Bidding in an Electricity Pay-as-Bid Auction*

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Abstract

One of the main elements of the current reform of electricity trading in the UK is the change from a uniform price auction in the wholesale market to discriminatory pricing. We analyse this change under two polar market structures (perfectly competitive and monopolistic supply), with demand uncertainty.

We find that under perfect competition there is a trade-off between efficiency and the level of consumer surplus between the two auction rules. We also establish that a move from uniform to discriminatory pricing under monopoly conditions has a negative impact on profits and output (weakly), a positive impact on consumer surplus, and ambiguous implications for welfare and average prices.

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1 Introduction

The recent reform of electricity trading arrangements in England & Wales is the main motivator for this paper. The electricity regulator's *New Electricity Trading Arrangements* (*NETA*), which have been introduced in March 2001, consist mainly of replacing the existing day-ahead *system-marginal-price* (SMP) auction (i.e. a uniform price auction) with a *pay-as-bid* (PAB) auction (i.e. a discriminatory auction) in a balancing market preceded by bilateral contracting (Ofgem (1999)). This paper is inspired by this reform, and seeks to provide some analytical insights on its potential impact.[1]

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[1]The issue of whether to introduce PAB electricity pricing has also been raised in the Californian wholesale market, and the U.S. federal electricity regulator (FERC) has introduced PAB for "high" bids (above \$150/MWh) as a temporary market power mitigation measure (FERC (2000)).

The choice between uniform and discriminatory pricing has been a controversial issue in other multi-unit auction markets, most notably in securities auctions.

In this paper we compare the effects of a change in the price rule from SMP to PAB by considering a sealed-bid multi-unit procurement auction, under two polar market structures: *perfect competition* (i.e. each bidder can only sell one infinitesimal unit of output) and *perfect collusion* (i.e. monopoly bidding). We model this auction under conditions of demand uncertainty and of complete information over costs, both of which broadly characterise electricity wholesale markets.

We find that under both market structures a switch from SMP to PAB leads to a fall in average output and an increase in consumer surplus (as long as demand is elastic). Under competitive conditions the reduction in output always implies a fall in welfare, so that there is a clear 'efficiency-consumer surplus' trade-off in moving from SMP to PAB. Under monopoly bidding the introduction of PAB has ambiguous implications for efficiency: we find that if (and only if) demand uncertainty is contained and the marginal cost schedule flat (relative to demand), PAB can lead to both higher consumer surplus and higher efficiency. However, with relatively high demand uncertainty and steep costs, welfare falls with PAB, and average prices might also rise (notwithstanding the increase in consumer surplus). Our results therefore show that the exercise of market power is harder under PAB, and that firms with market power may react in inefficient ways to a switch from SMP to PAB.

In discussing our results we elaborate on the links between monopoly SMP and PAB bidding and price discrimination. We argue that SMP allows the monopolist to neutralise the effects of demand (or "type") uncertainty and obtain an optimal price for each demand realisation (subject to the "no-fixed-fees" constraint implied by the auction rules we model). Monopoly SMP bidding is therefore analogous to third-degree price discrimination. Under PAB bidding on the other hand the monopolist suffers from the presence of demand uncertainty, and its problem is essentially equivalent to finding the optimal non-linear pricing schedule, with the additional presence of a "no-fixed-fees" and "no-quantity-discounts" constraint relative to the standard case. PAB can therefore be thought of as "fourth-degree price discrimination" (i.e. constrained non-linear pricing, or third degree price-discrimination with type-uncertainty).

We also comment on the impact of a change of price rule in the context of oligopolist interaction (i.e. the intermediate case between perfect competition and perfect collusion). We argue that, on the basis of existing results from multi-unit auction theory, switching from SMP to PAB may have strong effects in this case, by changing the nature of competition from "Cournot" to "Bertrand" with an associated reduction in market power.

The rest of this introductory section proceeds as follows: we firstly describe the nature of electricity auctions and of the *New Electricity Trading Arrangements* (*NETA*) introduced by the UK industry regulator Ofgem; we also briefly examine the relevant literature on multi-unit auction theory, in particular on the comparison between uniform and discriminatory price auctions, and then outline the rest of the paper.

1.1 Electricity Auctions, the *Pool* and *NETA*

In liberalised energy markets at least some wholesale electricity is typically traded in an auction-like environment, where producers (or generators) submit supply schedules to a System Operator (SO hereafter) which is responsible for the real-time balancing of aggregate supply and demand.

Most of the demand in electricity auctions (or *Pools*) is bid by the SO itself, which aggregates it from downstream distribution companies. Direct (and price-responsive) demand-side bids are often allowed in these auctions, but typically make up a small proportion of total demand.[2]

Depending on the market design the proportion of total production which electricity auctions trade varies from 100% (as in the original "gross" England and Wales Pool) to small proportions of energy (as in systems which rely on bilateral contracting with a balancing pool, e.g. Norway and Sweden).[3]

Electricity auctions are typically repeated very frequently (e.g. every day) and work as follows: suppliers (i.e. generators) submit price-quantity bids for production and an auctioneer (i.e. the SO) then constructs a non-decreasing aggregate bid function and crosses demand and supply. Under SMP-pricing, producers who are "in merit" (i.e. whose bid is below the marginal or "stop-out" price at which aggregate supply equals demand) earn the marginal price times the quantity bid. Under PAB-pricing, producers earn their own bid times their bid quantity, as long as they are "in merit".

The aggregate marginal cost structure in wholesale electricity markets tends to be well known to market participants, as generation technology is relatively standard. Marginal costs of production can however vary substantially across generation units of different technology. Given the non-storability of electricity and the need to meet significant demand peaks, the optimal industry plant-mix is usually characterised by an upward sloping industry marginal cost schedule which is associated with a downward-sloping fixed costs profile. For example the current England and Wales plant mix ranges from nuclear generators (high fixed cost, low marginal cost) to open cycle gas turbines (low fixed cost, high marginal cost).

The trading arrangements in England and Wales before the introduction of *NETA* were based around a day-ahead "Pool", which was introduced when the industry was liberalised in 1990. All generators wishing to produce in the Pool needed to place their bids for the next day to the SO. They did this once per day, specifying three components of cost,[4] which the SO then used to set prices for each half-hour of the following day, using an algorithm based on a SMP-type pricing rule.

The performance of the Pool since its set-up has been far from ideal, mainly because of the presence of market power by the incumbent generators.[5] Prices in the Pool have risen steadily (by about 25% in real terms) in the first five years of the market (1990-1995), in spite of falling fuel costs and considerable entry by independent producers. Despite further falls in the cost of generation and of entry and a substantial reduction in concentration relative to the first years of the market, current prices are still above those of 1990.[6,7]

[2] Wolfram (1999a) estimates price elasticities of around 0.1 on England and Wales pool data.

[3] See Wilson (1999) for a discussion of issues relating to market design and decentralisation in electricity markets.

[4] These included one "pure" marginal cost (i.e. a fuel cost) and two partially "fixed" costs (i.e. no-load heat and start-up costs).

[5] When the industry was privatised the two main generation companies, National Power and PowerGen, controlled almost 80% of total capacity in the wholesale electricity market (Ofgem (1998)).

[6] Econometric work has confirmed the existence of market power in the England & Wales Pool (Wolfram (1999)).

[7] The issue of market power has dominated regulatory activity vis-à-vis the wholesale market, prompting the

The dissatisfactory performance of the UK wholesale electricity market since 1990 has been partially attributed to its market design. The industry regulator and many commentators have argued that the auction has been easy to manipulate, given the presence of a highly public price signal, the high frequency of interaction between producers, and the effects of the uniform-price rule, which allowed generators with market power to guarantee a satisfactory level of output with their infra-marginal bids and set high prices using their marginal bids (Ofgem (1999)). The Pool has also been criticised for inducing excess entry by independent producers, which have been able to easily 'free-ride' on the market power of incumbent generators thanks to the properties of the uniform price rule.

The dissatisfaction with the Pool prompted a major review of electricity trading arrangements by the industry regulator, which started in 1998. This culminated in March 2001 with the introduction of a radically different market design, i.e. *NETA*.

The *NETA* reforms have abolished the day-head Pool, which has been substituted by three separate markets: a long-term contract market, a short-term (e.g. on-the-day) screen-based Power Exchange (PX), and a "real-time" balancing market (BM). The last of these markets is operated by the SO from 3 and 1/2 hours (or less) before real-time until real-time. In this market the SO calls for half-hourly demand and supply bids to balance the market.[8] The BM settles bids and offers which are "in merit" according to a pay-as-bid price rule, and charges/pays players which are out-of-balance after the contract markets the demand-weighted average of offers to produce (or decrease consumption) or of bids to decrease production (or increase consumption), according to whether the player is short or long of energy.[9]

To model the switch from the original Pool to NETA, this paper focuses on *NETA*'s BM, abstracting from the presence of the markets which precede it. We do so for reasons of tractability, and because backwards induction arguments suggest that the design of the BM will have a significant impact on earlier trading (especially in the PX), and is therefore a central element of *NETA* (even though it may involve relatively limited amounts of energy). We recognise in our modelling the residual nature of the BM market, and therefore emphasise the issue of demand uncertainty, which is likely to be a prominent feature of this market.

1.2 Insights from the Auction Theory Literature

The central issue we model in this paper is the comparison between uniform and discriminatory multi-unit sealed-bid procurement auctions. We do so in a setting which broadly corresponds to an electricity auction. That is, we model an environment where demand is endogenous (even though on aggregate relatively inelastic) and uncertain, due to the presence of stochastic shocks and, in the case of the real-time balancing market, unknown ex-ante contract position by market

UK regulator Ofgem to impose a cap on Pool prices during the period 1994-1996 and ordering plant divestment by the two incumbent generators in both 1996 and 1999 (as a result of which approximately 25% of the total capacity in the market changed ownership) (Competition Commission (2000)).

[8] Balancing requirements may arise because of the need to meet unexpected changes in players' positions, e.g. a generator may have an outage, or players may simply wish to change their production or consumption schedules relative to their commitments in the contract markets.

[9] The system therefore displays so-called "dual imbalance pricing". We abstract from this feature in our modelling.

participants. In addition costs are assumed to be known to all market participants, which is broadly the case in wholesale electricity markets.

The set-up just described is therefore not the typical auction-theory environment, where what is uncertain is the distribution of costs (or values) across bidders or the common value of the object(s) being auctioned. There are however insights which can be gained from the auction theory literature in relation to the three central features of the environment we model: the comparison between pricing rules in divisible goods auctions; demand elasticity (or endogenous quantity); and demand uncertainty.

Pricing rules in multi-unit auctions are examined by a number of authors, most notably Wilson (1979), Back and Zender (1993), Wang and Zender (1999) and Ausubel and Cramton (1998). These papers examine auctions of divisible goods ("share auctions") in a multi-player context. Both Wilson (1979) and Back and Zender (1993) show that in a common-value auction uniform pricing can enable strategic bidders to obtain seemingly collusive outcomes.[10] As Back and Zender show this can lead to much lower revenue for the seller by comparison with discriminatory auctions in equilibrium. For example in the case of no uncertainty over a common and constant value of the objects for sale v and no capacity constraints, bidders will bid flat bid functions at v in any pure strategy equilibrium of the discriminatory auction (i.e. a "Bertrand" outcome will prevail). By contrast uniform price auctions can sustain a multiplicity of equilibria, some of which have prices well below v.[11,12]

Ausubel and Cramton (1998) extend this analysis to a context with private values, showing that in many "reasonable" cases (e.g. i.i.d. values, flat and symmetric demand schedules) uniform pricing always implies an inefficiency relative to a pay-as-bid auction and leads to lower revenues (i.e. higher prices, in an auction to sell goods, as the one we model in this paper). This is due to the effects of market power in a uniform price auction which arises from the fact that large players have incentives to bid strategically to affect their profits on infra-marginal units. This result however does not carry over to asymmetric cases, and the comparison between pay-as-bid and uniform pricing in efficiency terms is in general ambiguous in a private values setting.[13]

Most papers in the auction theory literature deal with fixed quantities (for sale or purchase). Endogenous quantity changes results, as shown by Hansen (1988). He considers a procurement auction with elastic demand, and a winner–takes-all context (i.e. an indivisible-good situation).

[10] These papers therefore show that the multi-unit uniform price auction is not analogous to the single-unit second-price auction and that, in particular, it does not induce truthful bidding, as many commentators have informally argued (especially in the context of the U.S. securities auctions).

[11] This is because uniform price auctions allow bidders to submit very steep demand schedules, which imply a high cost of deviation from a quantity-withdrawal (or market-sharing) equilibrium for rivals, thus enforcing a low-price equilibrium which is qualitatively similar to a "Cournot" outcome. With discriminatory auctions "out-of-equilibrium" bids of this kind have a direct impact on the price, and are therefore not optimal. We discuss this point further in Section 4.2.

[12] When players receive independent signals about the common value of the objects for purchase a Winner's Curse effect will be present. This is likely to be stronger in a discriminatory price environment relative to a uniform price setting, but the trade-off betwen this effect and the strategic bidding effect on the seller's revenue does not seem to be well understood yet (see Wang and Zender (1999) and their "conjecture" (p. 28)).

[13] This arises because of the "first-price" features of discriminatory price auctions, which tend to reduce efficiency in the presence of asymmetries between bidders.

Quantity endogeneity implies that a first-price auction (i.e. pay-as-bid) yields lower prices than a second-price auction, since in the latter prices are determined by the producer with the second-lowest cost, which reduces quantity and increases the deadweight loss. This result is however derived in a single-winner setting, and is not directly applicable to the comparison between uniform and discriminatory pricing in multi-unit auctions that we model in this paper. This is because in our set-up both pricing rules determine market-clearing quantity at the intersection of the aggregate bid curve and demand, eliminating the quantity reduction effect of second-price rules.

Demand uncertainty in auctions is examined by Klemperer and Meyer (1989), who consider a multi-unit uniform-price procurement auction with uncertain and downwards sloping demand. They show that uncertainty makes the exercise of market power harder in an oligopolistic context, lowering profits relative to a Cournot outcome and making the most implicitly collusive strategies described by Back and Zender in uniform price auctions unfeasible. A related insight is provided by McAdams (2000) who shows that the seemingly collusive equilibria of the uniform price auction are eliminated in both the adjustable-supply auction (where the auctioneer sets quantity after the bids have been made, to maximise revenue) and in the increasable-supply auction (which is like the adjustable-supply format, with an additional minimum-quantity constraint). This is due to the fact that in both of these cases demand-uncertainty is used by the auctioneer to unravel strategic bidding.

Finally, Back and Zender (1993) and Wang and Zender (1999) find that with (bounded) uncertainty over the quantity sold and no uncertainty (or symmetric signals) over the common value of the objects on sale, discriminatory auctions still outperform uniform-price auctions in terms of seller revenues for all but one of the equilibria of the uniform-price auction.

1.3 Approach and Structure of the Paper

In this paper we model the difference between SMP and PAB auctions for two polar cases of market structure-conduct: perfect competition and perfect collusion. We therefore abstract from strategic interaction between players. Under complete information over costs and in the absence of demand uncertainty the two pricing regimes yield the same result in these settings: under perfect competition all players would be able to identify the marginal production unit on the system for any given level of demand, so that under PAB they would submit a bid equal to the marginal cost of that unit, achieving the same outcome they would obtain under SMP by simply bidding at cost. Similarly, the monopolist under PAB and with no demand uncertainty can bid all of its quantity at the price which satisfies the inverse elasticity rule (so that $MR = MC$). Under SMP the same outcome can achieved by submitting a marginal bid which satisfies this condition. However, as we show in what follows, a shift from SMP to PAB has an impact on both market prices and quantities in the presence of demand uncertainty. As we argued above, demand uncertainty is likely to characterise the electricity Balancing Mechanism under *NETA* given its role as a residual market.[14]

[14] Demand uncertainty can be a significant factor also in "gross" pools (i.e. even when aggregate hourly demand is known with precision ex-ante) if generators can only offer a single bid for multiple demand periods (e.g. as in the original England and Wales market design). Our analysis, which assumes demand uncertainty, is therefore

Sections 2 and 3 present our results on perfect competition and perfect collusion respectively, deriving the SMP and PAB equilibrium bid functions in each case, and comparing output, price/consumer surplus and welfare outcomes across the two auction rules. In Section 4 we discuss three aspects of our results: the relationship between monopoly PAB bidding and price discrimination (with a particular focus on non-linear pricing); the implications of the choice between PAB and SMP on strategic interaction; and the impact of PAB on market dynamics (and entry in particular). Section 5 summarises our results and concludes drawing some implications of our analysis for electricity market design.

2 Perfect Competition

2.1 Set-up

In this section of the paper we model a multi-unit procurement auctions under conditions of perfect competition. This is an interesting case given its nature as a benchmark of bidding behaviour in electricity auctions, and also its potential relevance as the long-run market structure of a de-regulated industry (e.g. following entry by independent producers, and the "commoditisation" of the market).[15]

We therefore assume an atomistic market structure, with many independently-owned electricity producers and with each producer supplying one infinitesimal unit of output dq at a cost of γq, where q can be interpreted as an index for an individual producer supplying dq. The qth producer's position in the industry's aggregate marginal cost curve corresponds precisely to this index. The industry marginal cost function is thus given by $MC(q) = \gamma q$, with $\gamma \geq 0$.

The demand-side is represented by a linear income-inelastic inverse demand curve, $p(q) = \mu - \rho q$, where $\mu \sim U[\underline{\mu}, \bar{\mu}]$ and $\rho \geq 0$, which is bid truthfully into the market by an auctioneer (or System Operator) under both auction rules.

