workers and firms. If overlapping holds then it means that investment is specific to a group of at least two workers or firms but, among these workers and firms, it is general. There exists, therefore, an immediate competitor for each firm and worker. Bertrand competition allow us to avoid making this assumption and still obtain a form of efficiency.

The paper by de Meza and Lockwood, (de Meza and Lockwood 1998), instead analyzes a matching environment in which both sides of the market can undertake match specific investments but focuses on a setup that delivers inefficient investments. As a result, the presence of asset ownership and asset trading may enhance welfare as in Grossman and Hart (1986). They focus on whether one would observe asset trading before or after investment and match formation. In our setting, given that we obtain efficiency within the match we do not need to explore the efficiency enhancing role of asset ownership.

Finally Felli and Roberts (1999) is a paper that can be viewed as companion to the present one. There we are concerned with an environment in which both workers and firms invest prior to the Bertrand competition that determines the equilibrium matches. In this case we show that sellers' investments are inefficient. However, we show that, if matching is assortative, then the extent of the inefficiency is small in a well defined sense. In particular, the overall inefficiency in the market is less than that which could result from the under-investment by one seller in the market with all other sellers making efficient investments. This result depends on the assortative nature of the matching process and assortative matching is shown to be a critical factor for competition to be able to solve the hold-up problems due to the lack of fully contingent contracts.

3. The Framework

We consider a simple matching model: S workers match with T firms, we assume that the number of workers is higher than the number of firms $S > T$.[1] Each firm is assumed to match only with one worker. Workers and firms are labelled, respectively, $s = 1, \ldots, S$ and $t = 1, \ldots, T$. Both workers and firms can make match specific investments, denoted respectively x_s and y_t, incurring costs $C(x_s)$ respectively $C(y_t)$.[2] The cost function $C(\cdot)$ is strictly convex and $C(0) = 0$. The surplus of each match is then a function of the identities of the worker and the firm involved and of both specific investments $v(t, s, y_t, x_s)$.

[1] In Section 6 below we discuss the case in which $S = T$.

[2] For simplicity we take both cost functions to be identical, none of our results depending on this assumption.

We assume that workers, firms and matches are heterogeneous and we do not impose any structure on the heterogeneity of matches. Notice that match heterogeneity gives a particular meaning to the term specific investments we used for x_s and y_t. Indeed, in our setting, the investments x_s and y_t have a use and value in matches other than (t, s); however, these values differ with the identity of the partner implying that at least one component of this value is specific to the match in question, since we consider a discrete number of firms and workers. We impose no upper limit on the degree of specificity.

We also assume that the surplus of each match is increasing and concave in match specific investments: $v_3 > 0$, $v_4 > 0$, $v_{33} \leq 0$, $v_{44} \leq 0$ and $(v_{33}v_{44} - v_{34}^2) \geq 0$.[3] If the worker investment is zero then the surplus is assumed to be zero.

Denote, for convenience, each match surplus, net of the firm's investment cost as

$$w(t, s, y_t, x_s) = v(t, s, y_t, x_s) - C(y_t). \tag{1}$$

Our assumption of concavity of the surplus function $v(\cdot, \cdot, \cdot, \cdot)$ with respect to firm's and worker's investments imply the appropriate concavity properties for the net surplus function $w(\cdot, \cdot, \cdot, \cdot)$ as defined in (1).

The timing of the model can be described as follows. First, each worker chooses his match-specific investment, then workers Bertrand compete for the firms so as to determine the equilibrium matches and, at the same time, the share of the match surplus accruing to each party to the match. Firms then choose their match-specific investment so as to maximize their profits.

We assume the following extensive form of the Bertrand competition game in which the T firms and the S workers engage. Workers Bertrand compete for firms. All workers simultaneously and independently make wage offers to every one of the T firms. Notice that we allow workers to make offers to more than one, possibly all firms. Each firm observes the offers she receives and decides which offer to accept. For sake of simplicity, we assume that this decision is taken sequentially. In other words, firm 1 decides first which offer to accept. This commits the worker selected

[3]Notice that in our setting we assume concavity for the sake of simplicity. Indeed, if the surplus function is concave each worker's and firm's investment decision is globally concave and hence first order conditions are both necessary and sufficient. All our results hold if we do not assume concavity.

to work for firm 1 and automatically withdraws all offers this worker made to other firms. All other firms and workers observe this decision and then firm 2 decides which offer to accept. This process is repeated until firm T decides which offer to accept. Notice that since $S > T$ even firm T, the last firm to decide, can potentially choose among multiple offers.[4]

We look for the *trembling-hand-perfect* equilibrium of such a game.

