Fiscal Contracts for a Monetary Union

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ABSTRACT. This paper suggests that in a monetary union: (i) fiscal policies should be delegated with optimal contracts, perhaps written over the deficit; (ii) policymakers would have no incentives to deviate by forming coalitions from the resulting equilibrium when exchange of information is allowed for. In a model of a monetary union with decentralized fiscal authorities and both fiscal-fiscal and fiscal-monetary spillovers, individual policymaking is inefficient whereas binding agreements are unfeasible. A centralised equilibrium is optimal and time consistent if the policymaker shares the social preferences and uses non-distortionary fiscal instruments. When policy is decentralized with heterogeneous preferences of authorities and fiscal policy is distortionary, the resulting equilibrium is always inefficient and stable to incentives to collude. The optimal policy mix can however be implemented in the decentralized game, via delegating all policies (by the same principal) where the resulting equilibrium is efficient and coalition-proof.

1. Introduction

Monetary unification, together with fiscal decentralization, raises new issues for optimal design of monetary and fiscal policy. Firstly, given fiscal spillovers, a regime in which fiscal authorities are not cooperating (or do not have a mechanism designed to act as a substitute for cooperation) is bound to lead to inefficiencies. Secondly, given that fiscal policy is not exogenous to optimal monetary policy, the optimal policy mix should be designed such that these interactions are taken into account. Thirdly, given that the number of authorities is larger than two, incentives to deviate from an overall Pareto optimal policy might also come from incentives of some authorities to collude at the expense of others. Fourthly, the usual argument for delegation of monetary policy (based on dynamic inconsistency) may be misfocused in such a setup, but delegation might still act as a solution, though to a different problem. These issues give rise to a debate in the literature related to whether fiscal constraints (of the type imposed by the Stability and Growth Pact

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the EMU) are necessary or not. We argue that given the current setup fiscal rules as well as an independent central bank are unequivocally necessary. Moreover, we also argue that they are not sufficient unless they are designed optimally.

This paper proposes an optimal policy design mechanism that deals with these issues. Delegation by inflation targeting for the common central bank and linear excessive deficit contracts for the fiscal authorities (for example) by one principal of all policies helps achieving an optimal policy mix (defined below). It does so without requiring any binding agreements (neither policy cooperation nor commitment with respect to private sectors) and preserving independence of the policy authorities. It is important to note that the need for delegation in this model comes rather from the impact of fiscal expansion on the other governments’ welfares and on the common inflation rate as well as of the monetary decisions on the fiscal authorities, rather than duly from time inconsistency of policies. Notably, the marginal penalties or targets we propose are state-independent, which increases their credibility and enforces implementation. Another contribution of this paper is imposed by the number of players being larger than two. The literature concerned with monetary unions usually does not consider coalition formation; nor do we, as we suppose binding agreements are unfeasible. Still, preserving the non-cooperative setup, we argue that the appropriate equilibrium concept to be used in this game is the Coalition Proof Nash Equilibrium (Bernheim et al, 1987). We show that our ‘Nash’ equilibrium is coalition-proof (in the sense that it is immune to incentives of authorities to deviate as coalitions). This makes the delegated equilibrium be coalition proof as well, which is another desirable feature for enforcement.

The literature on optimal monetary policy design and central bank independence (analyzed, i.a. by Barro and Gordon 1983, Rogoff 1985, Persson and Tabellini 1993, Walsh 1995, Svensson 1997) usually ignores fiscal policy and regards time inconsistency as the only incentive to deviate from optimality. This happens as the natural rate of output (employment) is too low due to distortions in the goods or labour market and, absent a non-distortory tax (subsidy) the authority is tempted to cheat the private sector by creating surprise inflation. Delegation by inflation contracts or targets is seen as a solution to this type of problem. However, should the use a non-distortory fiscal instrument be available, the problem would be solved without the need for delegation or binding commitments, which hints to the importance of modelling fiscal and monetary policy together. The first attempt to do this is Alesina and Tabellini (1987), where it is shown how commitment of monetary policy can even harm if monetary and fiscal policies are not coordinated.