Producers are assumed to be risk-neutral. Each producer submits a bid for its entire (infinitesimal) unit of capacity into the market. Aggregating these individual bids in "merit order", that is, from cheapest to most expensive, yields the industry's non-decreasing bid function, $\beta(q)$.[16]

The market clearing process is the same under SMP and PAB, and determines equilibrium quantity at the intersection of realised demand with the aggregate bid function. Payments by the auctioneer to the producers however differ across the two auction regimes: under SMP all producers which bid below or at the market clearing price obtain this price (i.e. the marginal and average price paid by demand coincide), whilst under PAB producers are paid their bid,

also relevant to this market design option.

[15] Arguably the current market structure in England and Wales is already moving towards competitive conditions, given the large sales of power plants by the incumbent producers (especially National Power and PowerGen) which have occurred over the course of 1999 and 2000.

[16] Both in this case and in the monopoly case modelled below, we assume that the industry cost and bid functions are smooth, following the approach introduced by Klemperer and Meyer (1989), and first applied to electricity markets by Green and Newbery (1992). This differs from the discrete step functions approach introduced by von der Fehr and Harbord (1993), especially in the context of static strategic interaction (see Section 4.2).

as long as this is below or equal to the market clearing price (i.e. the marginal price is always above the average price paid by demand).[17]

2.2 SMP

Under SMP each producer optimally bids its output at cost, since the price it receives is exogenous to its bid, given that it is never price-setting by assumption (this is the standard second-price auction result, in a context where no player is selling more than one unit). The industry's optimal bid function therefore corresponds to the industry marginal cost function γq, which implies that all profitable gains from trade are exhausted.

2.3 Pay-as-Bid[18]

In the PAB equilibrium each producer's bid needs to maximise expected profits (assuming risk-neutrality), which are given by:

$$E(\pi) = Pr_q(\text{“in merit”})(\beta(q) - \gamma q) = (1 - F(\beta(q)))(\beta(q) - \gamma q)$$

where Pr_q(“in merit”) indicates the probability of each generator being called to produce (i.e. being “in merit”). This is given by $1 - F(\beta(q)))$, where $F(\cdot)$ denotes the cumulative distribution function of the marginal bid. Expected profits are therefore the product of the probability of a bid being accepted and of the mark-up on cost associated with this bid. In contrast with the SMP auction, bidders face a trade-off between the profitability and the probability of producing. This is because higher bids reduce the likelihood of being called to produce, but also increase the mark-up over cost earned if the unit is “in merit”.

Each producer maximises profit given other producers' bids, as captured by $F(\cdot)$. We therefore look for bids that are consistent with this maximisation. Taking first-order conditions with respect to β we find that:

$$\beta(q) - \gamma q = \frac{1 - F(\beta(q))}{f(\beta(q))} =: \frac{1}{h(\beta(q))} \tag{1}$$

where $f(\beta)$ denotes the probability density function of β, and $h(\cdot)$ is the hazard rate of β. Immediately we have two results: a producer at the margin for the highest demand realisation (i.e. where $F(\beta(q)) = 1$) bids at cost, and the bid price exceeds cost everywhere ‘below’. The “no distortions at the top” result therefore holds in this setting, even though for reasons which differ from those present in the standard optimal mechanism design problem.[19]

[17] This is equivalent to assuming that also under PAB demand pays the marginal bid on all the units purchased, and that the surplus earned by the auctioneer due to the fact that generators are paid-as-bid is rebated to final consumers in a lump-sum manner. This market-clearing assumption allows us to “fix” the level of demand (for a given bid function) across the two auction formats, and focus the analysis on the supply-side effects of the change in pricing rule. An alternative approach, which we discuss for the case of monopoly, would be to assume that consumers purchase electricity as long as the average bid they pay is below their marginal benefit.

[18] This section is partially based on Federico and Rahman (1998).

[19] A similar result is shown by Nautz (1995).

Assume now that $\beta(q)$ is linear.[20] It follows that $\frac{1}{h(\cdot)}$ must be a linear function of μ. Given that μ is uniform we can rewrite $\frac{1}{h(\cdot)}$ as:

$$\frac{1}{h(\cdot)} = \bar{\beta} - \beta(q) \tag{2}$$

where $\bar{\beta}$ is defined by the intersection of the industry bid function with maximum demand.

Substituting (2) into (1) yields the optimal linear bid function $\beta_c^*(q)$ (where subscript c stands for 'competitive'):

$$\beta_c^*(q) = \frac{\gamma q + \bar{\beta}}{2} = \frac{\gamma}{2}(q + \bar{q}) \tag{3}$$

where $\bar{q}$ is defined as the intersection of the industry bid function (and of the aggregate marginal cost schedule) with maximum demand. The second expression for $\beta_c^*(q)$ follows from the fact that the producer who is in the merit order only if demand is at its highest level must bid at cost, as shown above.[21]

Equation (3) defines the optimal linear bid function for $q > \underline{q}_{PAB,c}$, where $\underline{q}_{PAB,c}$ is given by the intersection of $\beta_c^*(q)$ with the demand schedule at $\mu = \underline{\mu}$, as long as this is positive, and is zero otherwise (i.e. $\underline{q}_{PAB,c} = \max(0, q(\underline{\mu}, \beta_c^*(q)))$. Producers with marginal costs lower than $\gamma \underline{q}_{PAB,c}$ find it optimal to bid at $\underline{\beta} = \beta_c^*(\underline{q}_{PAB,c})$ (which is higher than $\beta_c^*(q)$) given that they run with probability equal 1 even when they raise their bids to $\underline{\beta}$. The optimal bid function is therefore horizontal up to $\underline{q}_{PAB,c}$ and then it is upwards sloping, increasing with q at half the slope of the marginal cost curve. Producers with cost higher than $\gamma \underline{q}_{PAB,c}$ find it therefore optimal to bid at the average of their cost and the marginal cost at maximum demand (see Figure 1). This is the unique equilibrium bid function in a competitive PAB market, as proven in Appendix A.1.

2.4 SMP-PAB Comparison

2.4.1 Output and Efficiency

Under competitive conditions output is always lower in the PAB auction than in the SMP one, given that under PAB all players (except for the one which produces only if demand is at its maximum level) mark-up their cost. This induces a deadweight loss for all demand realisations, except for the one corresponding to $\mu = \bar{\mu}$. This in turn implies that expected welfare is lower under PAB than under SMP, in the competitive benchmark.[22]

[20] We prove in Appendix A.1 that this is so in equilibrium.

[21] Substituting for the demand curve, yields the optimal bid function in terms of the underlying parameters of the model, i.e.:

$$\beta_c^*(\mu) = \frac{\gamma}{2\rho + \gamma}\left(\mu + \frac{\rho}{\rho + \gamma}\bar{\mu}\right)$$

[22] The difference in welfare between SMP and PAB is due to lower *allocative* efficiency under PAB. Both the SMP and PAB rules achieve *productive* efficiency, i.e. the aggregate cost of producing the equilibrium level of output is minimised. This result, for the PAB case, relies on the assumption that all producers have the same attitude to risk (namely, risk neutrality). Allowing for differential attitudes to risk among producers would lead to the possibility of inefficient production with PAB, but not with SMP.

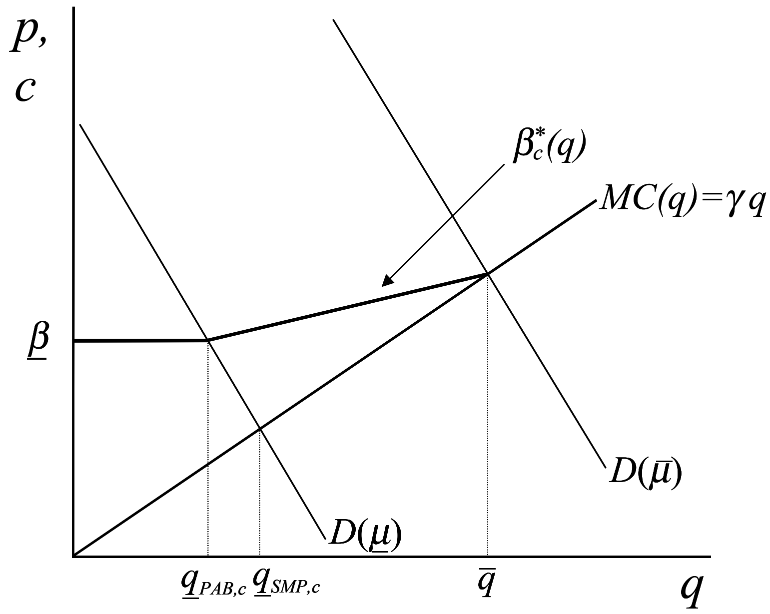

Figure 1: The competitive PAB equilibrium (for $\frac{\bar{\mu}}{\underline{\mu}} \leq \frac{2(\gamma+\rho)}{\gamma}$).

Note also that it is possible that under PAB the bid function lies above the demand schedule for some low-demand realisations (i.e. if $\beta^*_c(0) \equiv \frac{\gamma}{2}\bar{q} > \underline{\mu}$). This is the case if uncertainty is high (namely, $\frac{\bar{\mu}}{\underline{\mu}} > \frac{2(\gamma+\rho)}{\gamma}$), in which case the output-contraction and welfare-reduction effect due to PAB is even stronger.

2.4.2 Profits

From the PAB equilibrium bid function we obtain the following Lemma, which describes the impact of a switch from SMP to PAB on producers' expected profits.

Lemma 1 *All producers except for the one with marginal cost $\gamma\bar{q}$ earn lower expected profits under PAB than SMP, as long as demand is elastic. The absolute reduction in profits due to the introduction of PAB is decreasing in the producers' marginal cost.*

Proof. The fact that producers lose out from PAB follows from the fact that (i) $\underline{\beta}$ is below average prices under SMP (i.e. $\underline{\beta} = \frac{\gamma}{2}(\underline{q}_{PAB,c} + \bar{q}) < \frac{\gamma}{2}(\underline{q}_{SMP,c} + \bar{q})$ given that $\underline{q}_{SMP,c} > \underline{q}_{PAB,c}$ (where $\underline{q}_{SMP,c}$ is given by the intersection of minimum demand with the industry marginal cost schedule) (see Figure 1). This implies that low-cost (or base-load) producers (i.e. those with marginal costs below $\gamma\underline{q}_{PAB,c}$, and which therefore always produce under both price rules) suffer a fall in expected profits as a result of the shift to PAB, given that under SMP they earn the average price, whilst under PAB they earn $\underline{\beta}$; (ii) producers with marginal costs between $\gamma\underline{q}_{PAB,c}$ and $\gamma\underline{q}_{SMP,c}$ earn lower prices when they produce under PAB than SMP (following the same line of reasoning of case (i)) and also produce less frequently under PAB; (iii) producers with marginal costs between $\underline{q}_{SMP,c}$ and $\bar{q}$ earn the same expected revenue *when they produce* under the two price regimes (i.e. the average of their own costs and of $\gamma\bar{q}$) but are called to produce less frequently under PAB.

The fact that low-cost producers suffer more than other producers from the introduction of PAB follows directly from the computation of expected profits, and is proved in Appendix A.2. ■

Lemma 1 shows that PAB forces producers to give up expected rents relative to SMP. This is because marking-up bids over cost hurts them under conditions of demand elasticity, by reducing total output. The result that low-cost producers suffer more than others in a PAB auction arises because these producers face a higher opportunity cost (in term of foregone profits) of not being dispatched than other producers, which "forces" them to bid aggressively to run with certainty and prevents them from "free-riding" on the marginal prices set by producers with higher marginal costs (i.e. mid-merit and peak producers).[23] The possible dynamic implications of this result, in terms of the industry's market structure, are discussed in Section 4.3.

2.4.3 Prices and Consumer Surplus

When evaluating a change of market rules, utility regulators typically place more weight on its impact on consumers (and, indirectly, on market power), than on overall efficiency considerations. It is therefore important, from the point of view of regulatory policy, to assess the effects on consumers of a switch from SMP to PAB. We do so by reference to two measures: consumer surplus and demand-weighted (DW) average prices (i.e. the average unit expenditure of buyers in the auction). We compute the latter because this is what is typically reported as an indicator of market power in electricity markets (given its ease of measurement). However, as we show for the case of monopoly modelled below (see Section 3.5.3), consumer surplus and DW average prices *can both increase* as a result of a switch from SMP to PAB, implying that it is not always valid to assess the impact of a reform on consumers by simply considering its effect on DW average prices.

Expected consumer surplus is given as the difference between expected gross surplus and expected revenue (or expenditure). For the purpose of the welfare comparison between SMP and PAB, it is convenient to express both expected gross surplus and expected revenues in terms of equilibrium q rather than in terms of the underlying parameter μ. Expected gross surplus is given by:

$$E(GS_{j,c}) = \begin{cases} \int_{\underline{q}_{j,c}}^{\bar{q}} q_{j,c}(\mu - \frac{\rho}{2}q_{j,c})f(q_{j,c})dq_{j,c} & \text{for } \underline{q}_{j,c} > 0 \\ \Pr(\mu > \beta_c^*(0)) \int_0^{\bar{q}} q_{PAB,c}(\mu - \frac{\rho}{2}q_{PAB,c})f(q_{PAB,c})dq_{PAB,c} & \text{for } \underline{q}_{PAB,c} = 0 \end{cases} \tag{4}$$

where $q_{j,c}$ indicates equilibrium quantity in the competitive case and $j \in \{SMP, PAB\}$.

Expected SMP revenue is as follows:

$$E(R_{SMP,c}) = \int_{\underline{q}_{SMP,c}}^{\bar{q}} \gamma q_{SMP,c}^2 f(q_{SMP,c})dq_{SMP,c}$$

[23] Wolfram (1999b) draws similar implications in discussing NETA.

Under PAB, expected revenue is:

$$E(R_{PAB,c}) = \Pr(\mu > \beta_c^*(0)) \left[\beta_c^*(\underline{q}_{PAB,c})\underline{q}_{PAB,c} + \int_{\underline{q}_{PAB,c}}^{\bar{q}} \beta_c^*(q)\left(1 - F(q_{j,PAB})\right) dq_{j,PAB} \right]$$

Under both price regimes, DW average prices are given by the ratio of expected revenue to expected quantity.

The following Proposition states our results on consumers surplus and DW average prices under competitive conditions.

Proposition 1 *Consumer surplus is lower and demand-weighted average prices are higher under SMP than under PAB in a competitive setting, as long as demand is elastic. If demand is vertical, the two price rules are equivalent, in terms of both expected revenue and expected consumer surplus.*

Proof. See Appendix A.3.

This result shows that consumers always benefit from a switch from SMP to PAB under competitive conditions, as long as demand is elastic. This implies that some of the reduction in industry profits brought about by this reform translates into an increase in consumer surplus, and not just into a reduction in overall welfare.

This 'market power mitigation' effect is due to the fact the with PAB the average price paid by demand falls relative to SMP for high demand realisations, i.e. PAB reduces the relatively high price-quantity correlation which characterises SMP pricing. This leads to a reduction in demand-weighted prices (by making prices less peaky),[24] and it boosts consumer surplus for high demand realisations (especially given the fact that consumption is relatively undistorted with PAB if μ is close to $\bar{\mu}$). These effects are strong enough to outweigh the negative effects of PAB on consumers at low demand realisations (i.e. higher average prices and distortions in consumption), even when demand uncertainty is high (which implies that the output contraction brought about by PAB is relatively large).

Proposition 1, combined with the results on the output comparison, reveals that *there is a direct trade-off between efficiency and consumer surplus when comparing PAB and SMP pricing rules in a competitive setting.* A switch from SMP to PAB reduces average output and welfare, but it also reduces average expenditure by the auctioneer, and raises consumer surplus. The magnitude of this changes is directly related to the price elasticity of demand.

Our results therefore suggest that introducing PAB in electricity auctions under competitive conditions can enable policy-makers (e.g. the industry regulator) to "deliver" lower consumer prices (and therefore mitigate 'market power'),[25] but that this comes at the cost of lowering

[24]Note that simple (or time-weighted) average prices are lower under SMP than under PAB if demand is sufficiently inelastic. For instance if demand is vertical demand-weighted average prices are the same under the two pricing rules, which implies that simple average prices are lower under SMP given that higher price-demand correlation induced by this pricing rule.

[25]'Market power' can be said to be present also under perfectly competitive conditions, since generators earn inframarginal rents due to the presence of an upwards sloping industry marginal cost schedule. However, in the competitive long-run equilibrium, these rents are needed to cover the fixed costs of production, which implies that PAB may lead to the exit of some producers. We discuss the possible implications of this point in Section 4.3.

the overall efficiency of the market. However, given the relative inelastic nature of electricity demand, the size of these effects is likely to be limited.

3 Monopoly (Perfect Collusion)

3.1 Motivation

What happens to the exercise of market power when we switch from SMP to PAB? We examine the monopoly case, as the benchmark case of pricing behaviour under conditions of market power.

In an electricity context one can think of the monopolist as being either a large generator with a competitive (and price-taking) fringe, or a group of large generators which, thanks to the incentives provided by frequently repeated interaction, perfectly collude and act as a monopolist in the market (i.e. they succeed collectively to extract monopoly profits).[26]

Given the high frequency of interaction in electricity auctions, the insights gained by examining the monopoly case are relevant to understanding the impact of changes in pricing rules in wholesale electricity auctions in the presence of market power.

3.2 Set-up

The set-up we employ to model monopoly bidding in an electricity auction is based on the same assumptions made in the competitive case (see Section 2.1), with the key difference that the atomistic producers indexed q are now assumed to be under joint ownership, implying an aggregate cost function for the monopolist given by $C(q) = \frac{1}{2}\gamma q^2$.

We also explicitly assume here that bidding rules of the electricity auction we model are such that the monopolist needs to bid each of its units of production separately, and cannot offer different payment-quantity bundles (e.g. as under second-degree price discrimination).[27] This means that the monopolist bid function under both SMP and PAB needs to be non-decreasing in quantity, since the auctioneer can always "pick" the cheap bids first.[28] As we discuss below, this assumption therefore rules out both fixed fees and quantity-discounts constraining the monopolist in its pricing behaviour.