4. Firms' Investments

We proceed to solve our extensive form game backwards. We start therefore from the firms' investment choices. Consider firm t and let (t, s) be the match in which this firm is involved. Let x be the worker's investment choice, clearly both the match and the worker's investment are given when firm t chooses her investment. Further denote $\tilde{y}$ the level of the firm's investment foreseen by all the workers and firms in the early stages of the game when worker s chooses his investment and when the equilibrium match is determined by the Bertrand competition game. Denote $\pi^W(t, s, \tilde{y}, x)$ the worker's payoff determined by the Bertrand competition game and stated in the contract. Firm t's investment $y(t, s, x)$ is then the solution to the following maximization problem:

$$y(t, s, x) = \operatorname*{argmax}_{y} w(t, s, y, x) - \pi^W(t, s, \tilde{y}, x). \tag{2}$$

Notice that worker s's payoff $\pi^W(t, s, \tilde{y}, x_s)$ is independent of firm t's investment at this stage of the game. The firm is then residual claimant of the match surplus in excess of the worker's given payoff. Firm t's investment $y(t, s, x)$ is implicitly defined by the following first order condition:

$$w_3(t, s, y(t, s, x), x) = 0. \tag{3}$$

[4]An alternative extensive form of the Bertrand competition game that would lead to a similar equilibrium characterization can be described as follows. All workers submit simultaneously and independently offers to all firms. Firms simultaneously and independently decide which offer to accept. If a worker's offer is accepted by one firm only the worker is committed to work for that firm. If instead the same worker offer is accepted by more than one firm then the bidding process is repeated among the firms and workers who are not committed to a match yet. This process continues until all firms are matched.

We now provide the relevant benchmark for our analysis: the characterization of the constrained efficient level of investments by the worker and the firm where the constraint is represented by the given match.

Let the match (t, s) be given, then the constrained efficient levels of investments x_s^* and y_t^* are defined by the solution to the following social planner's problem:

$$\max_{x,y} w(t, s, y, x) - C(x) \tag{4}$$

These efficient levels are implicitly and uniquely defined by the following pair of first order conditions:

$$w_4(t, s, y_t^*, x_s^*) = \frac{dC(x_s^*)}{dx} \tag{5}$$

$$w_3(t, s, y_t^*, x_s^*) = 0 \tag{6}$$

We have now the necessary elements to be able to prove the efficiency properties of the firms' investment choices. Indeed, for any given investment by the worker $\bar{x}$ and any given match $(t, s,)$ we can now prove that each firm's investment is constrained efficient.[5] This result is stated in the following proposition.

Proposition 1. *Each firm's investment choice $y(t, s, \bar{x})$ is constrained efficient.*

Proof: Firm t's constrained efficient level of investment is given by the solution to the central planner's problem (4). This solution is implicitly defined by (6) where we need to substitute x_t^* with $\bar{x}$. Comparison of (6) with (3) concludes the proof. ■

We can now define a reduced form of the net surplus function in (1) substituting firm t's optimal investment choice (best reply) $y(t, s, x)$. This is

$$\omega(t, s, x) = w(t, s, y(t, s, x), x). \tag{7}$$

[5] Notice that here more constraints are imposed on efficiency than in the definition of x_s^* and y_t^*. These are the given match (t, s) in which worker and firm are involved and, in addition, the level of the worker's investment $\bar{x}$.

Notice that, using the definition (7), condition (3), and the concavity properties of the net surplus function $w(\cdot,\cdot,\cdot,\cdot)$ we can show that: $\omega_3 > 0$, $\omega_{33} \leq 0$.

5. Bertrand Competition

In this section we solve the extensive form of the Bertrand competition game, as described in Section 3 above, in which workers and firms engage so as to determine the equilibrium matches and the share of the match surplus accruing to each party to a match.