Dixit and Lambertini (2001a) analyse a model with monopolistic competition and a monetary and fiscal authority, as well as a private sector. They focus on time inconsistency of both monetary and fiscal policy, arguing that ‘fiscal discretion destroys monetary commitment’ and that constraints on the fiscal authority are needed to improve on the inefficient Nash equilibrium.

A different argument for the non-exogeneity of fiscal policy to monetary policy is provided by Sargent and Wallace (1981). There, an open market operation

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1The normative importance of this research in the focus of the ECB’s status on price stability and independence (aside ambiguities surveyed, e.g. in Svensson 2000) is obvious.

2Inefficient of coalitions of a subset of players is a recurring result in policy games: Rogoff (1985b), Oudiz and Sachs (1985) and Canzoneri and Henderson (1991) provide examples in international policy frameworks.
raising debt (and resulting in higher deficit) and reducing money supply results in a higher steady-state inflation and in an at best temporarily lower price level. This is sometimes labeled as a 'fiscal theory of inflation' as opposed to 'The Fiscal Theory of the Price Level' (FTPL) of e.g. Woodford 1995 or Sims 1994, which has stronger implications. For example, in order to determine the equilibrium price level, the money supply should not necessarily be exogenous. This theory can lead to the conclusion that fiscal policy is not merely but exclusively important in determining the price level.

When there are many fiscal authorities, the problem is naturally more complicated. Some studies (e.g. Canzoneri and Diba 1991, Chari and Kehoe 1997) focus on interactions between the common central bank and national governments related to the incentive of the latter to increase debt in order to acquire seigniorage from the former. Bergin (1997) argues that studies based on seigniorage might be misfocused, as the inflation tax on nominal bonds is large (if debts are large) even if seigniorage is small (an usual FTPL argument). He also presents arguments for fiscal rules in a monetary union, showing how a rise in debt in one country (unbacked by future taxes by one government) raises the price level through the monetary union, although he emphasises that fiscal solvency of all governments is not strictly necessary for a stable price level. A number of studies address the problem at the European Union directly. Beetsma and Uhlig (1999) show how the incentive for excessive debt accumulation is exacerbated in a monetary union, and hence an individual increase in debt spills over to the other participants through a common monetary policy. They propose a penalty for debt (related to the Stability and Growth Pact - SGP) in order to internalize this debt externality. Beetsma and Bovenberg (1999, 2001a) show that a monetary union without a fiscal union might be optimal as fiscal collusion might in fact weaken the strategic position of the common central bank. Dixit and Lambertini (2001b,c) show, in a model with fiscal spillovers on both the monetary authority and the other fiscal authorities, how discretionery policymaking is suboptimal, fiscal discretion destroys monetary commitment and fiscal restrictions modelled as constraints are needed to improve the discretionary equilibrium. The two policies are 'symbiotic' if they share the same (and socially optimal) loss function, that is they reach the social optimum without the need to commit and independent of the sequence of moves.

This paper extends the analysis in Bilbiie (2001) concerning the one-country case. There, it is argued for delegation through an inflation target (or contract) to the Central Bank and a contract/target on either deficit or output for the fiscal authority as a mean to achieving a desired equilibrium. Moreover, this delegation should be done by the same principal for implementation reasons (delegation by two principals suffers from an incentive compatibility problem: there is nothing to ensure that the two principals would choose the optimal contracts at the delegation stage, which we have shown in Bilbiie 2000). We did this in a model where fiscal policy influences both inflation and output, fiscal and monetary authorities both care about inflation and output deviations and the private sector forms rational expectations over inflation.

Beetsma and Bovenberg (2001b) make a proposal similar to ours (though in a different model based again on excessive accumulation of public debt) in favour of ceilings on public debt and debt targets as well as an inflation target for the ECB. Casella (2001) makes a proposal based on tradable deficit permits, similar in spirit
to ours in that it draws on microeconomic solutions to externalities/provision of public goods problems.