[26] In the presence of high discount factors the monopoly equilibrium is typically a sustainable outcome of the repeated game, as long as players are sufficiently similar (so that for all colluding players the industry monopoly outcome is superior to their Minimax payoff). In the presence of strong cost asymmetries side payments between players may be required to sustain monopoly pricing in equilibrium.

Note also that producers in electricity markets may restrain from extracting monopoly prices for fear of regulatory intervention, or because of the threat of entry.

[27] This bidding restriction is present in most electricity auctions, and it applies both to the balancing mechanism under NETA and to the original Pool design.

[28] Note that this does not lead to inter-temporal arbitraging by demand, since the System Operator cannot delay consumption and needs to meet all of its demand in the corresponding session of the market.

3.3 SMP

Optimal monopoly quantities and prices are given by the locus of points where marginal revenue and marginal cost coincide for each demand realisation μ, i.e.:

$$q_{SMP,m} = \frac{\mu}{2\rho + \gamma}; \quad p_{SMP,m} = \frac{\rho + \gamma}{2\rho + \gamma}\mu$$

where, like in the competitive case, the subscript SMP, m indicates equilibrium outcomes in the SMP monopoly case (the same notation is used below in the PAB case).

Under SMP, and in the presence of uncertain demand, this optimal outcome can be achieved by bidding the following linear Supply Function:[29]

$$SF^*_m(q) = (\rho + \gamma)q \tag{5}$$

With SMP the monopolist therefore secures maximum profits (given by the $MR = MC$ condition) at every realisation of demand, since the uniform pricing mechanism permits it to optimally price-discriminate between different demand-states (subject to the "no fixed fees" constraint), and to *neutralise* the effects of demand uncertainty. SMP effectively enables the monopolist to practice *third-degree price discrimination*, where each demand-realisation can be thought of as a different market which can be optimally priced separately from the others.

Note that the assumption that each demand realisation (or marginal benefit schedule) is bid truthfully by the auctioneer, and that the market clears where demand crosses the monopolist's bid function, plays an important role in generating this result. If consumers were allowed and able to engage in strategic behaviour to maximise consumer surplus (that is, consume until the point where the marginal benefit equals the *marginal bid* in the market), the monopolist's problem under SMP would boil down to the choice of an optimal marginal payment schedule. With this alternative assumption the SMP and PAB pricing regimes would therefore be outcome-equivalent, given that under PAB the monopolist can only bid a marginal payment schedule.

Similarly, as we emphasize at the end of the next sub-section, if demand were allowed to consume until its marginal benefit equals the *average bid* in the market under both pricing regimes (and not only under SMP, as it is the case in our set-up), the PAB equilibrium with monopoly bidding would converge, in terms of equilibrium prices and quantities, to the SMP one.

3.4 Pay-as-Bid

In a PAB pricing regime with demand uncertainty the monopolist can no longer rely on a unique bid function to maximise profits for each demand realisation.

Marginal bids which are optimal (i.e. equalise marginal revenue and cost) for low demand realisations now have an impact on the average price charged to high demand realisations, given that the average price received by the monopolist for each demand level is no longer only a function of the marginal bid (as under SMP). This means that the PAB monopolist can price

[29] This corresponds to the Supply Function Equilibrium for a monopolist, introduced by Klemperer and Meyer (1989).

discriminate across demand states less effectively than under SMP, since it cannot price each "market" (i.e. each demand schedule) separately by setting an optimal marginal bid.

Under PAB the monopolist essentially needs to engage in second-degree price discrimination with no fixed fees and no quantity discounts (which can be therefore thought of as "fourth-degree price discrimination", as we discuss in Section 4.1). This necessarily induces a fall in profits relative to SMP, as long as there is some demand uncertainty.[30]

Formally the monopolist faces the following stochastic control problem:

$$\max_{\beta}\left(\beta(\underline{q})\underline{q}-\frac{1}{2}\gamma\underline{q}^{2}+E_{\mu}\int_{\underline{q}}^{q(\mu,\beta)}(\beta-\gamma\theta)\,d\theta\right) \tag{6}$$

which reflects the fact the monopolist submits a flat function for any quantity that is supplied with certainty (i.e. $\underline{q}$), and is subject to the constraint that the slope of the bid function $\beta(q)$ be non-negative. $\underline{q}$ in equation (6) indicates, as in the competitive case, minimum monopoly output for a given bid function, and is short-hand for $\max(0, q(\underline{\mu},\beta(q)))$.[31]

Proposition 2

(i) If $\gamma > \rho$ (i.e. the cost function is steep relative to demand) the optimum bid function for the monopolist in terms of μ, $\beta_m^*(\mu)$, is linear and upwards-sloping and is given by:

$$\beta_m^*(\mu)=\begin{cases}\frac{\gamma-\rho}{\gamma+\rho}\mu+\frac{\rho}{2\rho+\gamma}\left(\bar{\mu}+\frac{2\rho}{\rho+\gamma}\underline{\mu}\right) & \text{for } \frac{\bar{\mu}}{\underline{\mu}}\leq 2\Leftrightarrow \underline{q}_{PAB,m}>0\\ \frac{\gamma-\rho}{\gamma+\rho}\mu+\frac{\rho}{\gamma+\rho}\bar{\mu} & \text{for } \frac{\bar{\mu}}{\underline{\mu}}>2\Leftrightarrow \underline{q}_{PAB,m}=0\end{cases} \tag{7}$$

This bid function is flatter than under SMP and it results in marginal prices at the minimum demand realisation which are higher than under SMP (i.e. $\beta_m^*(\underline{\mu}) > SF_m^*(\underline{\mu})$), and in marginal prices at maximum demand which are below the corresponding SMP prices (i.e. $\beta_m^*(\bar{\mu}) < SF_m^*(\bar{\mu})$). The value of μ at which $\beta_m^*(\mu) = SF_m^*(\mu)$ is greater than $E(\mu)=\frac{\underline{\mu}+\bar{\mu}}{2}$.

In terms of q the bid function is as follows, for $q \geq \underline{q}_{PAB,m}$:

$$\beta_m^*(q)=\begin{cases}\frac{\gamma-\rho}{2}q+\frac{(\rho+\gamma)\bar{\mu}+2\rho\underline{\mu}}{2(2\rho+\gamma)} & \text{for } \frac{\bar{\mu}}{\underline{\mu}}\leq 2\Leftrightarrow \underline{q}_{PAB,m}>0\\ \frac{\gamma-\rho}{2}q+\frac{\bar{\mu}}{2} & \text{for } \frac{\bar{\mu}}{\underline{\mu}}>2\Leftrightarrow \underline{q}_{PAB,m}=0\end{cases}. \tag{8}$$

(ii) If $\gamma \leq \rho$ (i.e. the cost function is flat relative to demand) the monopolist bids a flat bid function at a price $\hat{\beta}_m$ given by:

$$\hat{\beta}_m=\begin{cases}\frac{\rho+\gamma}{2\rho+\gamma}E(\mu)\equiv SF_m^*(E(\mu)) & \text{for } \frac{\bar{\mu}}{\underline{\mu}}\leq\frac{3\rho+\gamma}{\rho+\gamma}\Leftrightarrow \underline{q}_{PAB,m}>0\\ \frac{\rho+\gamma}{3\rho+\gamma}\bar{\mu} & \text{for } \frac{\bar{\mu}}{\underline{\mu}}>\frac{3\rho+\gamma}{\rho+\gamma}\Leftrightarrow \underline{q}_{PAB,m}=0\end{cases} \tag{9}$$

Bids are above costs everywhere except for the case $\gamma > \rho$ and $\frac{\bar{\mu}}{\underline{\mu}} > 2$, where we have "no distortions at the top" (i.e. $\beta_m^*(\bar{\mu}) = \gamma\bar{q}$).

Proof. See Appendix A.4.

[30] If the level of demand is certain, under PAB the monopolist can perfectly replicate the prices obtained under SMP, by offering a flat bid function at the price given by the SMP bid function at that level of demand.

[31] Similarly, as in the competitive case, $\bar{q}$ stands for $q(\bar{\mu},\beta(q))$, and the equilibrium levels of $\bar{q}$ and $\underline{q}$ are indicated as $\bar{q}_{j,m}$ and $\underline{q}_{j,m}$ respectively, for $j \in \{SMP, PAB\}$.

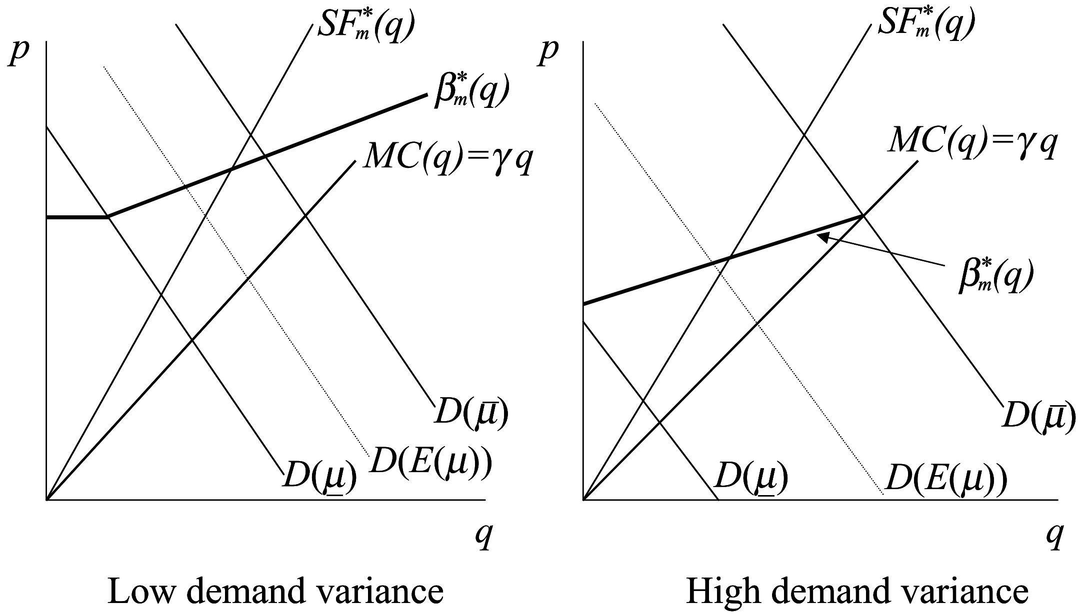

Figure 2: The monopoly bid functions, for $\gamma > \rho$.

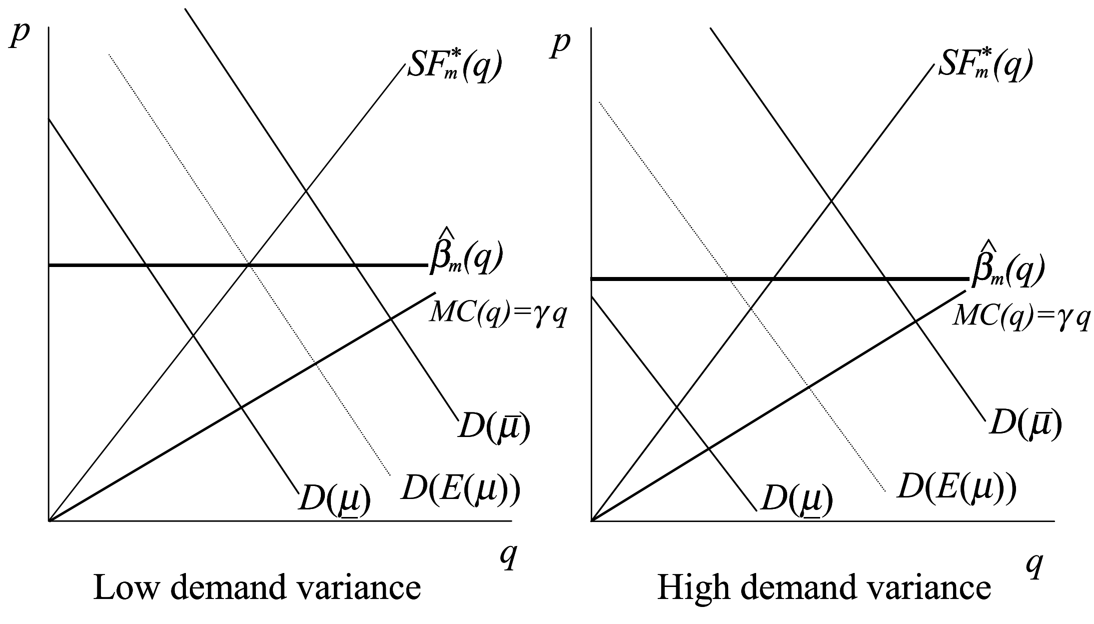

Figure 3: The monopoly bid functions, for $\gamma \leq \rho$.

Proposition 2 shows that the monopolist reacts to the introduction of PAB by reducing output relative to SMP for low demand realisations, and doing the opposite for high demand realisations. This is because the bids called under low-demand realisation are those which are (almost) always called, and therefore have an externality over the monopolist's profit level when demand is high. This externality induces it to raise these bids above the level implied by $MR = MC$ for low levels of demand. On the other hand the incentives for the monopolist to price up its output for high demand realisations is reduced with PAB since higher marginal prices yield higher profit margins only over the marginal units and not over all units sold (which is the case under SMP).[32]

The monopoly PAB bid function is therefore flatter than the corresponding bid function under SMP, given the presence of an externality from low bids to high bids (which induces the monopolist to raise the former), and the absence of an inframarginal quantity effect on the profitability of high bids (which induces the monopolist to lower them relative to SMP). The monopoly PAB function is also flatter than the competitive PAB function, given that under competitive conditions bids do not take into account neither of these two externalities (i.e. PAB bids by low-cost units are "too low" with perfect competition).

Monopoly PAB pricing corresponds partially to the standard optimal mechanism design result under hidden information over consumer types (e.g. as applied to non-linear pricing problems): consumption is distorted the most for low-consumption "types" (i.e. low-demand states in our set-up), in order to minimise the 'information rents' given to high-consumption types.[33] However, given that the monopolist can price discriminate between demand states only by offering different unit prices (rather than different payment-quantity bundles), the standard "no distortions at the top" result does not always apply. We elaborate on this point in the Section 4.1 of the paper, exploring the similarities between PAB and non-linear pricing further.

Note also that only if marginal costs increase "fast enough" with q (i.e. $\gamma > \rho$), the monopolist still finds it optimal to price-discriminate across demand-states and to bid an upwards sloping function, given that cost of meeting each marginal demand increment is relatively high; otherwise it does not find it optimal to increase its bids with q, and instead bids a flat function (i.e. the constraint that the bid function be non-downwards sloping binds). This flat bid function is at the optimal expected SMP price (i.e. $SF_m^*(E(\mu))$) if demand variance is relatively limited; otherwise the monopolist raises its bids relative to expected SF_m^* and does not supply some low-demand realisations. Similarly, in the $\gamma > \rho$ case if the spread between maximum and minimum demand is relatively high ($\frac{\bar{\mu}}{\underline{\mu}} > 2$), the monopolist prefers not to supply some demand realisations, so that $\underline{q}_{PAB,m} = 0$.

Finally, as pointed out also in Section 3.3, the assumption that the market clearing quantity

[32]There is therefore a structural change in the monopolist's bid function from SMP to PAB. When solving the firm's PAB problem, the stock of lower bids affects the bidding incentives in higher-demand states of the world. On the other hand, an SMP auction entirely removes this externality across units of production. Thus the Euler-Lagrange equation that applies in the SMP auction boils down to the usual $MR = MC$ result. Under PAB, the marginal increment in profit from increasing the stock of bids must be equated to increments in the marginal profit associated with the flow of bids.

[33]See, e.g., Mussa and Rosen (1978).

is set at the intersection of the industry marginal benefit schedule and the monopolist's PAB bid function plays a key role in leading to this result. If this is relaxed, and demand consumes where the *average* price function implied by the PAB bid function equals its marginal benefit, then the monopolist can maximise profits for each demand realisation, as under SMP. To achieve this outcome under PAB it would be sufficient for the monopolist to submit a (marginal) bid function which is such that the average price schedule it implies coincides with the optimal SMP bid function derived above. Under this alternative assumption a perfectly collusive industry would therefore be indifferent to the choice of auction rule, and so would be consumers and the regulator. This equivalence result would not however apply to more competitive market structures, as players would fail to fully internalise the effect each (infra-marginal) bid has on the overall level of demand, via its impact on the average price.

We draw out the implications of our market-clearing assumption on the consumer surplus comparison between SMP and PAB in Section 3.5.3 below.

3.5 SMP-PAB Comparison

In comparing the effects of a switch from SMP to PAB under monopoly conditions it is convenient to distinguish between four cases, depending on whether the optimal PAB bid function is upwards sloping or flat, and whether equilibrium minimum monopoly output under PAB ($\underline{q}_{PAB,m}$) is above or equal to 0. We therefore have: Case I, where $\gamma > \rho$ and $\frac{\bar{\mu}}{\underline{\mu}} \leq 2$; Case II, where $\gamma > \rho$ and $\frac{\bar{\mu}}{\underline{\mu}} > 2$; Case III, with $\gamma \leq \rho$ and $\frac{\bar{\mu}}{\underline{\mu}} \leq \frac{3\rho+\gamma}{\rho+\gamma}$; and Case IV, with $\gamma \leq \rho$ and $\frac{\bar{\mu}}{\underline{\mu}} > \frac{3\rho+\gamma}{\rho+\gamma}$.

Figure 4 illustrates these four cases.