The surplus that is shared in each potential match is described in (1) above. Indeed, in this case, since the firm's investment has yet to be chosen, the cost of the firm's investment will be deducted from the match surplus: Bertrand competition among the workers determines the share of the surplus, net of the firm's investment cost, that the worker and firm in the match receive, as in (1).

The other feature of the Bertrand competition subgame is that it occurs after the workers have chosen their investment levels $x_1, \ldots, x_S$. Depending on these investment levels and the innate characteristics of both workers and firms we may observe possibly very different equilibrium matches. In other words, given that we have not made any assumption on the assortative nature of the matching process, the Bertrand competition subgame may have multiple equilibria. An equilibrium of this subgame is a T-ple of matches and a specification of the shares of the match net surplus accruing to the T matched workers and firms. We can provide a characterization common to all these equilibria. For any T-ple of equilibrium matches this is the equilibrium shares of surplus that firms and workers receive.

Let an equilibrium of the Bertrand competition subgame be given. Denote $\alpha(t)$ the identity of the worker that in this equilibrium matches with firm t for all $t = 1, \ldots, T$, and $\alpha(T+1), \ldots, \alpha(S)$ the identities of the workers that in equilibrium are left unmatched. We can now state the following proposition.

Proposition 2. *The equilibrium share of the surplus that each firm t and each worker $\alpha(t)$, matched with firm t, receive for every $t = 1, \ldots, T$ are:*

$$\pi_t^F = \max_{k=1,\ldots,S-T} \left[\omega(t, \alpha(t+k), x_{\alpha(t+k)}) - \omega(t+k, \alpha(t+k), x_{\alpha(t+k)}) + \pi_{t+k}^F\right] \qquad (8)$$

$$\pi_t^W = \omega(t, \alpha(t), x_{\alpha(t)}) - \pi_t^F \tag{9}$$

where

$$\pi_h^F = 0 \qquad \pi_h^W = 0 \qquad \forall h = T+1, \ldots, S. \tag{10}$$

Proof: We characterize the equilibrium proceeding by induction. Denote by t the class of subgames that starts with firm t having to choose among the submitted bids. These subgames differ depending on the bids previously accepted by firms $1, \ldots, t-1$. We first solve for the equilibrium of the T-th (the last) subgame in which all firms but firm T have selected a worker's bid.

Without loss of generality, we take $S = T + 1$. This subgame is then a simple decision problem for firm T that has to choose between the bids submitted by the two remaining workers: $\alpha(T)$ and $\alpha(T+1)$. Denote $B_{\alpha(T)}$ and $B_{\alpha(T+1)}$ these workers' bids. Firm T clearly chooses the highest of these two bids.

Worker $\alpha(T+1)$ generates net surplus $\omega(T, \alpha(T+1), x_{\alpha(T+1)})$ if selected by firm T while worker $\alpha(T)$ generates net surplus $\omega(T, \alpha(T), x_{\alpha(T)})$ if selected. This implies that $\omega(T, \alpha(T+1), x_{\alpha(T+1)})$ is worker $\alpha(T+1)$'s maximum willingness to bid while $\omega(T, \alpha(T), x_{\alpha(T)})$ is worker $\alpha(T)$'s maximum willingness to bid. By assumption worker $\alpha(T)$ matches in equilibrium with firm T. Therefore

$$\omega(T, \alpha(T), x_{\alpha(T)}) \geq \omega(T, \alpha(T+1), x_{\alpha(T+1)}).$$

Worker $\alpha(T)$ then submits a bid equal to the minimum necessary to outbid worker $\alpha(T+1)$. In other words, the equilibrium bid of worker $\alpha(T)$ coincides with the equilibrium bid of worker $\alpha(T+1)$:

$$B_{\alpha(T)} = B_{\alpha(T+1)}.$$

Worker $\alpha(T+1)$, on his part, has an incentive to deviate and outbid worker $\alpha(T)$ for any bid $B_{\alpha(T)} < \omega(T, \alpha(T+1), x_{\alpha(T+1)})$. Therefore both workers' equilibrium bids are:

$$B_{\alpha(T)} = B_{\alpha(T+1)} = \omega(T, \alpha(T+1), x_{\alpha(T+1)}).$$

These equilibrium bids are uniquely defined.[6]

The payoff of firm T is then $\pi_T^F = \omega(T, \alpha(T+1), x_{\alpha(T+1)})$ while worker $\alpha(T)$'s payoff is $\pi_T^W = \omega(T, \alpha(T), x_{\alpha(T)}) - \pi_T^F$ and worker $\alpha(T+1)$'s payoff is $\pi_{T+1}^W = 0$. Notice that if we define $\pi_{T+1}^F = 0$ then these payoff coincide with the ones in (8), (9) and (10) above.