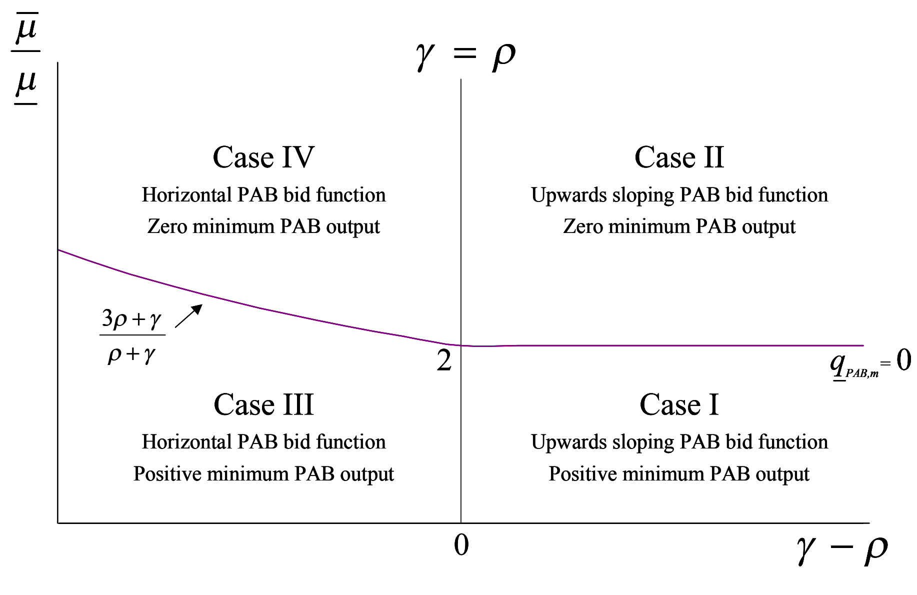

Figure 4: Illustration of the four cases in the comparison between PAB and SMP under monopoly conditions.

3.5.1 Output

Proposition 2 implies that expected output falls under PAB relative to SMP if $\gamma > \rho$. This is because the monopolist's PAB bid function crosses the SMP bid function at a level of μ which is greater than $E(\mu) = \frac{\bar{\mu}+\underline{\mu}}{2}$. Also, if the ratio of maximum and minimum demand is sufficiently high (i.e. Case II) the monopolist prefers not to supply some low-demand realisations, and sets $\beta_m^*(\underline{\mu}) > \underline{\mu}$, which implies $\underline{q}_{PAB,m} = 0$ and therefore an even stronger output-reduction effect.

In the $\gamma \leq \rho$ case expected output is the same under PAB and SMP if $\underline{q}_{PAB,m} > 0$, which requires $\frac{\bar{\mu}}{\underline{\mu}} \leq \frac{3\rho+\gamma}{\rho+\gamma}$ (Case III). If this last condition does not hold, there is output contraction under PAB also in the $\gamma \leq \rho$ case, since some demand realisations are not supplied and bids are increased above the average SMP price.

Therefore, similarly to our results under perfect competition, the switch from PAB to SMP leads to a reduction in output (weakly). This effect is particularly strong in the presence of high demand uncertainty, which implies that under PAB it is too costly to supply low-demand realisations (in terms of their externality over the profits made when demand is high), whilst this is "costless" (in terms of its externality on other sales) under SMP.

Even when demand uncertainty is relatively limited but costs are steep (i.e. Case I), PAB induces output-contraction. This is due to the fact that under PAB the monopolist finds it optimal to bid a flatter function than the SMP one and therefore finds it relative costly to keep average output unchanged relative to SMP, given the convexity of the cost function. Supplying the same expected level of output as under SMP would in fact require the expansion of output at high levels of demand exactly matching the contraction of output at low-levels of demand, which in turn would lead to an excessive increase in total costs from the monopolist's point of view (given that marginal costs increase relatively fast with quantity when γ is high).

The output contraction effect due to PAB is closely related to the "market opening" effect identified in the literature on third-degree price discrimination (see e.g. Varian (1985)). This refers to the fact that allowing for third-degree price discrimination (which corresponds to SMP-pricing in our set-up, as discussed above), as opposed to forcing the monopolist to set a unique price for all markets, may allow some markets which the monopolist would have otherwise excluded to be supplied. This effect is present in exactly the same form in our set-up under Case IV, where PAB leads to a unique price being charged by the monopolist to all market (i.e. demand levels) and to the exclusion of some markets, whilst SMP is characterised by price-discrimination (i.e. an upwards sloping bid function), and no-exclusion.

3.5.2 Efficiency

Whilst the efficiency comparison between SMP and PAB under competitive conditions is immediate (i.e. welfare is higher under SMP given that PAB is characterised by lower output for all demand realisations except for maximum demand), this comparison is less straightforward under monopoly conditions.

This is so because, even though average output is (weakly) lower under PAB relative to SMP (i.e. there is a "total output" effect which favours SMP from a welfare point of view), a PAB price rule leads to a more efficient *allocation* of output across demand realisations. This is because under PAB the monopolist offers a flatter bid function, which re-allocates output from low marginal utility consumption to high marginal utility consumption (i.e. output is reduced for low demand realisations, and increased for high ones). By narrowing the difference between marginal utilities across demand realisations (and, in the $\gamma \leq \rho$ case, equalising marginal utilities) PAB increases efficiency.

This "unequal marginal utilities" in favour of PAB is however balanced by both the "total output" effect (described above) and by an additional "cost saving" effect in favour of SMP. The latter arises from the presence of cost convexity, which implies that the total cost of producing a given level of output is lower under SMP than PAB given that under the former the distribution of output across demand realisations has lower variance (because of the steeper bid function).[34]

As Proposition 3 below shows, the welfare gains due to the utility-enhancing output allocation obtained by switching to PAB do not always outweigh the efficiency loss due to both the "total output" and "cost saving" effects, leading to an ambiguous overall welfare impact of a change in the pricing rule.

Welfare for each demand realisation μ (defined as $W_{j,m}(\mu)$) under each pricing rule is given by the integral of demand minus the integral of the marginal cost schedule, until equilibrium quantity $q_{j,m}(\mu)$, with $j \in \{SMP, PAB\}$. That is:

$$W_{j,m}(\mu) = \mu q_{j,m}(\mu) - \frac{\gamma + \rho}{2} \left(q_{j,m}(\mu)\right)^2 \tag{10}$$

Expected welfare under each price rule (defined as $E(W_{j,m})$) is therefore given by:

$$E(W_{j,m}) = \begin{cases} \int_{\underline{\mu}}^{\bar{\mu}} W_j(\mu) f(\mu) d\mu & \text{for } \underline{q}_{j,m} > 0 \\ \Pr(\mu > \hat{\mu}) \int_{\hat{\mu}}^{\bar{\mu}} W_j(\mu) f(\mu) d\mu & \text{for } \underline{q}_{j,m} = 0 \end{cases} \tag{11}$$

where $\hat{\mu}$ equals $\beta_m^*(\hat{\mu})$ (for PAB case II) or it equals $\hat{\beta}_m$ (for PAB case IV).

Comparing expected welfare under SMP and under the four PAB cases identified above, we obtain the following Proposition.

Proposition 3

Under monopoly conditions expected welfare is higher under SMP than under PAB as long as $\gamma > \rho$ (i.e. for Cases I and II).

If $\gamma \leq \rho$ the welfare comparison is ambiguous: under Case III, $E(W_{PAB,m}) > E(W_{SMP,m})$ if and only if $\gamma < \bar{\gamma}(\rho) \equiv \left(\sqrt{2} - 1\right) \rho$ (i.e. if costs are sufficiently flat); under Case IV $E(W_{PAB,m}) > E(W_{SMP,m})$ if and only if $\gamma < \bar{\gamma}(\rho)$ and demand uncertainty is not excessive.

Proof. See Appendix A.5.

This Proposition shows that the SMP-PAB welfare comparison is ambiguous. If costs are steep enough relative to demand (and therefore the monopolist's PAB bid function is upwards

[34] Both the "total output" and "unequal marginal utilities" effect are familiar from the literature on the welfare effects of banning third-degree price discrimination (e.g. Varian (1985)). The "cost saving" effect present with price discrimination (i.e. with SMP, in our setting, as argued above) is typically not analysed in this literature given the assumption that marginal costs are constant or that total costs depend only on total output, and not on its distribution. This latter condition is not satisfied in our set-up (i.e. we have that $E_\mu(C(q)) > C(E_\mu(q))$, giving rise to a "cost saving" effect in favour of SMP.

sloping), expected welfare is *always reduced* by a switch from SMP to PAB. This is because under this case both the "total output" and "cost saving" effects associated with SMP are relatively strong.

Otherwise the comparison is ambiguous. If and only if costs are relatively flat ($\gamma < \bar{\gamma}(\rho)$) and demand uncertainty is not excessive welfare is higher with PAB. This is because if these two conditions hold the "total output" and "cost saving" effect due to SMP are weak, and are dominated by the "unequal marginal utilities" effect which favours PAB. Therefore, even when average output falls with PAB (as under Case IV), welfare might increase if costs are flat enough. However the negative effect of the contraction in average output induced by PAB bidding will eventually outweigh the "unequal marginal utilities" effect as demand uncertainty increases (see Figure 5 for an illustration).

Note finally that, contrary to the standard result on the effects of not allowing for third-degree price-discrimination (or SMP, in our setting), we find that an increase in (average) output is *not* a necessary condition for SMP to be welfare enhancing. That is, welfare under SMP can be higher than welfare under PAB also under Case III, where expected output is the same across the two regimes, if the "cost saving" effect due to SMP is sufficiently strong.

3.5.3 Prices and Consumer Surplus

As in the competitive case considered in Section 2, we assess the impact of a switch from SMP to PAB on consumers by examining both consumer surplus and demand-weighted (DW) average prices. The following Proposition summarises our results.

Proposition 4

(i) Expected consumer surplus is always higher under PAB than under SMP.

(ii) Expected DW prices are lower under PAB than under SMP in Cases I and III. In Cases II and IV, where the monopolist prefers not to supply some demand realisations in a PAB auction, DW average prices can be higher under PAB. This is the case if demand variance is relatively high.

Proof. See Appendix A.6.

This Proposition partially confirms the competitive results given in Proposition 1: consumers always benefit from the switch from SMP to PAB. However, unlike the competitive case, average prices may actually increase under PAB, giving the *impression* that consumers are suffering from the change in price rule.

If demand uncertainty is relatively limited, the latter is not the case: the introduction of PAB lowers the relatively high price-quantity correlation present under SMP and it both reduces prices and increases consumption at high demand realisations. Both of these effects reduce DW average prices and increase expected consumer surplus.

However, if some demand realisations are not supplied under PAB, as it is the case with high demand uncertainty, then DW average prices may actually increase relative to SMP. This is because the DW average of SMP prices is reduced by the (accepted) bids which are submitted when demand is relatively low, and which are not offered by the monopolist under PAB.

This increase in average prices with PAB does not however hurt consumers in terms of expected consumer surplus, since PAB also implies output expansion and price reduction "at the top" (for high μ). This boosts expected consumer surplus and it outweighs the relative loss in surplus which realises at low μ realisations, given the convexity of consumer surplus in μ. The fact that the gain in consumer surplus due to a switch to PAB increases with μ faster than the reduction in DW average prices does, implies that a change in the price rule can lead to an increase in both consumer surplus and DW average prices.

Our results on the consumer surplus comparison between SMP and PAB are partially driven by the market clearing rule we have assumed in our set-up. By clearing the market where demand crosses the monopolist's bid function we are effectively 'allowing' consumers to maximise their surplus under PAB (by consuming where their marginal benefit equals the marginal purchase cost), *for a given monopoly bid function*. On the contrary, under SMP demand consumes until its marginal benefit equals the average (purchase) cost, implying that consumer surplus is not being maximised. Under PAB demand therefore enjoys a strategic advantage which, even allowing for the change in the monopolist's bidding behaviour induced by a PAB price rule, leads to an increase in consumer surplus. However, allowing demand to behave strategically, as PAB does, may come at a cost in terms of overall efficiency, as shown by Proposition 3.

3.5.4 Summary of the PAB-SMP Comparison under Monopoly Conditions

Our comparison between the SMP and PAB price rules under monopoly broadly confirms the ones obtained under a competitive market structure: a switch from SMP to PAB makes the exercise of market power harder, and it leads to an increase in consumer surplus. On the other hand it can also induce inefficient behaviour (i.e. a reduction in output and/or an inefficient distribution of output), leading to a trade-off between the price level (lower with PAB) and welfare (also lower with PAB).

Contrary to the competitive case there are situations where this price-efficiency trade-off does not apply. If demand uncertainty is relatively high PAB is 'bad' in terms of both welfare and prices, reducing the former and increasing the latter. The increase in prices due to PAB is not however associated with a reduction in consumer surplus, implying that the basic 'efficiency-consumer surplus' trade-off between SMP and PAB still applies. However, the increase in prices under PAB may give the impression that the market power increases as a result of the switch to PAB, and that consumers are worse off.

Moreover, there are circumstances where neither the 'price-efficiency' nor the 'consumer surplus-efficiency' trade-offs apply under monopoly conditions. This is the case if marginal costs are relatively flat, and demand uncertainty is limited. Under these conditions a switch from PAB to SMP reduce prices and increases both consumer surplus and welfare, as shown in Propositions 3 and 4. These conditions would therefore make a switch from PAB to SMP under perfectly collusive conditions an appealing policy option from a regulatory point of view.[35]

Figure 5 summarises and illustrates the various cases of the comparison between PAB and SMP, in terms of price and welfare effects. The figure plots three schedules, in terms of demand

[35] On the other hand, given that demand uncertainty is small in this case, the absolute size of the welfare and consumer surplus gains obtained by switching to PAB would also be limited.

uncertainty ($\bar{\mu}/\underline{\mu}$) and relative cost steepness ($\gamma - \rho$): a $\underline{q}_{PAB,m} = 0$ schedule, above which minimum monopoly output under PAB is zero (i.e. some demand realisations are not supplied), and below which minimum monopoly PAB output is always positive;[36] a $\Delta P = 0$ schedule, above which DW average prices are higher under PAB than SMP, and below which the converse is true;[37] and a $\Delta W = 0$ function, below which expected welfare is higher under PAB than under SMP, and above which the converse is true.[38]

The $\Delta P = 0$ and $\Delta W = 0$ schedules jointly determine three areas in uncertainty-cost steepness space: a "good" area (G), under which a switch from SMP to PAB leads to both lower prices and higher welfare; an 'apparently' "bad" area (B), where PAB leads to both higher prices and lower welfare relative to SMP; and a "mixed" area (M) where PAB leads to a trade-off between prices (which are lower than SMP) and welfare (which is lower too). As the figure shows the presence of relatively low demand uncertainty and flat costs can make PAB superior to SMP under both price/consumer surplus and welfare considerations.

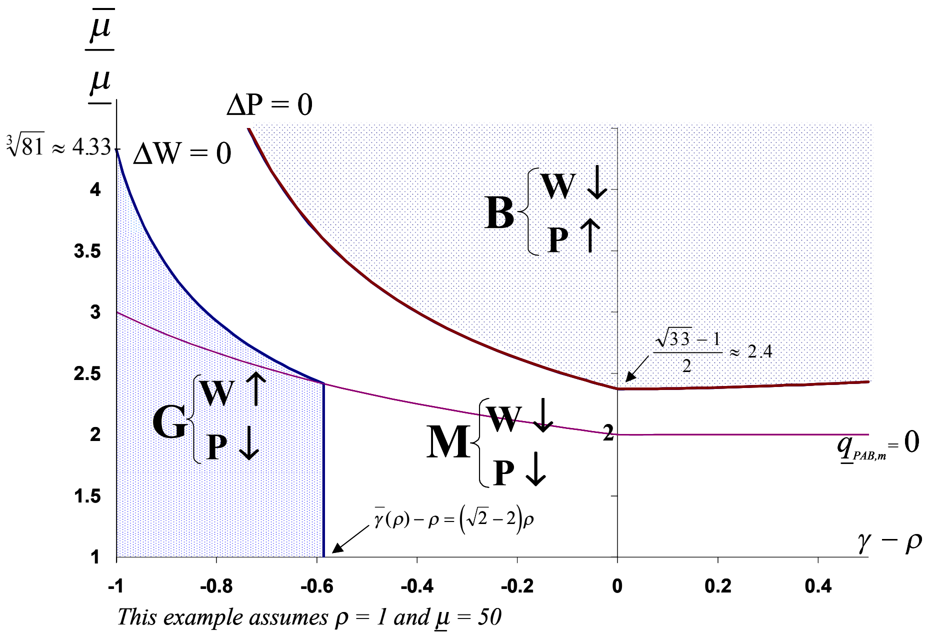

Figure 5: Illustration of the effects of a switch from SMP to PAB under monopoly conditions in terms of expected prices (P) and expected welfare (W).

[36] This schedule is given by $\frac{\bar{\mu}}{\underline{\mu}} = \max(2, \frac{3\rho+\gamma}{\rho+\gamma})$ (see Proposition 2).

[37] This schedule is obtained from the Proof of Proposition 4 (equations C1 and C2).

[38] The $\Delta W = 0$ function is obtained from the conditions set out in Proposition 3 and its Proof.

4 Discussion

In this discussion we address three issues arising from the results we have presented so far: the relationship between PAB bidding and the theory of price discrimination (and non-linear pricing in particular); strategic interaction under PAB; and market dynamics with PAB.

4.1 Pay-as-Bid and Price Discrimination

Throughout the previous section we have referred to and used results and terminology from the price-discrimination literature to describe and analyse monopoly bidding behaviour in PAB and SMP auctions. In this discussion section we make the link between price discrimination and the comparison between SMP and PAB auctions more explicit, and argue that monopoly bidding under PAB can be conveniently thought of as "fourth-degree price discrimination" (whilst, as argued above, SMP is analogous to third-degree price discrimination).

The first point to note in discussing the links between PAB and price discrimination is that the monopoly's profit function under PAB with demand uncertainty is closely related to the one faced by a monopolist seeking to second-degree price discriminate across different consumers, under asymmetric information over the consumer "type" μ. That is, each demand realisation can be interpreted as a different consumer type. Equation (6) corresponds directly to the maximand in the standard non-linear pricing (NLP) problem:

$$E(\pi) = \int_{\underline{\mu}}^{\bar{\mu}} T(q(\mu)) - C(q(\mu))f(\mu)d\mu \tag{12}$$

where $T(q(\mu))$ indicates the monopolist's total charge for the consumption of q units.

However in the PAB setting we model the monopolist enjoys less freedom in its pricing than under the more general NLP problem. As discussed above, in a typical electricity auction each unit of production needs to be priced separately from the others, which implies that the overall bid function needs to be non-decreasing and that fixed fees cannot be charged (i.e. the $T(q(\mu))$ function needs to go through the origin and has to be weakly convex). The impossibility of charging fixed fees or offering quantity discounts limits the monopolist's ability to extract rents from consumers, which in turn affects its optimal marginal price (or bid) schedule.

Under NLP on the other hand, the monopolist is free to bid a decreasing price function (if it finds it optimal to do so), since this can be enforced by an appropriate design of the $\{T(q), q\}$ bundles, and it also can appropriate all of the surplus of the lowest-demand consumers by setting a fixed fee.

The effect of the no-fixed-fees and no-quantity-discounts constraints present under PAB can easily been seen in the case of no demand uncertainty. The unconstrained monopolist in this case can price at marginal cost and extract the whole of consumer surplus by means of a fixed fee (i.e. it will practice *first degree price discrimination*), whilst in the auction we model (i.e. where fixed fees and quantity discounts are not allowed) it will settle for pricing according to the inverse elasticity rule (or $MR = MC$) for each demand "type". This leaves some surplus to demand and leads to higher marginal payments (or prices) (i.e. as in *third degree price* *discrimination*).[39,40] As uncertainty is introduced, the monopolist is forced to depart from its first best in both cases: in the unconstrained case the monopolist will practice non-linear pricing (i.e. *second degree price discrimination*), whilst if fixed fees and decreasing price schedules are not allowed it will engage in PAB-pricing, which can therefore be thought of as "*fourth degree price discrimination*" (i.e. price-discrimination with type uncertainty and unit-by-unit bidding).

Figure 6 illustrates the optimal monopoly tariff schedules $T(q)$ under the four degrees of price-discrimination, for Case III of our PAB-SMP comparison and for $\mu = \bar{\mu}$. The figure assumes that there is no demand uncertainty under first and third degree price discrimination (the monopolist knows that μ equals $\bar{\mu}$, and bids accordingly), whilst the level of demand is ex-ante uncertain under second and fourth degree price discrimination.[41]

The impact of the higher degree of discretion in pricing afforded by the possibility of charging fixed fees and offering quantity discounts present under second degree price discrimination can be seen explicitly by solving for the NLP marginal payment schedule, and comparing it to the PAB bid function, under the same parameter assumptions. The FOC implied by (12) after substituting for the incentive compatibility constraint is:[42]

$$\frac{\partial GS(\mu, q)}{\partial q} = \frac{dC(q)}{dq} + \frac{1 - F(\mu)}{f(\mu)} \frac{\partial^2 GS(\mu, q)}{\partial \mu \partial q}$$

where $GS(\mu, q)$ indicates the consumer gross surplus function.

Making the same parameter assumptions of the previous section (i.e. $\mu \sim U[\underline{\mu}, \bar{\mu}]$, $C(q) = \frac{\gamma}{2}q^2$ and $GS(\mu, q) = \mu q - \frac{\rho}{2}q^2$) we obtain the following price (or marginal payment) function $p(\mu)$:

$$p^*(\mu) = \frac{\gamma - \rho}{\gamma + \rho}\mu + \frac{\rho}{\gamma + \rho}\bar{\mu} \tag{13}$$

This shows that also under NLP the relative size of the slopes of the marginal cost and demand schedules (i.e. γ and ρ) determine the monopolist's incentive to price up or down with quantity. Equation (13) also shows that the NLP and PAB marginal price (or bid) functions

[39] As we discuss in the previous section, this outcome can be implemented in a uniform price (SMP) auction even when demand is uncertain, as long as it is "non-strategic". If demand is strategic (i.e. it maximises consumer surplus for a given bid function), the third degree price discrimination outcome can be achieved by the monopolist only if demand is certain, by bidding a flat bid profile at the price which satisfies the inverse elasticity rule.

[40] Note that the standard definition of third-degree price discrimination assumes that different consumer types can be separated (i.e. there is no type uncertainty) and that the monopolist has to charge a constant unit price to each type. This second assumption is stronger than the one we impose on the monopolist in our modelling (namely, no fixed fees and no quantity discounts) but, in the case of no demand uncertainty, is outcome-equivalent.

[41] As the figure indicates, the slope of the schedule for first degree price discrimination is equal to the marginal cost at $\mu = \bar{\mu}$ (i.e. $\frac{\gamma}{\rho+\gamma}\bar{\mu}$). Consumption is therefore undistorted (i.e. $q = \bar{q}^{FB}$), and the whole of consumer surplus is captured by a high fixed fee ($F^{FIRST}(\bar{\mu})$). The corresponding schedule for third degree discrimination is steeper and goes through the origin, because of the impossibility of charging a fixed fee. The tariff for fourth degree price discrimination also goes through the origin, but is flatter than the one for third degree discrimination, given the assumption of type uncertainty, which induces the monopolist to price at the average optimal "SMP" price. Finally, the schedule for second degree discrimination is steeper than marginal costs everywhere except at the top, and allows for a fixed fee (shown as $F^{SECOND}(\mu)$) which extracts the rents of low-value types. $\bar{\mu}$-types obtain information rents under this schedule relative to first degree discrimination, as shown by the gap between $T^{FIRST}(q(\bar{\mu}))$ and $T^{SECOND}(q(\mu))$ at $q = \overline{q}^{FB}$.

[42] See Tirole (1988), p. 157.

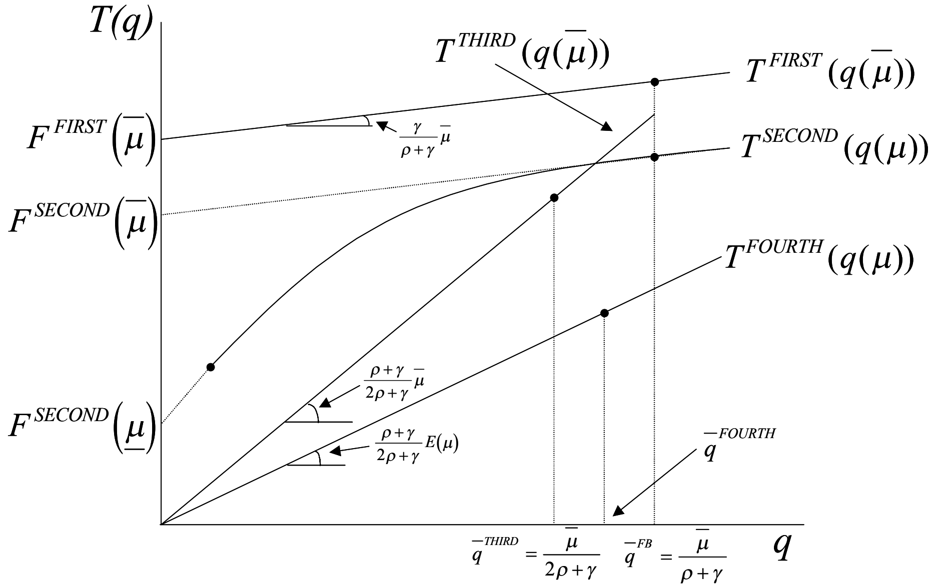

Figure 6: Illustration of the four degrees of price discrimination at $\mu = \bar{\mu}$, for PAB case III (i.e. $\gamma \leq \rho$ and $\frac{\bar{\mu}}{\underline{\mu}} \leq \frac{3\rho+\gamma}{\rho+\gamma}$).

are identical in Case II of the PAB-SMP comparison, i.e. with $\gamma > \rho$ and $\frac{\bar{\mu}}{\underline{\mu}} > 2$ (compare equations (7) and (13)). This is because only for this parameter combination the constraints implied by the pricing rules we impose on the PAB monopolist do not bind: the NLP monopolist wants to bid an increasing price function,[43] and does not charge a fixed fee, since the surplus of lowest-demand type is set to zero by virtue of the fact that this type is not supplied (recall that for $\frac{\bar{\mu}}{\underline{\mu}} \geq 2$, $\underline{q}_{PAB,m} = 0$). Therefore only in this case the PAB function implies "no distortions at the top" (i.e. $\beta^*_m(\bar{\mu}) = p^*(\bar{\mu}) = \frac{\gamma}{\gamma+\rho}\bar{\mu}$), which is always the case under NLP.

For the other three cases of the SMP-PAB comparison PAB implies distortions everywhere: for $\gamma > \rho$ and $\frac{\bar{\mu}}{\underline{\mu}} \leq 2$, PAB forces the monopolist to grant rents to the $\underline{\mu}$-type, which induces her to increase marginal prices (or bids) relative to the NLP schedule, to appropriate the optimal amount of consumer surplus; for $\gamma \leq \rho$, the monopolist under PAB cannot bid the optimal NLP price schedule, which is decreasing in quantity, and is forced to submit a flat bid function, which implies that prices cannot converge to cost for the highest realisation of μ.

Finally, as emphasised in the previous section, our welfare comparison between SMP and PAB partially confirms the standard results on the effects of allowing for third-degree price discrimination. This is because SMP-bidding is like third degree price-discrimination across μ-types, whilst PAB bidding corresponds either to a situation with *less* price-discrimination (i.e. with a flatter, but still upwards sloping, bid function, as under cases I and II) or with no price-discrimination (i.e. with a horizontal bid function, as under cases III and IV). Therefore the effects of switching from SMP to PAB are analogous to the effects of banning (or reducing)

[43]This is always enforceable if we assume that consumers need to satisfy all of their demand requirements in one purchase - i.e. there is no repeat purchasing or, as in the case of electricity, demand is instantaneous.

third-degree price-discrimination.[44]

As in the literature on third-degree price discrimination, we find that the welfare effects of "banning" SMP are ambiguous: this is because the (beneficial) total output and cost saving effects due to SMP can be outweighed by the efficient narrowing of marginal utilities across demand realisations brought about by PAB. Our results show that the latter effect might outweigh the former two, if costs are sufficiently flat and demand-dispersion sufficiently small; and that, given our assumptions on costs,[45] an increase in expected output is not a necessary condition for SMP to be welfare-superior to PAB.

4.2 Strategic Interaction

As noted in the introductory section of this paper there are some strong results from multi-unit auction theory on the issue of strategic interaction under SMP and PAB (Back and Zender (1993); Wang and Zender (1999)): in settings which are not radically different from electricity auctions pay-as-bid encourages Bertrand outcomes, whilst uniform pricing allows for "seemingly collusive" outcomes.[46]

This sharp difference between the two price rules arises because PAB forces players to compete in "prices", making it harder for them to defend their market share by placing low inframarginal bids. This in turn raises the benefits of one-shot deviations from any strategic/high-price outcome leading to aggressive equilibrium behaviour, and more competitive pricing.[47]

Under SMP on the other hand players can use the whole of their bid schedule to achieve the double objective of setting high prices (which is obtained with appropriate marginal bids) and minimising the incentives for rivals to deviate from a high-price outcome (which is achieved by

[44] We recognise that from the point of view of terminology, this analogy might be confusing. This is because SMP (i.e a *uniform* price auction) leads to (third degree) price *discrimination* across demand types; whilst PAB (i.e. a *discriminatory* price auction) implies (more) *uniform* pricing across demand realisations.

[45] Namely, that $C(q)$ is convex, which implies that expected costs (over different realisations of the type μ) are not only a function of expected output.

[46] In a procurement auction the results of Back and Zender (1993) and Wang and Zender (1999) apply directly if all players have the same marginal cost of production, and if there is an upper bound on prices.

[47] Quoting from Wang and Zender (1999, p. 22):

> "With risk-neutral bidders, discriminatory pricing intensifies bidder competition to the fullest extent, the bidders compete by submitting flat demand curves and thus lose any strategic advantage derived from asset divisibility.[...] Simply using a reserve price of zero together with discriminatory pricing eliminates all of the bidders' strategic advantage."

The UK regulator Ofgem has indeed relied on this kind of arguments to explain the rationale behind the abolition of the uniform-price Pool (1999, p. 174):

> "A factor which has clearly provided incentives for strategic bidding is the use of marginal bids by generators to set Pool prices, which then apply to all output. For example, this allows a generator to bid relatively highly at the margin for higher cost supplies whilst protecting its volume position by bidding lower prices for lower cost supplies. If a generator's marginal bid is undercut by a rival, the resulting volume loss is relatively small. The generator, knowing that rivals will be adopting the same bidding strategy, will anticipate that, if it cuts prices, its volume gain will be relatively small. Price cutting is therefore made less profitable, and higher prices encouraged."

placing low "quantity-protecting" inframarginal bids). This outcome essentially corresponds to a Cournot equilibrium, where players dump their output in the market (i.e. bid very aggressively for infra-marginal output) and they let the price be set by the intersection of demand with a vertical aggregate supply schedule. This bidding behaviour favours high prices (compared to Bertrand, or PAB, competition) given that it leads to steep residual demand functions in equilibrium for each bidder, which eliminates incentives to deviate from the high-price outcome. Therefore, whilst SMP allows for Cournot-like equilibria (even though these are not unique), PAB leads players to behave in a Bertrand fashion.

This relatively stark result needs to be qualified by a number of considerations, all of which are of some relevance to electricity markets. These are: the presence of capacity constraints; the impact of demand uncertainty; the possibility of incomplete information about costs; the impact of repeated interaction; and discreteness in the bid functions producers can submit. All of these factors mitigate the difference between SMP-Cournot and PAB-Bertrand price outcomes, possibly reversing it.

The role of the first factor (the impact of capacity constraints) is straightforward and well-known: if players are capacity constrained the PAB-Bertrand equilibrium is less competitive, given that players' incentives to deviate from a high-price outcome are reduced by the inability to supply the whole of residual demand. The difference between Bertrand and Cournot price outcomes therefore is smaller, and it disappears if demand is at a level which implies that the Cournot equilibrium quantities are greater than the players' capacities.

As discussed in the introductory section of this paper, the impact of the second factor, demand uncertainty, on strategic interaction has also been analysed in the multi-unit auction literature. Klemperer and Meyer (1989) show that the Cournot outcome is no longer attainable under SMP with uncertain demand (except at the maximum demand realisation) given that some of the low infra-marginal bids necessary to sustain the Cournot outcome can now become price-setting. This induces players to raise them, which in turn weakens their role as "threats" against deviations from the Cournot outcome. This in turn leads to lower prices and therefore narrows the difference between the Cournot and Bertrand outcomes (Back and Zender (1993)).

The third factor, incomplete cost information, can lead to Winner's Curse effects, which are stronger under PAB than SMP, given the "first-price auction" properties of PAB. This in turn can raise the level of prices in PAB relative to SMP, partially compensating for the stronger strategic advantage enjoyed by players under SMP (Wang and Zender (1999)).[48]

Fourthly a static analysis of competition in electricity auctions may be of partial relevance given the high frequency with which these are repeated. As argued above, tacit collusion may be a more likely state of affairs than one-shot strategic interaction in electricity markets, making the results on monopoly behaviour under SMP and PAB an important benchmark for these markets. The results presented in Section 3 show that, even though a switch from SMP to PAB mitigates market power (and therefore confirms our intuition from static oligopoly models), this does not imply that welfare will be higher under SMP, if demand is uncertain.

[48] In electricity markets incomplete information over costs may arise in a context with sequential markets. For instance, players bidding in the balancing mechanism under NETA may be uncertain over who has contracted in the preceding Power Exchange, and may therefore face uncertainty over the costs of their rival bidders.

An additional insight regarding the relationship between collusion and the price rule is that SMP may facilitate the attainment of a collusive outcome relative to PAB. This is the case for reasons which are similar to those put forward above, in the context of static interaction: by allowing for aggressive infra-marginal bids SMP can deter deviation from collusive outcomes more effectively than PAB (see Fabra (2001)).

Fifth, and final, assumptions on the shape of the bid functions players can submit also matter for the comparison between PAB and SMP. This is because in a setting with discrete step bid function (e.g. as in von der Fehr and Harbord (1993)), there is always a discrete unit of price-setting output at the margin. This provides an incentive to players to deviate from any high-price outcome, even in the presence of aggressive infra-marginal bids, as long as players are not capacity constrained. Therefore, if capacity constraints do not bind, competition at the margin will drive prices to cost under both SMP and PAB, eliminating the bidders' strategic advantage due to SMP. On the other hand, if demand is sufficiently high relative to the players' capacities, some of this strategic advantage is restored. This is because SMP allows for asymmetric equilibria, where one player sets the price (acting as a monopolist over residual demand) whilst the others submit lower infra-marginal bids and are capacity constrained. This kind of equilibria are not present under PAB, where placing low infra-marginal quantity-protecting bids is not profitable, and the equilibrium is therefore more competitive (see e.g. Fabra (2001)).

Existing results on strategic interaction under SMP and PAB therefore partially confirm the results presented in this paper on the two benchmarks cases of perfect competition and perfect collusion. A switch from SMP to PAB will generally reduce market power, and lower industry profits. Whether this will be welfare-enhancing, or at least price-reducing, depends on the specific circumstances of the auction, and cannot be established a priori.

4.3 Dynamics and Entry

Both our modelling of the SMP-PAB comparison and the discussion presented above on strategic interaction under the two price rules have abstracted from the issue of market dynamics and entry/exit considerations. This is likely to be a major determinant of the impact of a switch from SMP to PAB in electricity auctions, where entry barriers are relatively low. In this section we briefly highlight two implications of our analysis on market dynamics.