We now move to the t-th subgame, $(t < T)$. In this case firm t has to choose among the potential bids of the remaining $(T - t + 2)$ workers: $\alpha(t), \ldots, \alpha(T+1)$. Our induction hypothesis is that the continuation equilibrium payoffs of the workers $\alpha(t+1), \ldots, \alpha(T+1)$ and the firms $t+1, \ldots, T$ are described in (8), (9) and (10) above. Firm t clearly chooses the highest bid she receives.

By assumption firm t will select worker $\alpha(t)$. Therefore $\alpha(t)$, by definition, has the highest willingness to pay and submits a bid $B_{\alpha(t)}$ equal to the minimum necessary to outbid workers $\alpha(t+1), \ldots, \alpha(T+1)$. Let $j = t+1, \ldots, T+1$. Worker $\alpha(j)$'s maximum willingness to bid for firm t is then

$$\omega(t, \alpha(j), x_{\alpha(j)}) - \pi_j^W.$$

Indeed, worker $\alpha(j)$ is willing to pay the surplus he will be able to generate if matched with firm t in excess of the payoff π_j^W he can guarantee himself, from our induction hypothesis, by not competing for firm t and moving to subgame j the only one in which his bid will be selected by firm j. Therefore the equilibrium bid of worker $\alpha(t)$ is such that:

$$B_{\alpha(t)} = \max_{k=1,\ldots,S-t} \left\{ \omega(t, \alpha(t+k), x_{\alpha(t+k)}) - \pi_{t+k}^W \right\}. \tag{11}$$

Firm t equilibrium payoff is then $\pi_t^F = B_{\alpha(t)}$. Substituting the expression of π_{t+k}^W, as in (9), into (11) we obtain π_t^F as in (8). Worker $\alpha(t)$'s payoff is instead $\omega(t, \alpha(t), x_{\alpha(t)}) - \pi_t^F$ as in (9). ■

The characterization of the workers' and firms' equilibrium shares of surplus in Proposition 2 allow us to prove a partial efficiency condition that needs to be satisfied

[6] This is just one of a whole continuum of subgame perfect equilibrium bids of this simple Bertrand game *but* the unique trembling-hand-perfect equilibrium bids. Trembling-hand-perfection is here used in a completely standard way to insure that worker $\alpha(T+1)$ does not choose an equilibrium bid (not selected by firm T) in excess of his maximum willingness to pay.

in equilibrium. This is a limited equilibrium assortative matching condition. Let the workers' investments be given and denote $\alpha(t+k^*)$ the identity of the worker that, according to (8), determines the remuneration of firm t and is the immediate competitor of worker $\alpha(t)$ for the match with t:

$$k^* = \underset{k=1,\ldots,S-t}{\operatorname{argmax}} \left[\omega(t,\alpha(t+k),x_{\alpha(t+k)}) - \pi^W_{t+k}\right] \tag{12}$$

We can now prove that the total surplus generated by the two equilibrium matches $(t,\alpha(t))$ and $(t+k^*,\alpha(t+k^*))$ strictly dominates the surplus obtained if, for equal investment choices, these two workers and these two firms are mismatched in the following way: $(t+k^*,\alpha(t))$ and $(t,\alpha(t+k^*))$.