The first is related to our results on the impact of PAB in a competitive environment. We have shown that low marginal cost (i.e. base-load) producers suffer more than others in a PAB environment relative to SMP, given that they face a higher opportunity cost of not producing and therefore need to bid more aggressively to ensure they produce with certainty (see Lemma 1). This may in turn have an entry-deterrence (or exit-inducing) impact on baseload producers by not allowing them to recover their (high) fixed costs, leading to a shift in the plant mix in the market and to a generally flatter aggregate marginal cost schedule. Also, if, as it is arguably currently the case in electricity markets, technological conditions are such that most of the profitable independent entry in the market is likely to be base-load, this effect of PAB might reduce aggregate independent entry and strengthen the market power of incumbent producers.[49]

[49] Baseload generators have been responsible for most of the independent entry into the England and Wales market over the last decade, significantly contributing to the erosion of market power in the industry.

The second, related, point refers to the interaction between strategic (i.e. large) and competitive (i.e. small) players under the two auction rules. Under SMP small players find it relatively easy to free-ride on the market power of larger players. By simply bidding at cost they can obtain the price set by the strategic players, who may jointly act as a residual monopolist (by tacitly colluding). This outcome cannot be replicated under PAB. Under PAB competitive players will have to raise their bids above costs (i.e. as shown in our competitive model), which will affect the shape of the residual demand faced by the strategic players. In particular this will become flatter at the margin, inducing the residual monopolist to increase output and reduce the market share of the non-strategic bidders.[50] Therefore, for reasons which are distinct from the point made above, this second effect also suggests that a switch from SMP to PAB may discourage entry by smaller bidders, by making it harder for them to free-ride on the relatively high prices set by incumbent (and large) players.[51]

5 Conclusion

This paper has analysed the change from uniform to discriminatory pricing in an electricity auction with demand uncertainty. We have analysed two benchmark cases, perfect competition and monopoly, showing how the introduction of PAB leads to an increase in consumer surplus *and* a reduction in output in both.

In the monopoly case however demand-weighted average prices may actually increase relative to SMP, if the output-contraction effect due to PAB is strong because of high demand-uncertainty. In addition, whilst efficiency is always reduced by a switch from SMP to PAB under competitive conditions, this effect is ambiguous under monopoly pricing.

We have also discussed both why the monopoly case is a relevant benchmark to consider in electricity auctions, and how the SMP-PAB monopoly comparison relates to existing results on one-shot multi-unit auctions (which suggest that a switch away from SMP can significantly erode bidders' strategic advantage). Our monopoly results partially confirm the insights from the static strategic analysis, and therefore lend a degree of support to the UK electricity regulator's claim that SMP facilitates the exercise of market power. However they also show that players with market power may react to PAB in ways which are inefficient, leading to lower output and lower welfare. In addition, PAB may be associated with dynamic effects (e.g. on entry) which may even strengthen market power in the medium-run.

This last point has significant implications for electricity market design: the presence of a uniform-price "gross" pool allows players to compete in "supply-functions" and achieve mutually beneficial price outcomes even under static interaction, and potentially maximum profits in a

[50] The presence of demand uncertainty might mitigate this effect, given that the monopolist has incentives to reduce output under PAB as shown by our monopoly results.

[51] An additional entry deterrence effect of PAB might be due to the bigger "premium on information" this creates relative to SMP. This will favour large players with access to better information about market conditions relative to small players, and it may even induce strategic players to create "strategic uncertainty" in the market (e.g. by randomising their bids). This is an effect of discriminatory price auctions which is often discussed in the context of securities auctions, and has been stressed in the electricity context by the Blue Ribbon Panel Report of the California Power Exchange (2001).

repeated interaction (even with uncertain demand). Forcing players to compete in prices by introducing PAB pricing rules or abolishing gross pools and allowing for continuous bilateral contracting can potentially remove these equilibria. This however comes at the cost of rendering entry by independent players less attractive, and possibly slowing down the changes in market structure which are arguably the key driver of prices in deregulated electricity markets in the medium term.

A Appendix

A.1 Proof that the linear bid function is the unique equilibrium in the competitive PAB case.

Note first that equation (1) must be satisfied for all q. In particular, it must hold for $\bar{q}$. Also, q must be consistent with realised demand, so we may rewrite (1) as:

$$\beta - \tfrac{\gamma}{\rho}(\mu - \beta) = \frac{1}{h(\beta)} = \frac{1}{h(\mu)}\frac{d\beta}{d\mu} \tag{14}$$

where the last equality follows from the properties of the hazard rate, and we are expressing β as a function of μ. Taking a Taylor series expansion of β around $\bar{\mu}$, we obtain:

$$\beta(\mu) = \gamma\bar{q} + \sum_{n=1}^{\infty}\frac{a_n}{n!}(\mu - \bar{\mu})^n$$

We can take the first derivative of this expansion to obtain:

$$\frac{d\beta(\mu)}{d\mu} = \sum_{n=1}^{\infty} n\frac{a_n}{n!}(\mu - \bar{\mu})^{n-1} \quad \Rightarrow \quad (\bar{\mu} - \mu)\frac{d\beta(\mu)}{d\mu} = -\sum_{n=1}^{\infty} n\frac{a_n}{n!}(\mu - \bar{\mu})^n$$

Since μ is uniformly distributed, it follows that $\frac{1}{h(\mu)} = \bar{\mu} - \mu$. Thus, substituting these two results, we obtain:

$$(1 + \tfrac{\gamma}{\rho})\beta - \tfrac{\gamma}{\rho}\mu = \frac{1}{h(\mu)}\frac{d\beta}{d\mu} \quad \Rightarrow \quad (1 + \tfrac{\gamma}{\rho})\gamma\bar{q} + \sum_{n=1}^{\infty}(1 + n + \tfrac{\gamma}{\rho})\frac{a_n}{n!}(\mu - \bar{\mu})^n = \tfrac{\gamma}{\rho}\mu \tag{15}$$

This can only hold for any μ if $a_1 = \frac{\gamma}{\rho}/(2 + \frac{\gamma}{\rho})$ and $a_n = 0$ for every $n > 1$. Notice that, since $\bar{q}$ solves for pricing at marginal cost, it follows that

$$\gamma\bar{q} = \bar{\mu} - \rho\bar{q} \quad \Rightarrow \quad \bar{q} = \bar{\mu}/(\rho + \gamma).$$

Notice that this equality, together with the restrictions on a_n, satisfies (15). Substituting all this into (14) yields:

$$\beta_c^*(\mu) = \frac{\gamma}{\rho + \gamma}\bar{\mu} + \frac{\gamma}{2\rho + \gamma}(\mu - \bar{\mu})$$

Substituting this equation back for q in the demand curve yields the optimal bidding function in terms of q: $\beta_c^*(q) = (\gamma q + \gamma\bar{q})/2$.

A.2 Proof of Lemma 1

Lemma 1 states that all producers (except for the one indexed $\bar{q}$) earn lower expected profits under PAB than under SMP (as proven in the text) and that the absolute loss from the introduction of PAB is decreasing with marginal costs. In what follows we prove the second part of this statement.

We define as $\Delta(q)$ the difference in expected profits between SMP and PAB for a producer with marginal cost equal to γq.We distinguish between three cases of $\Delta(q)$: $\Delta_1(q)$, which indicates the level of $\Delta(q)$ for producers whose marginal cost is below $\gamma \underline{q}_{PAB,c}$; $\Delta_2(q)$, which indicates the loss for producers with marginal costs between $\gamma \underline{q}_{PAB,c}$ and $\gamma \underline{q}_{SMP,c}$; and $\Delta_3(q)$, which relates to producers with costs above $\gamma \underline{q}_{SMP}$.

From the SMP and PAB bid functions and the distributions of demand we obtain that:

$$\Delta(q) = \begin{cases} \Delta_1(q) = \frac{\gamma^2 \Delta\mu}{2(\gamma+\rho)(\gamma+2\rho)} & \text{for } q < \underline{q}_{PAB,c} \\ \Delta_2(q) = \frac{\gamma(\bar{\mu}+\underline{\mu})}{2(\gamma+\rho)} - \gamma q - \frac{\gamma(\gamma+\rho)}{4\Delta\mu}(\bar{q}-q)^2 & \text{for } q \in [\underline{q}_{PAB,c}, \underline{q}_{SMP,c}] \\ \Delta_3(q) = \frac{\gamma^2(\bar{q}-q)}{4\Delta\mu} & \text{for } q \in (\underline{q}_{SMP,c}, \bar{q}] \end{cases}$$

where $\Delta\mu = \bar{\mu} - \underline{\mu}$. It is straightforward to show that both $\frac{\partial \Delta_2(q)}{\partial q}$ and $\frac{\partial \Delta_3(q)}{\partial q}$ are negative, which in turn implies that $\Delta_1(q) \geq \Delta_2(q) > \Delta_3(q)$.

A.3 Proof of Proposition 1

We firstly demonstrate the DW average price result and then prove the consumer surplus result.

A.3.1 DW Average Prices[52]

SMP Consider firstly the SMP case. Expected revenue is given by the following expression, where we integrate over equilibrium q rather than over μ:[53,54]

$$\begin{aligned} E(R_{SMP,c}) &= \int_{\underline{q}_{SMP,c}}^{\bar{q}} \gamma q^2 f(q) dq = \frac{\gamma}{3\Delta q_{SMP,c}} \left(\bar{q}^3 - (\bar{q} - \Delta q_{SMP,c})^3 \right) \\ &= \gamma \left(\bar{q}\,(\bar{q} - \Delta q_{SMP,c}) + \frac{\Delta q_{SMP,c}^2}{3} \right) = \gamma \left(\left(\bar{q} - \frac{\Delta q_{SMP,c}}{2} \right)^2 + \frac{\Delta q_{SMP,c}^2}{12} \right) \end{aligned} \tag{16}$$

[52] This is partially follows the approach of Green and McDaniel (1999), who establish expected revenue equivalence between SMP and PAB for the case of vertical demand.

[53] In this Appendix we denote equilibrium quantity as simply q in the various expressions for expected revenue and gross/net surplus, for notational convenience.

[54] Integrating over μ, yields the following:

$$\begin{aligned} E(R_{SMP,c}) &= \frac{\gamma}{3(\gamma+\rho)^2} \frac{\bar{\mu}^3 - \underline{\mu}^3}{\Delta\mu} \\ &= \frac{\gamma}{3(\gamma+\rho)^2} \left(\bar{\mu}^2 + \bar{\mu}\underline{\mu} + \underline{\mu}^2 \right) \end{aligned}$$

which is a result we use below.

where $\Delta q_{SMP,c} = \bar{q} - \underline{q}_{SMP,c}$. Given that expected quantity equals $\frac{\bar{q}+\underline{q}_{SMP,c}}{2} = \bar{q} - \frac{\Delta q_{SMP,c}}{2}$, demand-weighted average prices equal:

$$E(P_{SMP,c}) = \gamma\left(\bar{q} - \frac{\Delta q_{SMP,c}}{2}\right) + \frac{\gamma \Delta q_{SMP,c}^2}{6\left(2\bar{q} - \Delta q_{SMP,c}\right)} \tag{17}$$

PAB Under PAB expected revenue is as follows:

$$\begin{aligned}
E(R_{PAB,c}) &= \Pr\left(\mu > \beta_c^*(0)\right)\left[\begin{array}{c} \beta_c^*\left(\underline{q}_{PAB,c}\right)\underline{q}_{PAB,c} + \\ + \int\limits_{\underline{q}_{PAB,c}}^{\bar{q}} \frac{\gamma}{2}\left(\bar{q}+q\right)(1-F(q))dq \end{array}\right] \qquad (18) \\
&= \Pr\left(\mu > \beta_c^*(0)\right)\left[\begin{array}{c} \frac{\gamma}{2}\left(\bar{q} - \Delta q_{PAB,c}\right)\left(2\bar{q} - \Delta q_{PAB,c}\right) + \\ + \frac{\gamma}{2\Delta q_{PAB,c}} \int\limits_{\bar{q}-\Delta q_{PAB,c}}^{\bar{q}} \left(\bar{q}^2 - q^2\right) dq \end{array}\right] \\
&= \Pr\left(\mu > \beta_c^*(0)\right)\gamma\bar{q}\left(\left(\bar{q} - \Delta q_{PAB,c}\right) + \frac{\Delta q_{PAB,c}^2}{3}\right) \\
&= \Pr\left(\mu > \beta_c^*(0)\right)\gamma\left(\left(\bar{q} - \frac{\Delta q_{PAB,c}}{2}\right)^2 + \frac{\Delta q_{PAB,c}^2}{12}\right)
\end{aligned}$$

where $\Delta q_{PAB,c} = \bar{q} - \underline{q}_{PAB,c}$. Note that this implies that expected PAB revenue differs from the corresponding SMP value only if $\Delta q_{PAB,c} \neq \Delta q_{SMP,c}$.

Dividing (18) by expected quantity we obtain the following expression for DW average prices under PAB:

$$E(P_{PAB,c}) = \begin{cases} \gamma\left(\bar{q} - \frac{\Delta q_{PAB,c}}{2}\right) + \frac{\gamma \Delta q_{PAB,c}^2}{6\left(2\bar{q} - \Delta q_{PAB,c}\right)} & \text{for } \frac{\bar{\mu}}{\underline{\mu}} < \frac{2(\gamma+\rho)}{\gamma} \Leftrightarrow \underline{q}_{PAB,c} > 0 \\ \frac{2}{3}\gamma\bar{q} & \text{for } \frac{\bar{\mu}}{\underline{\mu}} \geq \frac{2(\gamma+\rho)}{\gamma} \Leftrightarrow \underline{q}_{PAB,c} = 0 \end{cases} \tag{19}$$

SMP-PAB Comparison Consider the $\underline{q}_{PAB,c} > 0$ case first. Comparing (17) and (19), after substituting for $\Delta q_{SMP,c}$ and $\Delta q_{PAB,c}$ in terms of the underlying parameters of the model (i.e. $\Delta q_{SMP,c} = \frac{\bar{\mu}-\underline{\mu}}{\gamma+\rho}$ and $\Delta q_{PAB,c} = \frac{\bar{\mu}-\underline{\mu}}{\frac{\gamma}{2}+\rho}$), yields:

$$\begin{aligned}
E(P_{SMP,c}) &> E(P_{PAB,c}) \text{ iff} \\
\frac{(2\rho-\gamma)\bar{\mu} + 4(\gamma+\rho)\underline{\mu}}{(\gamma+2\rho)(\rho\bar{\mu} + (\gamma+\rho)\underline{\mu})} &> \frac{\bar{\mu} + 2\underline{\mu}}{(\gamma+\rho)(\bar{\mu}+\underline{\mu})}
\end{aligned}$$

which gives a threshold ratio of the maximum and minimum demand intercepts ($\left(\frac{\bar{\mu}}{\underline{\mu}}\right)^* \equiv r^*$) below which $E(P_{SMP,c}) > E(P_{PAB,c})$, which is given by the following expression:

$$r^*(\gamma,\rho) \equiv \frac{1}{\gamma}\left(\gamma + 2\rho + \sqrt{3\gamma^2 + 10\gamma\rho + \rho^2}\right)$$

The condition for $\underline{q}_{PAB} > 0$ (i.e. $\frac{\bar{\mu}}{\underline{\mu}} < \frac{2(\gamma+\rho)}{\gamma}$) implies that $\frac{\bar{\mu}}{\underline{\mu}} < r^*(\gamma,\rho)$, since $\sqrt{3\gamma^2+10\gamma\rho+\rho^2} > \gamma$. This in turn implies that $E(P_{SMP,c}) > E(P_{PAB,c})$.

Turning now to the average price comparison in the $\underline{q}_{PAB,c} = 0$ case, it is straightforward to obtain that $E(P_{SMP,c}) > E(P_{PAB,c})$ if $(\bar{q} - \Delta q_{SMP,c})^2 > 0$, which always holds.

Note finally that if demand is vertical, we have that $\Delta q_{SMP,c} = \Delta q_{PAB,c}$ which implies that $E(P_{SMP,c}) = E(P_{PAB,c})$. This in turn implies that $E(CS_{SMP,c}) = E(CS_{PAB,c})$.

A.3.2 Consumer Surplus

SMP Substituting for equilibrium q in (4) and integrating yields:

$$E\left(GS_{SMP,c}\right) = \frac{2\gamma+\rho}{6\Delta q_{SMP,c}}\left(\bar{q}^3 - \left(\bar{q} - \Delta q_{SMP,c}\right)^3\right) \tag{20}$$

Combining this result with the expression for expected revenue given above, we obtain:

$$E(CS_{SMP,c}) = \frac{\rho}{2}\left(\bar{q}\left(\bar{q} - \Delta q_{SMP,c}\right) + \frac{\Delta q^2_{SMP,c}}{3}\right) \tag{21}$$

PAB and Comparison (a) Consider firstly the low demand uncertainty case (i.e. $\underline{q}_{PAB,c} > 0$). Substituting for equilibrium q into (4) yields:

$$\begin{aligned} E(GS_{PAB,c}) &= \frac{1}{2\Delta q_{PAB,c}}\int_{\bar{q}-\Delta q_{PAB,c}}^{\bar{q}} q\left(\gamma\bar{q} + (\gamma+\rho)\,q\right)dq \\ &= \frac{1}{2}\left(\left(2\gamma+\rho\right)\bar{q} - \left(\frac{3}{2}\gamma+\rho\right)\bar{q}\Delta q_{PAB,c} + \frac{\gamma+\rho}{2}\Delta q^2_{PAB,c}\right) \end{aligned}$$

Combining this result with the expression for expected revenue given above, we obtain:

$$E(CS_{PAB}) = \frac{1}{2}\left(\bar{q}\left(\rho\bar{q} + \frac{\gamma-2\rho}{2}\Delta q_{PAB,c}\right) + \frac{\rho-\gamma}{3}\Delta q^2_{PAB,c}\right) \tag{22}$$

Defining the difference between consumer surplus under the two price regimes as $\Delta CS_c\left(\delta\right) = E(CS_{PAB,c}) - E(CS_{SMP,c})$, where $\delta = \Delta\mu$, and substituting for the underlying parameters of the model we have:

$$\begin{aligned} \Delta CS_c\left(\delta\right) &= \frac{1}{2}\left[\bar{q}\left((\frac{\gamma}{2}-\rho)\Delta q_{PAB,c} + \rho\Delta q_{SMP,c}\right) + \frac{1}{3}\left((\rho-\gamma)\Delta q^2_{PAB,c} - \rho\Delta q^2_{SMP,c}\right)\right] \\ &= \frac{\gamma^2\delta}{2(\gamma+\rho)^2}\left(\bar{\mu} + \frac{5\rho-4\gamma}{3(\gamma+2\rho)}\delta\right) \end{aligned}$$

This can only take a negative value if both δ and γ are high. Substituting for the maximum value δ can take given the restriction $\underline{q}_{PAB,c} \geq 0$ (i.e. $\delta = \delta^{\max} \equiv \frac{2\rho+\gamma}{2(\gamma+\rho)}\bar{\mu}$), we obtain

$$\Delta CS_c\left(\delta^{\max}\right) = \frac{\gamma^2\delta^{\max}}{2(\gamma+\rho)^2}\left(\frac{2\gamma+11\rho}{6(\gamma+\rho)}\right)\bar{\mu} > 0$$

which proves that consumer surplus is always higher under PAB than under SMP if $\underline{q}_{PAB,c} > 0$.

(b) Turning to the $\underline{q}_{PAB,c} = 0$ case, we have (by setting $\Delta q_{PAB,c} = \bar{q}$ in (22)):

$$\begin{aligned} E(CS_{PAB,c}) &= \Pr\left(\mu > \beta_c^*(0)\right) \frac{2\rho+\gamma}{12}\bar{q}^2 \\ &= \frac{(2\rho+\gamma)^2}{24(\gamma+\rho)^3} \frac{\bar{\mu}^3}{\Delta\mu} \end{aligned}$$

Noting that $E(CS_{SMP,c})$ can be re-expressed as $E(CS_{SMP,c}) = \frac{\rho}{6(\rho+\gamma)^2}\frac{\bar{\mu}^3-\underline{\mu}^3}{\Delta\mu}$, by integrating over μ rather than over q in the derivation of $E(GS_{SMP,c})$ and $E(R_{SMP,c})$, we obtain:

$$\begin{aligned} \Delta CS_c(\delta) &= \frac{1}{6(\gamma+\rho)^2\delta}\left(\frac{(\gamma+2\rho)^2\bar{\mu}^3}{4(\gamma+\rho)} - \rho(\bar{\mu}^3 - \underline{\mu}^3)\right) \\ &= \frac{1}{6(\gamma+\rho)^2\delta}\left(\frac{\gamma^2}{4(\gamma+\rho)}\bar{\mu}^3 + \rho\underline{\mu}^3\right) > 0 \end{aligned}$$

A.4 Proof of Proposition 2

A.4.1 Part (i): $\gamma > \rho$

Focus first on the integral in (6). This is the same as:

$$E_\mu \int_{\underline{\mu}}^{\mu} (\beta(\theta) - \gamma q(\theta, \beta(\theta)))\frac{dq}{d\theta}d\theta = \tfrac{1}{\rho}E_\mu \int_{\underline{\mu}}^{\mu} (\beta(\theta)(1+\tfrac{\gamma}{\rho}) - \theta\tfrac{\gamma}{\rho})(1 - \frac{d\beta}{d\theta})d\theta$$

where we have simply substituted for the demand curve.[55] Notice, first of all, that we are assuming that β is a function of μ, and not explicitly of q. If we multiply out the expression on the r.h.s., we obtain:

$$\tfrac{1}{\rho}E_\mu \int_{\underline{\mu}}^{\mu} (\beta(\theta)(1+\tfrac{\gamma}{\rho}) - \theta\tfrac{\gamma}{\rho})d\theta - \int_{\beta(\underline{\mu})}^{\beta(\mu)} \beta(\theta)(1+\tfrac{\gamma}{\rho})d\beta + \int_{\underline{\mu}}^{\mu} \theta\tfrac{\gamma}{\rho}\frac{d\beta}{d\theta}d\theta \tag{23}$$

Defining $B(\theta)$ as the antiderivative of $\beta(\theta)$ and solving for these three terms separately; from left to right, we obtain:

$$\int_{\underline{\mu}}^{\mu} (\beta(\theta)(1+\tfrac{\gamma}{\rho}) - \theta\tfrac{\gamma}{\rho})d\theta = \left[B(\mu) - B(\underline{\mu})\right](1+\tfrac{\gamma}{\rho}) - \tfrac{1}{2}\tfrac{\gamma}{\rho}(\mu^2 - \underline{\mu}^2).$$

The next term equals:

$$\int_{\beta(\underline{\mu})}^{\beta(\mu)} (1+\tfrac{\gamma}{\rho})\beta d\beta = (1+\tfrac{\gamma}{\rho})\tfrac{1}{2}\left[\beta(\mu)^2 - \beta(\underline{\mu})^2\right].$$

Finally, we need to integrate the last term by parts.

$$\int_{\underline{\mu}}^{\mu} \theta\tfrac{\gamma}{\rho}\frac{d\beta}{d\theta}d\theta = \tfrac{\gamma}{\rho}\left[\mu\beta(\mu) - \underline{\mu}\beta(\underline{\mu}) - \int_{\underline{\mu}}^{\mu}\beta(\theta)d\theta\right] = \tfrac{\gamma}{\rho}\left[\mu\beta(\mu) - \underline{\mu}\beta(\underline{\mu}) - (B(\mu) - B(\underline{\mu}))\right]$$

[55] Note that $q = (\mu - \beta)/\rho$ which implies $dq/d\mu = (1 - d\beta/d\mu)/\rho$.

Collecting the terms together we can conclude that (23) is equal to $\frac{1}{\rho}E_{\mu}F(\mu)$, where

$$F(\mu) = B(\mu) - B(\underline{\mu}) - \tfrac{1}{2}\tfrac{\gamma}{\rho}(\mu^2 - \underline{\mu}^2) - (1 + \tfrac{\gamma}{\rho})\tfrac{1}{2}\left[\beta(\mu)^2 - \beta(\underline{\mu})^2\right] + \tfrac{\gamma}{\rho}(\mu\beta(\mu) - \underline{\mu}\beta(\underline{\mu})).$$

Rearranging,

$$F(\mu) = B(\mu) - B(\underline{\mu}) - \tfrac{1}{2}\left[\beta(\mu)^2 - \beta(\underline{\mu})^2\right] - \tfrac{1}{2}\tfrac{\gamma}{\rho}\left[(\beta(\mu) - \mu)^2 - (\beta(\underline{\mu}) - \underline{\mu})^2\right]$$

We can apply the Euler-Lagrange condition to maximise the function $\frac{1}{\rho}E_{\mu}F(\mu)$ given that $F(\mu) = F(\mu, B, B')$, so that, at the optimum:

$$\frac{\partial F}{\partial B} - \frac{d}{d\mu}\frac{\partial F}{\partial \beta} = 0$$

Calculating the derivatives, we get, for each μ,

$$\frac{d}{d\mu}(\beta + \tfrac{\gamma}{\rho}(\beta - \mu)) \quad = \quad -1 \tag{24}$$

$$\Rightarrow \quad (1 + \tfrac{\gamma}{\rho})\frac{d\beta}{d\mu} = -(1 - \tfrac{\gamma}{\rho}) \tag{25}$$

The last condition implies that β is a linear function of μ. We need a transversality condition to pin down the function, but so far we have the following:

$$\beta(\mu) = \frac{\gamma - \rho}{\gamma + \rho}\mu + \text{ a constant.} \tag{26}$$

For the constraint that the bid function be upward-sloping not to bind we require that $\gamma > \rho$, which is intuitive.

We now need to solve for the constant.

Consider first the $\underline{q} > 0$ case. Maximising expected profits (given by equation (6)) relative to the constant, defined as c, we obtain the following:

$$\frac{\partial \text{ -}}{\partial c} = \underline{q} - \tfrac{1}{\rho}\underline{\beta} + \tfrac{\gamma}{\rho}\underline{q} + \tfrac{1}{\rho}(1 - \frac{\gamma - \rho}{\gamma + \rho})(1 + \tfrac{\gamma}{\rho})E_{\mu}\int_{\underline{\mu}}^{\mu} d\theta = 0$$

Rearranging and manipulating this equation yields:

$$\underline{\beta} = \underline{\mu}\frac{\rho + \gamma}{2\rho + \gamma} + \frac{\rho(\bar{\mu} - \underline{\mu})}{2\rho + \gamma} > \underline{\mu}\frac{\rho + \gamma}{2\rho + \gamma}.$$

Using (26) and the fact that $\underline{\beta} = \beta(\underline{\mu})$, it follows that

$$c = \frac{\rho}{2\rho + \gamma}\left[\bar{\mu} + \underline{\mu}\frac{2\rho}{\rho + \gamma}\right]$$

We can now offer a bid function for the $\underline{q}_{PAB,m} > 0$ case, as given by Proposition 2 - part (i). This bid function shows that $\underline{q} > 0$ if its intercept in q-space is greater than $\underline{\mu}$ (i.e. $\frac{(\rho+\gamma)\bar{\mu}+2\rho\underline{\mu}}{2(2\rho+\gamma)} > \underline{\mu}$), which holds if $\frac{\bar{\mu}}{\underline{\mu}} < 2$. If this last condition does not hold, the monopolist optimisation problem can be re-written as follows:

$$\max_c \Pr(\mu > \beta(\underline{q}) \left(\int_0^{\bar{q}} \left(c + \left(\frac{\gamma - \rho}{2} - \gamma \right) q \right) (1 - F(q) dq \right)$$

where we are expressing the bid function β in terms of q, and exploiting the fact that it is linear with a slope of $\frac{\gamma - \rho}{2}$ (in q-space), which is established above. This simplifies to:

$$\max_c \frac{\bar{\mu} - c}{\bar{\mu} - \underline{\mu}} \frac{\bar{q}}{2} \left(c - \frac{\gamma + \rho}{6} \bar{q} \right)$$

Substituting for $\bar{q}$, differentiating w.r.t. the constant c and equating to 0 yields $c = \frac{\bar{\mu}}{2}$.

The further results of part (i) of the proposition follow trivially.

A.4.2 Part (ii): $\gamma \leq \rho$

This follows from the simpler optimisation problem the monopolist faces if $\gamma \leq \rho$, namely:

$$\max_\beta E(\pi) = \max_\beta \Pr(\mu > \beta) \left(\beta \underline{q} - \frac{\gamma}{2} \underline{q}^2 + \int_{\underline{q}}^{\bar{q}} (\beta - \gamma q) (1 - F(q)) \, dq \right) \tag{27}$$

If $\underline{\mu} > \beta$ (or $\underline{q} > 0$) (27) simplifies to:

$$\max_\beta \beta \underline{q} - \frac{\gamma}{2} \underline{q}^2 + \Delta q \left(\frac{\beta - \gamma \bar{q}}{2} + \frac{\Delta q}{3} \right) \tag{28}$$

where $\Delta q = \bar{q} - \underline{q}$. Differentiating (28) w.r.t. β yields the value $\hat{\beta}_m$ given in Proposition 2- part (ii) for the $\underline{q} > 0$ case, which obtains if $\frac{\bar{\mu}}{\underline{\mu}} < \frac{3\rho + \gamma}{\rho + \gamma}$.

If $\underline{\mu} < \beta$ (27) simplifies to:

$$\max_\beta \left(\frac{\bar{\mu} - \beta}{\bar{\mu} - \underline{\mu}} \right) \frac{\bar{q}}{2} \left(\beta - \frac{\gamma}{3} \bar{q} \right) \tag{29}$$

which yields $\hat{\beta}_m = \frac{\rho + \gamma}{3\rho + \gamma} \bar{\mu}$, which is greater than $MP^*(E(\mu))$ for $\frac{\bar{\mu}}{\underline{\mu}} > \frac{3\rho + \gamma}{\rho + \gamma}$.

A.5 Proof of Proposition 3

Define firstly $\Delta W_m^i(\delta)$ as the difference between expected welfare under SMP and that under PAB under monopoly conditions, where $i \in \{I, II, III, IV\}$ indicates each of the four PAB cases, and $\delta = \Delta \mu$.

A.5.1 SMP

From equation (11), and substituting for equilibrium quantity, we obtain that under SMP:

$$\begin{aligned} E(W_{SMP,m}) &= \frac{3\rho + \gamma}{6(2\rho + \gamma)^2} \frac{\bar{\mu}^3 - \underline{\mu}^3}{\Delta \mu} \\ &= \frac{3\rho + \gamma}{6(2\rho + \gamma)^2} \left(\bar{\mu}^2 + \bar{\mu} \underline{\mu} + \underline{\mu}^2 \right) \end{aligned} \tag{30}$$

A.5.2 PAB

Case I Substituting for equilibrium quantity ($q_{PAB,m} = \frac{2\mu}{\gamma+\rho} - \frac{\bar{\mu}+\frac{2\rho}{\rho+\gamma}\underline{\mu}}{2\rho+\gamma}$) into (10), and simplifying we have:

$$W_{PAB,m}(\mu) = \kappa\left(\mu - \frac{\rho+\gamma}{2}\kappa\right)$$

where $\kappa = \frac{\bar{\mu}+\frac{2\rho}{\rho+\gamma}\underline{\mu}}{2\rho+\gamma}$. Expected PAB welfare is therefore as follows (where the second line is obtained after some simplification, and substituting for κ in terms of underlying parameters):

$$\begin{aligned} E\left(W_{PAB,m}\right) &= \frac{\kappa}{\Delta\mu}\int_{\underline{\mu}}^{\bar{\mu}}\left(\mu - \frac{\rho+\gamma}{2}\kappa\right)d\mu \\ &= \frac{\left(\rho\bar{\mu}+\gamma\underline{\mu}\right)\left((\rho+\gamma)\bar{\mu}+2\rho\underline{\mu}\right)}{2\left(\rho+\gamma\right)\left(2\rho+\gamma\right)^2} \end{aligned}$$

After straightforward manipulation this yields:

$$\Delta W_m^I(\delta) = c_I\left[\gamma(\gamma+\rho)\bar{\mu}^2 + (\gamma^2+3\rho^2-2\gamma\rho)\underline{\mu}^2 + (\gamma\rho-2\gamma^2-3\rho^2)\bar{\mu}\underline{\mu}\right]$$

where $c_I = \frac{1}{6(\rho+\gamma)(2\rho+\gamma)^2} > 0$. Notice that this implies $\Delta W^I(\delta = 0) = 0$, i.e. welfare is the same across two regimes under conditions of no demand uncertainty.

Differentiating $\Delta W_m^I(\delta)$ w.r.t. δ we obtain:

$$\frac{\partial \Delta W_m^I(\delta)}{\partial \delta} = 3\rho(\gamma-\rho)\underline{\mu} + 2\gamma(\gamma+\rho)\delta > 0$$

given that $\gamma > \rho$.

Case II Equilibrium PAB quantity in this case is given by $q_{PAB,m} = \frac{2\mu}{\gamma+\rho} - \frac{\bar{\mu}}{\gamma+\rho}$, so that expected welfare is as follows:

$$\begin{aligned} E\left(W_{PAB,m}\right) &= \frac{\bar{\mu}}{2\Delta\mu}\int_{\frac{\bar{\mu}}{2}}^{\bar{\mu}}\frac{\bar{\mu}(2\mu-\bar{\mu})}{2(\gamma+\rho)}f(\mu)d\mu \\ &= \frac{\bar{\mu}^3}{8\Delta\mu(\gamma+\rho)} \end{aligned}$$

This yields:

$$\Delta W_m^{II}(\delta) = c_{II}\left[4(\gamma+\rho)\left(3\rho+\gamma\right)\left(\bar{\mu}^3-\underline{\mu}^3\right) - 3(4\rho^2+\gamma^2+4\rho\gamma)\bar{\mu}^3\right]$$

where $c_{II} = \frac{1}{24(\gamma+\rho)(2\rho+\gamma)^2\delta} > 0$. The term in the square brackets of $\Delta W_m^{II}(\delta)$ is increasing in δ, and is therefore at its minimum if $\delta = \delta_{\min} \equiv \underline{\mu}$. Evaluating $\Delta W_m^{II}(\delta)$ at $\delta_{\min}$, we obtain:

$$\Delta W_m^{II}(\delta_{\min}) = 4c_{II}(\gamma^2+\rho(4\gamma-3\rho))\underline{\mu}^3 > 0$$

Case III Equilibrium PAB quantity is given by $q_{PAB,m} = \frac{\mu}{\rho} - \frac{\rho+\gamma}{2\rho+\gamma}\frac{E(\mu)}{\rho}$, so that expected welfare is given by:

$$E\left(W_{PAB,m}\right) = \int_{\underline{\mu}}^{\bar{\mu}} \left[\frac{\rho-\gamma}{2\rho^2}\mu^2 - \left(\frac{2\gamma(\gamma+\rho)}{2\rho^2(2+\gamma)}\right)E(\mu)\mu - \frac{(\rho+\gamma)^3}{2\rho^2(2\rho+\gamma)^2}E(\mu)\right] f(\mu)d\mu$$

which, after some algebraic manipulation, simplifies to:

$$E\left(W_{PAB,m}\right) = \frac{(\rho-\gamma)\left(\bar{\mu}^3 - \underline{\mu}^3\right)}{6\rho^2\Delta\mu} + \frac{(\gamma+\rho)(\gamma^2-\rho^2+2\rho\gamma)}{8\rho^2(2\rho+\gamma)^2}\left(\bar{\mu}+\underline{\mu}\right)^2$$

Comparing this to the corresponding SMP value yields:

$$\Delta W_m^{III}(\delta) = c_{III}\left[(\gamma+\rho)(\gamma^2-\rho^2+2\rho\gamma)\left(\bar{\mu}^3 - \underline{\mu}^3 - 3\bar{\mu}\underline{\mu}\delta\right)\right]$$

where $c_{III} = \frac{1}{24\rho^2(2\rho+\gamma)^2\delta} > 0$. The last term in the square brackets of this expression is always positive (for $\delta > 0$) (and is 0 for $\delta = 0$), and the second term is positive iff $\gamma > \bar{\gamma}(\rho) \equiv \left(\sqrt{2}-1\right)\rho$. If the latter condition holds, $\Delta W_m^{III}(\delta) > 0$ (as in cases I and II)). Otherwise welfare is higher under PAB.