Proposition 3. *Let $(t,\alpha(t))$, $t=1,\ldots,T$, be a T-ple of equilibrium matches of the Bertrand competition game and for every t let k^* be defined as in (12). Then*

$$\begin{aligned} \omega(t,\alpha(t),x_{\alpha(t)}) \;\; &+ \;\; \omega(t+k^*,\alpha(t+k^*),x_{\alpha(t+k^*)}) \;\; \geq \\ &\geq \;\; \omega(t+k^*,\alpha(t),x_{\alpha(t)}) \;\; + \;\; \omega(t,\alpha(t+k^*),x_{\alpha(t+k^*)}) \end{aligned} \tag{13}$$

Proof: The characterization of the equilibrium of the Bertrand competition payoffs in Proposition 2 above is such that in equilibrium both the worker that actually matches with a firm and his immediate competitor submit the same bid. Consider now the worker $\alpha(t)$'s payoff when matching with firm $t+k^*$:

$$\omega(t+k^*,\alpha(t),x_{\alpha(t)}) - \pi^F_{t+k^*}. \tag{14}$$

Worker $\alpha(t)$'s equilibrium payoff in (9) is instead:

$$\begin{aligned} \pi^W_t \;\; &= \;\; \omega(t,\alpha(t),x_{\alpha(t)}) - \pi^F_t = \\ &= \;\; \omega(t,\alpha(t),x_{\alpha(t)}) - \omega(t,\alpha(t+k^*),x_{\alpha(t+k^*)}) \;\; + \\ &\qquad\qquad + \;\; \omega(t+k^*,\alpha(t+k^*),x_{\alpha(t+k^*)}) - \pi^F_{t+k^*}. \end{aligned} \tag{15}$$

In equilibrium the payoff in (15) needs to be higher that the payoff in (14) because $\alpha(t)$ could choose to bid low at stage t and then mimic $\alpha(t+k)$, winning at $t+k$ (the identities of the successful bidder at the other stages are unaffected). This condition gives us (13). ■

As mentioned above the result in Proposition 3 provides us with a pairwise equilibrium assortative matching condition. In other words, Proposition 3 shows that total surplus cannot be improved by some pairwise re-matching.

Notice however that there is no guarantee that the equilibrium characterized in Proposition 2 above maximizes total surplus. This equilibrium, however, satisfies a weaker efficiency property: namely there exist no division of surpluses resulting from other matches that makes all firms and workers *strictly* better off. This result is demonstrated in the following proposition.

Proposition 4. *There exists no matching of workers to firms and resulting division of surpluses to firms* $(\hat{\pi}_1^F, \ldots, \hat{\pi}_T^F)$ *and to workers* $(\hat{\pi}_{\alpha(1)}^W, \ldots, \hat{\pi}_{\alpha(S)}^W)$ *such that:*

$$\hat{\pi}_t^F > \pi_t^F \qquad \forall t = 1, \ldots, T \tag{16}$$

and

$$\hat{\pi}_{\alpha(s)}^W > \pi_s^W \qquad \forall s = 1, \ldots, T; \tag{17}$$

where π_t^F *and* π_s^W *are defined in (8), (9) and (10) above.*

Proof: Assume by way of contradiction that there exists a new matching of workers to firms, denoted $(t, \beta(t))$ for $t = 1, \ldots, T$, and a resulting division of surpluses to firms $(\hat{\pi}_1^F, \ldots, \hat{\pi}_T^F)$ and to workers $(\hat{\pi}_{\beta(1)}^W, \ldots, \hat{\pi}_{\beta(S)}^W)$ satisfying (16) and (17). As everybody is better-off, everybody must be rematched compared to the equilibrium. In particular, at stage 1 of the Bertrand competition game, worker $\beta(1)$ could submit a bid

$$B_{\beta(1)} = \hat{\pi}_1^F > B_{\alpha(1)} = \pi_1^F \tag{18}$$

that yields him payoff $\hat{\pi}_{\beta(1)}^W$. By (18) firm 1 would select this bid and by (16) and (17) this bid would represent a profitable deviation of worker $\beta(1)$ and firm 1 and hence a contradiction of the fact that $(t, \alpha(t))$ for $t = 1, \ldots, T$ is an equilibrium of the Bertrand competition game. ■

The sense in which Proposition 4 demonstrates only a weaker efficiency property of the equilibrium of the Bertrand competition game is the fact that in re-matching workers to firms we only allow transfers between the worker and the firm involved in the same match. In other words, we rule out redistribution of surplus among matches. Moreover we insist on strict Pareto improvements.

Notice that Propositions 3 and 4 are particularly surprising since we did not make any assumption on the assortative nature of the matching process.

The equilibrium payoffs presented in Proposition 2 above can be solved recursively starting from π^F_{T+1} and π^W_{T+1} as computed in (10).

Worker $\alpha(t)$'s equilibrium payoff π^W_t is the sum of the net social surplus, as in (7), and an expression $\mathcal{W}_t$ that does not depend on worker t's match specific investment x_t:

$$\pi^W_t = \omega(t, \alpha(t), x_{\alpha(t)}) + \mathcal{W}_t. \tag{19}$$

Similarly, the firm's equilibrium payoff π^F_t is the sum of the surplus generated by the match of firm t with worker $\alpha(t + k^*)$, where this worker is the immediate competitor in the bidding for the match with firm t, and an expression $\mathcal{P}_t$ that does not depend on firm t's match-specific investment y_t:

$$\pi^F_t = \omega(t, \alpha(t + k^*), x_{\alpha(t+k^*)}) + \mathcal{P}_t. \tag{20}$$

Notice that firm t's profit is unaffected by the investment of the worker with whom she is matched.

6. Workers' Investments

We consider now the worker's investment choice that precedes the Bertrand competition game. For any selected equilibrium of the Bertrand competition game recall that Proposition 2 above characterizes the equilibrium bids and payoffs to each worker and firm. Denote, as in Section 5 above, $(t, \alpha(t))$ the equilibrium match in which worker $\alpha(t)$ will be involved. Then worker $\alpha(t)$'s investment choice $x_{\alpha(t)}$ is the solution to the following problem:

$$x_t = \operatorname*{argmax}_x \omega(t, \alpha(t), x) + \mathcal{W}_t - C(x) \tag{21}$$

and is implicitly defined by the following first order conditions:

$$\omega_3(t, \alpha(t), x_{\alpha(t)}) = w_4(t, \alpha(t), y(t, \alpha(t), x_{\alpha(t)}), x_{\alpha(t)}) = \frac{dC(x_{\alpha(t)})}{dx}. \tag{22}$$

We can now characterize the efficiency properties of worker $\alpha(t)$'s investment choice: namely that each worker's investment choice is constrained efficient.[7] This result is stated in the following proposition.

Proposition 5. *For any given match implied by the equilibrium of the Bertrand competition game* $(t, \alpha(t))$, $t = 1, \ldots, T$, *worker* $\alpha(t)$*'s investment choice is constrained efficient.*

Proof: The result is proved by a comparison of the implicit definition (22) of the equilibrium investment $x_{\alpha(t)}$ with the characterization of worker $\alpha(t)$'s efficient investment choice (5) for the given match $(t, \alpha(t))$. ■

The two Propositions 1 and 5 provide a full characterization of the efficiency properties of the workers' and firms' investment choices in our environment: namely that for any selected equilibrium of the Bertrand competition game both workers and firms choose constrained efficient investments. This is true despite the fact that these investments are — at least in part — match specific and that the competition game gives rise to outcomes where each worker and each firm does not capture the full return from his/her investment decision. The rationale behind this result can be described as follows. In a dynamic setting, such as the one we consider, it is possible for both parties to a match to be residual claimants of the match surplus at different times. This is what happens in this sequential case for workers and firms.

Contracts are simple agreements that specify a constant (non-contingent) remuneration for the worker. Once each worker's compensation, determined by the Bertrand competition game, is specified in the contract, each firm is residual claimant of any return of her investment choice in excess of the remuneration she has promised the worker. At the same time, when matches and remunerations are determined,

[7] Once again the constraints are represented by the selected equilibrium of the Bertrand competition game.

workers Bertrand compete for firms in an environment in which the match surplus, and hence each worker's willingness to bid, differ across possible matches. The result is then that, in equilibrium, each worker's remuneration is the difference between the surplus generated in the match and the bid that the immediate competitor submits for the firm involved in the match. Therefore, when choosing investment, before the Bertrand competition game, each worker is also the residual claimant of the returns from his investment in excess of the competitor bid that does not depend on this investment. In this way the marginal incentives of both workers and firms to invest are efficient for any given equilibrium match.