Case IV PAB welfare in this case is given by:

$$\begin{aligned} W_{PAB,m}(\mu) &= \left(\frac{\mu}{\rho} - \frac{\rho+\gamma}{3\rho+\gamma}\frac{\bar{\mu}}{\rho}\right)\left(\frac{\rho-\gamma}{2\rho}\mu + \frac{(\rho+\gamma)^2}{2\rho(3\rho+\gamma)}\bar{\mu}\right) \\ &= \frac{1}{2\rho^2}\left((\rho-\gamma)\mu^2 - \frac{(\rho+\gamma)^3}{(3\rho+\gamma)^2}\bar{\mu}^2 + \frac{2\gamma(\rho+\gamma)}{3\rho+\gamma}\bar{\mu}\mu\right) \end{aligned}$$

After some simplification, this gives:

$$E(W_{PAB,m}) = \frac{\bar{\mu}^3}{2\rho^2\Delta\mu\,(3\rho+\gamma)^3}\left[\frac{\rho-\gamma}{3}\left((3\rho+\gamma)^3 - (\rho+\gamma)^3\right) + 2\rho(\gamma+\rho)\left(\gamma^2+2\gamma\rho-\rho^2\right)\right] \tag{31}$$

Comparing equations (30) and (31) we can derive that $\Delta W_m^{IV}(\delta) > 0$ if the following condition on demand uncertainty holds:

$$\frac{\bar{\mu}}{\underline{\mu}} \geq \sqrt[3]{\frac{\rho^2(3\rho+\gamma)^4}{(3\rho+\gamma)^3(\gamma^3+3\rho\gamma^2+\rho^2\gamma-\rho^3)+(\rho+\gamma)(2\rho+\gamma)^2(7\rho^3-11\rho^2\gamma-7\rho\gamma^2-\gamma^3)}}$$

Therefore, even if $\gamma \leq \bar{\gamma}(\rho)$, eventually the output contraction effect due to PAB will outweigh the beneficial ‘equalisation of marginal utilities’ effect, leading to a reduction in welfare. See Figure 5 for a plot of the $\Delta W_m^i(\delta) = 0$ schedule for PAB cases III and IV.

A.6 Proof of Proposition 4

This proof follows the approach adopted in the competitive case to compute expected demand-weighted prices and consumer surplus. We start by proving the average price results.

A.6.1 DW Average Prices

SMP Recalling that under monopoly condition SMP equilibrium quantities are given by $q_{SMP,m} = \frac{\mu}{2\rho+\gamma}$ and that the bid function is $SF^*(q) = (\gamma+\rho)q$, the expression for DW average prices given in Section A.3.1 (see footnote 54) generalises to the following:

$$E(P_{SMP,m}) = \frac{2}{3}\frac{\rho+\gamma}{2\rho+\gamma}\frac{\bar{\mu}^2+\underline{\mu}^2+\bar{\mu}\underline{\mu}}{\bar{\mu}+\underline{\mu}} \tag{32}$$

PAB and Comparison Case I.

Applying the approach used in equation (18) to the monopoly case yields:

$$E(R_{PAB,m}) = \underline{q}_{PAB,m}\left(\alpha+\lambda\underline{q}_{PAB,m}\right) + \frac{1}{\Delta q_{PAB,m}}\int_{\bar{q}_{PAB,m}-\Delta q_{PAB,m}}^{\bar{q}_{PAB,m}}(\alpha+\lambda q)(\bar{q}_{PAB,m}-q)dq$$

where α and λ in the first expression indicate the intercept and the slope of the monopolist optimal bid function $\beta^*_m(q)$ given in equation (8). After some algebraic manipulation this yields:

$$\begin{aligned} E(R_{PAB,m}) &= (\bar{q}_{PAB,m}-\Delta q_{PAB,m})\left(\alpha+\lambda\left(\bar{q}_{PAB,m}-\Delta q_{PAB,m}\right)\right)+ \\ &\quad +\Delta q_{PAB,m}\left(\frac{\alpha+\lambda\bar{q}_{PAB,m}}{2}-\frac{\lambda}{3}\Delta q_{PAB,m}\right) \\ &= \alpha E(q_{PAB,m})+\lambda\left[\left(\bar{q}_{PAB,m}-\frac{4}{3}\Delta q_{PAB,m}\right)E(q_{PAB,m})+\frac{1}{3}\bar{q}_{PAB,m}\Delta q_{PAB,m}\right] \end{aligned} \tag{33}$$

where $E(q_{PAB,m}) = \bar{q}_{PAB,m} - \frac{\Delta q_{PAB,m}}{2}$. Therefore:

$$\begin{aligned} E(P_{PAB,m}) &= \alpha+\lambda\left(\bar{q}_{PAB,m}-\frac{4\Delta q_{PAB,m}}{3}\right)+\frac{\lambda}{3}\frac{\bar{q}_{PAB,m}\Delta q_{PAB,m}}{\left(\bar{q}_{PAB,m}-\frac{\Delta q_{PAB,m}}{2}\right)} = \\ &= \frac{(\gamma^3+3\gamma^2\rho+3\gamma\rho^2-\rho^3)\bar{\mu}^2+2(2\gamma^3+3\rho\gamma^2-2\rho^3)\underline{\mu}^2+2(4\rho^3+3\rho^2\gamma-\gamma^3)\bar{\mu}\underline{\mu}}{3(\gamma+\rho)(2\rho+\gamma)\left(\rho\bar{\mu}+\gamma\underline{\mu}\right)} \end{aligned} \tag{34}$$

where the second expression is obtained by substituting for $\bar{q}_{PAB,m}$ and $\Delta q_{PAB,m}$ in terms of the underlying parameters of the model, and simplifying.

Comparing (34) and (32) we obtain after some algebraic manipulation that $E(P_{SMP,m}) > E(P_{PAB,m})$ if:

$$\bar{\mu}^2\left[\gamma(\gamma^2+\rho(\gamma-\rho))(3\underline{\mu}-\bar{\mu})+\rho^3(3\bar{\mu}-5\underline{\mu})\right] > 2\underline{\mu}^2\left[(\gamma(\gamma^2+\rho(\gamma-\rho)\underline{\mu}+\rho^3(\bar{\mu}-2\underline{\mu})\right] \tag{35}$$

where the l.h.s. equals the r.h.s. for $\underline{\mu} = \bar{\mu}$ (i.e. the no uncertainty case).

To show that (35) holds it is sufficient to show that, assuming $\bar{\mu} = \underline{\mu}+\delta$, where $\delta \in (0, \underline{\mu})$, the l.h.s. of (35) increases with δ faster than the r.h.s. This is equivalent to the following condition:

$$3(\gamma^3(\underline{\mu}^2-\delta^2)-\rho^3(\underline{\mu}^2-3\delta^2))+3\gamma\rho(\gamma-\rho))(\underline{\mu}^2-\delta^2)+8\rho^3\delta\underline{\mu} > 0$$

which is always the case given that $\gamma > \rho$ and $\underline{\mu} > \delta$.

Case II.

Imposing $\Delta q_{PAB,m} = \bar{q}_{PAB,m}$ in the first line of (34) yields:

$$E(P_{PAB,m}) = \alpha + \frac{\lambda}{3}\bar{q}_{PAB,m} = \frac{2\gamma + \rho}{3(\gamma + \rho)}\bar{\mu} \tag{36}$$

where α and λ are defined as in Case I. Comparing (36) and (32) we obtain after some manipulation that $E(P_{SMP,m}) > E(P_{PAB,m})$ if and only if:

$$\begin{aligned} 2(\rho + \gamma)^2 \underline{\mu}^2 &> \rho\gamma \left(\bar{\mu} + \underline{\mu}\right) \bar{\mu} \\ &\text{or} \\ \frac{\bar{\mu}}{\underline{\mu}} &< \frac{\sqrt{(8\rho^2 + 8\gamma^2 + 17\rho\gamma)\,\rho\gamma} - \rho\gamma}{2\rho\gamma} \end{aligned} \tag{C1}$$

which does not always hold (e.g. if $\gamma = \rho$ it does not hold if $\frac{\bar{\mu}}{\underline{\mu}} > \frac{\sqrt{33}-1}{2} \approx 2.4$). See Figure 5 in the main text for a plot of this condition.

Case III.

Time-weighted average prices are the same between SMP and PAB in this case, given that the monopoly PAB bid function crosses the SMP bid function at expected demand. Demand-weighted average prices are however higher under SMP, given the positive correlation between demand and prices, due to the SMP bid function being upwards sloping.

Case IV.

Comparing $\hat{\beta}_m = \frac{\rho+\gamma}{3\rho+\gamma}\bar{\mu}$ with (32) yields the following condition for $E(P_{SMP,m}) > E(P_{PAB,m})$:

$$\begin{aligned} 2(3\rho + \gamma)\underline{\mu}^2 &> \gamma \left(\bar{\mu} + \underline{\mu}\right) \bar{\mu} \\ &\text{or} \\ \frac{\bar{\mu}}{\underline{\mu}} &< \frac{\sqrt{3\gamma\,(3\gamma + 8\rho)} - \gamma}{2\gamma} \end{aligned} \tag{C2}$$

which does not always hold, and in particular it fails to hold if the difference between ρ and γ is relatively low (e.g. if $\gamma = \rho$ it does not hold if $\frac{\bar{\mu}}{\underline{\mu}} > \frac{\sqrt{33}-1}{2} \approx 2.4$, which confirms the results obtained for Case II). See Figure 5 in the main text for a plot of this condition.

A.6.2 Consumer Surplus

SMP Using (4) and substituting for equilibrium quantity under SMP monopoly bidding, we have

$$E\left(GS_{SMP,m}\right) = \frac{2\gamma + 3\rho}{6\Delta q_{SMP,m}} \left(\bar{q}_{SMP,m}^3 - \left(\bar{q}_{SMP,m} - \Delta q_{SMP,m}\right)^3\right) \tag{37}$$

Expected revenues are given by equation (16), replacing the bid function γq with $(\gamma + \rho)\, q$. Expected consumer surplus is therefore given by:

$$E(CS_{SMP}) = \frac{\rho}{2} \left(\bar{q}_{SMP,m} \left(\bar{q}_{SMP,m} - \Delta q_{SMP,m}\right) + \frac{\Delta q_{SMP,m}^2}{3} \right) \tag{38}$$

This can also be expressed, in terms of μ, as:

$$E(CS_{SMP,m}) = \frac{\rho}{6(2\rho+\gamma)^2}\frac{\bar{\mu}^3 - \underline{\mu}^3}{\Delta\mu} \tag{39}$$

PAB and Comparison We express the difference in consumer surplus between PAB and SMP as $\Delta CS_m^i(\delta)$, with $i \in \{I, II, III, IV\}$, and $\delta = \Delta\mu$.

Case I.

Re-write the optimal bid function $\beta_m^*(q)$ as $\beta_m^*(q) = \alpha + \frac{\gamma-\rho}{2}q$. In equilibrium we therefore have $\mu = \alpha + \frac{\gamma+\rho}{2}q$. In terms of q expected gross surplus therefore equals:

$$\begin{aligned} E(GS_{PAB,m}) &= \int_{\bar{q}_{PAB,m}-\Delta q_{PAB,m}}^{\bar{q}_{PAB,m}} \frac{\gamma q + 2\alpha}{2} q f(q) dq \\ &= \alpha E(q_{PAB,m}) + \frac{1}{2}\left(\gamma \bar{q}_{PAB,m}^2 - \gamma \bar{q}_{PAB,m}\Delta q_{PAB,m} + \frac{\gamma \Delta q_{PAB,m}^2}{3}\right) \end{aligned}$$

Expected revenue is given equation (33). Combining these two equations we have:

$$E(CS_{PAB,m}) = \frac{1}{2}\left(\rho \bar{q}_{PAB,m}^2 - \frac{3\rho-\gamma}{2}\bar{q}_{PAB,m}\Delta q_{PAB,m} + \frac{2\rho-\gamma}{3}\Delta q_{PAB,m}^2\right) \tag{40}$$

Substituting for the underlying parameters of the model, and comparing with (39) yields, after some straightforward manipulation:

$$\Delta CS_m^I(\delta) = \frac{\Delta\mu}{6(\gamma+\rho)(\gamma+2\rho)^2}\left[\left(4\rho^2 - \gamma^2 + \gamma\rho\right)\bar{\mu} + \left(4\gamma^2 - \gamma\rho - 7\rho^2\right)\underline{\mu}\right]$$

Replacing $\bar{\mu}$ with $\underline{\mu} + \delta$, with $\delta \in [0, \underline{\mu}]$ yields:

$$\begin{aligned} \Delta CS_m^I(\delta) &= \frac{\Delta\mu}{6(\gamma+\rho)(\gamma+2\rho)^2}\left[\rho^2(4\delta - 3\underline{\mu}) + \gamma^2(3\underline{\mu} - \delta) + \gamma\rho\delta\right] \\ &= \frac{\Delta\mu}{6(\gamma+\rho)(\gamma+2\rho)^2}\left[(\gamma-\rho)\left(3(\gamma+\rho)\underline{\mu} - \gamma\delta\right) + 4\rho^2\delta\right] \end{aligned}$$

which is always positive given that $\gamma \geq \rho$ and $\delta < \underline{\mu}$.

Case II.

Also in this case, it is convenient to compute consumer surplus by integrating over q rather than over μ. The equilibrium bid function implies $\mu = \frac{\gamma+\rho}{2}\left(q + \bar{q}_{PAB,m}\right)$. Expected gross surplus is therefore as follows:

$$\begin{aligned} E(GS_{PAB,m}) &= \Pr(\mu > \frac{\bar{\mu}}{2}) \int_0^{\bar{q}_{PAB,m}} \frac{\gamma q + (\gamma+\rho)\bar{q}_{PAB,m}}{2} q f(q) dq \\ &= \frac{\bar{\mu}}{2\Delta\mu}\frac{\bar{q}_{PAB,m}^2}{12}(5\gamma + 3\rho) \end{aligned}$$

From the first part of this proof we have that:

$$E(R_{PAB,m}) = \frac{\bar{\mu}}{2\Delta\mu}\frac{\bar{q}_{PAB,m}^2}{12}(4\gamma + 2\rho)$$

which is obtained from equation (33), by imposing $\underline{q}_{PAB,m} = 0$ and substituting for the equilibrium bid function. Expected consumer surplus is therefore given by:

$$\begin{aligned} E(CS_{PAB,m}) &= \frac{\bar{\mu}}{2\Delta\mu}\frac{\bar{q}^2_{PAB,m}}{12}(\gamma+\rho) \\ &= \frac{1}{24(\gamma+\rho)}\frac{\bar{\mu}^3}{\Delta\mu} \end{aligned}$$

Comparing this value with the corresponding SMP one we have:

$$\Delta CS^{II}_m(\delta) = \frac{\gamma^2\bar{\mu}^3 + 4\rho(\gamma+\rho)\underline{\mu}^3}{24(\gamma+\rho)(2\rho+\gamma)^2\Delta\mu} > 0$$

Case III.

The consumer surplus comparison between PAB and SMP is trivial in this case, given that the introduction of PAB equalises the marginal benefit of consumption across demand realisations, and does not lead to a reduction in expected output. Consumer surplus is therefore necessarily higher under PAB.

Case IV.

In this case consumer surplus is simply given by $CS_{PAB,m}(\mu) = q(\mu - \frac{\rho}{2}q - \hat{\beta}_m) = \frac{\rho}{2}q^2$. Expected consumer surplus is therefore given by:

$$\begin{aligned} E(CS_{PAB,m}) &= \Pr(\mu > \hat{\beta}_m)\int_0^{\bar{q}_{PAB,m}} \frac{\rho}{2}q^2 f(q)dq \\ &= \frac{\rho^2}{3\rho+\gamma}\frac{\bar{\mu}}{\Delta\mu}\frac{\bar{q}^2_{PAB,m}}{3} \\ &= \frac{4\rho^2}{3(3\rho+\gamma)^3}\frac{\bar{\mu}^3}{\Delta\mu} \end{aligned}$$

Comparing this with expected consumer surplus under SMP yields:

$$\begin{aligned} \Delta CS^{IV}_m(\delta) &= \frac{\rho}{3\Delta\mu}\left[\left(\frac{4\rho}{(3\rho+\gamma)^3} - \frac{1}{2(2\rho+\gamma)^2}\right)\bar{\mu}^3 + \frac{\underline{\mu}^3}{2(2\rho+\gamma)^2}\right] \\ &= \frac{\rho}{3\Delta\mu}\left[\frac{5\rho^3 + 23\rho^2\gamma - \rho\gamma^2 - \gamma^3}{2(2\rho+\gamma)^2(3\rho+\gamma)^3}\bar{\mu}^3 + \frac{\underline{\mu}^3}{2(2\rho+\gamma)^2}\right] > 0 \end{aligned}$$

given that $\rho > \gamma$.

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