We conclude this section with two observations. Notice, first, that Propositions 1 and 5 holds even in the case in which $T = 1$ and $S = 2$. In other words, we do not need a large competitive market to guarantee that investment choices by the firms and the workers are efficient. Indeed, what guarantees efficiency is not the size of the market but the fact that at different times both the workers and the firms are residual claimants of the match surplus and hence have efficient marginal incentives to invest.

Further, notice that the same efficiency properties do not hold in the presence of only one worker, labelled s, and one firm, labelled t. In this case Bertrand competition does not help in determining how the worker and the firm share the surplus of the match: they are in a situation of bilateral monopoly. Assume therefore that the worker gets a proportion γ of the net surplus of the match, as defined in (1), while the firm gets a proportion $(1 - \gamma)$. The firm then chooses a constrained efficient level of investment $y(x)$ implicitly defined by

$$w_3(t, s, y(x), x) = 0$$

The worker, on his part, chooses an inefficiently low level of investment $\hat{x}$ defined by the following condition:

$$\gamma w_4(t, s, y(\hat{x}), \hat{x}) = \frac{dC(\hat{x})}{dx}.$$

Notice that the worker's investment $\hat{x}$ is optimal only in the case $\gamma = 1$.

There is a sense, however, in which the case we just described is a very special one. Consider the model in which the number of firms is the same as the number of workers, $S = T$. In this case the inefficiency generated by the under-investment of firm T and the constrained efficient under-investment of worker $\alpha(T)$ are the only inefficiencies present. The rest of the firms and workers will invest efficiently as in (3) and (22). In other words, our result, and in particular the fact that both sides of the market are residual claimants at different times, still holds for firms $1, \ldots, T-1$ and workers $\alpha(1), \ldots, \alpha(T-1)$. Notice that this implies that the inefficiency generated by the under-investment of firm T and worker $\alpha(T)$ will normally be associated with a low productivity match.

7. Concluding Remarks

When both sides to a market can undertake match specific investments Bertrand competition between these sides (workers and firms) for matches may solve the hold-up problems generated by the absence of fully contingent contracts. In this paper, we have shown that when workers' investments precede Bertrand competition that, in turn, precede firms' investments, constrained efficiency can be achieved.

There is a sense in which our result is very general. Indeed, no structure is imposed on the nature of the matching process. Imposing more structure would help to guarantee efficiency of the equilibrium matches observed but would not affect the constrained efficient nature of the workers' and firms' investments.

There is however a sense in which our result is very special. In our analysis two rather different asymmetries play a critical role. In the first place, Bertrand competition implies that the incentives of one side of the market to make *ex-ante* efficient investments are correct. Indeed, the side of the market that Bertrand competes is residual claimant of the match surplus in excess of the bid needed to outbid its immediate competitor. This result is independent of the degree of competition generated by the immediate competitor; in other words, it is independent of the degree of specificity of the investment undertaken.

Notice that if we change the identity of the side of the market that Bertrand competes for a match — we assume that firms compete for workers — then workers undertake inefficient investments. This is because workers are now paid the maximum willingness to bid of the immediate competitor for the match in which they are involved. Since we consider match specific investments and a discrete number of parties on both sides of the market, this willingness to pay differs from the surplus generated in the match. However, even these parties have an incentive to invest since their investment affects, although not to an efficient degree, this willingness to pay and hence their remuneration. Of course, now the degree of specificity matters. In Felli and Roberts (1999) we show that if we impose more structure on the matching process and we assume that matching is assortative, the aggregate inefficiency generated by workers' under-investment is small in the sense that is strictly dominated by the inefficiency that would be induced by the under-investment of one sole match in which the most efficient workers is paid the willingness to bid of the least efficient firm. In other words, the incentives of both sides of the market to undertake *ex-ante* investments are either efficient or near-efficient (inefficiencies are small).

The other asymmetry that plays a critical role concerns the content of the simple contract the parties write at the end of the Bertrand competition game. If this contract specifies the remuneration of the party to the match that does not have to take any ex-post investments, the worker in our case, then the other party, the firm, is residual claimant of the match surplus in excess of the amount she promised to the match partner in the contract. Therefore this firms' incentives to undertake match specific investments are efficient. The same is not true if, once again, we change the identity of the party to the match whose remuneration is specified in the contract. In other words firms' profits are specified in the contract. In this case firms' incentives to undertake investments are fully blunted. Notice that, in the absence of fully contingent contracts, this is the main source of inefficiency present in this environment and it is the inefficiency with which institutional mechanisms (such as private ownership) or contractual devices (such as options to own) should be concerned.

References

Acemoglu, D. (1997): "Training and Innovation in an Imperfect Labor Market," *Review of Economic Studies*, 64, 445–464.

Acemoglu, D., and R. Shimer (1998): "Holdups and Efficiency with Search Frictions," mimeo.

Aghion, P., M. Dewatripont, and P. Rey (1994): "Renegotiation Design with Unverifiable Information," *Econometrica*, 62, 257–82.

Aghion, P., and J. Tirole (1997): "Formal and Real Authority in Organizations," *Journal of Political Economy*, forthcoming.

Burdett, K., and M. Coles (1997): "Marriage and Class," *Quarterly Journal of Economics*, 112, 141–68.

Cole, H., G. Mailath, and A. Postlewaite (1998): "Efficient Non-Contractible Investments," mimeo.

Cooper, R., and A. John (1988): "Coordinating Coordination Failures in Keynesian Models," *Quarterly Journal of Economics*, 103, 441–63.

de Meza, D., and B. Lockwood (1998): "The Property-Rights Theory of the Firm with Endogenous Timing of Asset Purchase," mimeo.

Diamond, P. (1971): "A Model of Price Adjustment," *Journal of Economic Theory*, 3, 156–68.

——— (1982): "Wage Determination and Efficiency in Search Equilibrium," *Review of Economic Studies*, 49, 217–27.

Eeckhout, J. (1999): "Bilateral Search and Vertical Heterogeneity," *International Economic Review*, forthcoming.

Felli, L., and C. Harris (1996): "Learning, Wage Dynamics, and Firm-Specific Human Capital," *Journal of Political Economy*, 104, 838–68.

Felli, L., and K. Roberts (1999): "Does Competition Solve the Hold-up Problem?," mimeo.

Grossman, S. J., and O. D. Hart (1986): "The Costs and Benefits of Ownership: A Theory of Vertical and Lateral Integration," *Journal of Political Economy*, 94, 691–719.

Grout, P. (1984): "Investment and Wages in the Absence of Binding Contracts: A Nash Bargaining Solution," *Econometrica*, 52, 449–460.

Hart, O. (1979): "Monopolistic Competition in a Large Economy with Differentiated Commodities," *Review of Economic Studies*, 46, 1–30.

Hart, O. D., and J. Moore (1988): "Incomplete Contracts and Renegotiation," *Econometrica*, 56, 755–85.

——— (1990): "Property Rights and the Nature of the Firm," *Journal of Political Economy*, 98, 1119–58.

Holmström, B. (1999): "The Firm as a Subeconomy," *Journal of Law Economics and Organization*, 15, 74–102.

MacLeod, B., and J. Malcomson (1993): "Investments, Holdup and the Form of Market Contracts," *American Economic Review*, 83, 811–37.

Maskin, E., and J. Tirole (1999): "Two Remarks on the Property-Rights Literature," *Review of Economic Studies*, 66, 139–50.

Moen, E. (1997): "Competitive Search Equilibrium," *Journal of Political Economy*, 105, 385–411.

Mortensen, D., and C. Pissarides (1994): "Job Creation and Job Destruction in the Theory of Unemployment," *Review of Economic Studies*, 61, 397–416.

Nöldeke, G., and K. M. Schmidt (1995): "Option Contracts and Renegotiation: A Solution to the Hold-Up Problem," *RAND Journal of Economics*, 26, 163–179.

Roberts, K. (1996): "Thin Market Externalities and the Size and Density of Markets," paper presented at the Morishima Conference, Siena 1996.

Segal, I., and M. Whinston (1998): "The Mirrlees Approach to Implementation and Renegotiation: Theory and Applications to Hold-Up and Risk Sharing," mimeo.

Williamson, O. (1985): *The Economic Institutions of Capitalism*. New York: Free Press.