Before proceeding to the analysis, a brief review of the institutional situation in Europe is in order to provide further motivation for this paper. The monetary policy situation is reasonably clear: the ECB has been delegated with an arguably clear mandate to maintain price stability, abstracting from ambiguities regarding e.g. definitions of this, transparency and decision on exchange rate policy. Design of fiscal policies, reflecting fear of fiscal threats to price stability reviewed before, is also supposed to obey a set of rules. While the Maastricht Treaty formulated restrictions on public debt and deficits preceding accession, the Amsterdam Treaty (imposing the Stability and Growth Pact SGP) goes beyond and specifies penalties for excessive deficits, while national fiscal authorities are required to have a medium-term budgetary objective close to balance or in surplus and support stability-oriented monetary policies. The European Council of Economics and Finance Ministers (ECOFIN) can ask for a non-interest bearing deposit once it considers a country has an excessive deficit and does not take the measures to correct it. The deposit becomes a fine if the deficit persists, and the fine is increasing with the deficit. Sanctions are also waived under some special circumstances (exceptionally bad shock, fall of more than two percent in the GDP, etc.). A comparison between the existing reviewed theory and the policymaking situation in Europe immediately hints to a fertile area of research dealing with designing policy institutions in the EMU, and this is what this paper is about.

We will proceed as follows: Section 2 presents the model, in Section 3 we look at the centralised policy case and define the first best equilibrium, Section 4 derives the main results in the decentralized case solving for the Coalition Proof Nash Equilibrium and the optimal delegation parameters and Section 5 concludes and analyses some policy implications. Appendices contain proofs of the relevant results.

2. The model

We set up a linear-quadratic model in order to analyse the issues mentioned in the introduction. The reduced form equations are a modified version of Dixit and Lambertini (2001b). It resembles the Barro-Gordon (1983) model having added $n$ fiscal authorities and differs from Bilbiie (2001) in that it allows for many fiscal authorities and spillovers among them. There is a common Central Bank setting its instrument $m$ and one fiscal authority for each country $i = 1, ..., n$ setting its instrument $f_i$ at each time $t$. All policies influence macroeconomic outcomes, which are judged as a level of inflation $\pi_t$ and output $y_t$ for each country. The economy is described by the following reduced forms holding at every $t$, where bold letters denote vectors or matrices:

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3Svensson (1999) provides a discussion of these ambiguities and possible threats to the main objective.

4More details are provided by, i.a. Artis and Winkler (1997).

5Dixit and Lambertini (2001a) present a microfounded version of this model in the one-country case.
\[(2.1) \quad y_{it} = \sum_{i=1}^{n} a_{ij} f_{jt} + b_i (\pi_t - \pi_t^*) - \varepsilon_{it}, \quad \forall i = 1, \ldots, n \]

or, stacked in vector-matrix form

\[\mathbf{y}_t = \mathbf{A} f_t + \mathbf{b} (\mathbf{\pi}_t - \mathbf{\pi}_t^*) - \mathbf{\varepsilon}_t, \quad (1')\]

\[(2.2) \quad \pi_t = m_t + \sum_{j=1}^{n} c_{ij} f_{jt} + \nu_t = m_t + \mathbf{c}^t \mathbf{f}_t + \nu_t\]

Equation (1) is an aggregate supply function for each country, similar to an expectation-augmented Phillips curve where the natural rate of output $y_t$ is normalised to 0 for convenience in each country ($b_i$ representing the effect of surprise inflation on output and $\varepsilon_t$ an adverse supply shock). However, it differs from a Lucas supply since fiscal policy is allowed to influence output. Here, an increase in $f_t$ means a more expansionary fiscal policy, hence one can think of $f_t$ as the budget deficit $^6$(or a subsidy in Dixit and Lambertini 2001a). At a domestic level, it does so directly, the effect being captured by the coefficient $a_{ii}$, and indirectly through the common inflation rate (with coefficient $c_i$). Note that fiscal policy does not need ‘surprises’ to stimulate output. At the union level, fiscal cross-country spillovers are captured again through two terms: (i) direct impact $a_{ij}, i \neq j$ and (ii) indirectly through the common inflation rate (with elasticity $c_{ij}, j \neq i$). Hence, equation (2) captures the influence of the policy authorities on the common inflation rate (which, as explained before, leads to indirect cross-country externalities), and $\nu$ can be thought of as an aggregate demand shock. Monetary policy influences inflation rate, a higher $m$ meaning a more expansionary monetary policy (thus an increase in base money or a decrease in interest rates). All shocks are iid with mean zero and constant finite variance and let $\omega = (\varepsilon, \nu)$ be the vector of all shocks. There is a private sector forming rational expectations at $t - 1$ about policy instruments at $t$ over the distribution of shocks $\Phi(\omega)^7$:

\[(2.3) \quad m_t^\omega = E_{t-1} [m_t (\omega)] \equiv \int m_t (\omega) d\Phi(\omega)\]

\[f_t^\omega = E_{t-1} [f_t (\omega)] \equiv \int f_t (\omega) d\Phi(\omega)\]

Aside from the other (by now mainstream) modelling choices, one might ask why should fiscal policy be specified as such? We argue that this embeds a few arguments made in the literature that we reviewed before. Specifically, in equation (1), the impact of one country’s policy on its own output reflects the attempt of the government to increase output (employment) above the natural rate, as the latter

\(^6\)As fiscal restraints in Europe concern the overall fiscal stance we do not make the finer distinction between manipulating tax revenues as opposed to government spending (dividing these further in investment and consumption), though this distinction is important empirically (see e.g. Blanchard and Perotti, 1999).

\(^7\)This way to specify expectations is non-standard but this way we do not require the private sector to ‘know’ the equation determining inflation (4). However, note that once expectations are formed by these $n+1$ rules, expected inflation is also determined like $\pi_t^\omega = m_t^\omega + \mathbf{c}^t \mathbf{f}_t^\omega$.\]
is inefficiently low due to frictions in the labour/goods markets\footnote{Examples of such distortions include: a monopolistic trade union, monopolistic competition in the goods market, ‘efficiency wage’ considerations, etc.} (however, $a_{ij}$ can be negative allowing for crowding out). Cross-country fiscal spillovers (direct or indirect through the common inflation rate) are often argued to come from the following sources. First, if one country goes into budgetary problems, in order to avoid a debt run a debt bailout might arise, either fiscal (by the other countries) or monetary (by the common central bank) if it appears ex-post optimal although in EMU it is ex-ante forbidden. Second, in a monetary union the substitutability of public debts for investors increases, which implies that ceteris paribus, if a country increases its debt, investors will hold the other country’s debt if it is compensated by an increase in the interest rate. Another argument is that given fixed exchange rate in the monetary union, fiscal expansion puts upward pressure on prices as its effect on real variables (e.g. stimulate employment which might be tempting before elections) cannot be offset by exchange rate variations, leading the Common Central Bank to tighten its policy affecting all countries in the union. The same Mundell-Flemming type argument hints to more incentives to use fiscal policy as an aggregate demand management tool given that monetary union enhances trade integration (as this raises demand externalities). Other arguments could be similar to the ones listed in the Introduction, based on seigniorage, excessive debt accumulation, etc.\footnote{Note that the fiscal impact on inflation reflects the Sargent-Wallace unpleasant monetarist argument or even the FTPL arguments reviewed before.} As the sign of these spillovers is subject to debate (see Beetsma 2000 for more details on this debate) we do not restrict the signs of the coefficients. Note that many of the arguments for spillovers are based on debt, which although we do not model explicitly here can be thought of as something beyond the model with a certain impact on the reduced forms. However, the normative implications we study here would not be affected should debt be explicitly modelled.

3. Optimal and time consistent centralised policy

Here we study an ideal and far from realistic situation in order to use it as a benchmark for further analysis. We suppose the existence of a benevolent union-wide policymaker choosing all policy instruments (monetary and all fiscal) to minimize a particular loss function. As we look at normative implications, we specify a union social welfare function (i.e. aggregation of preferences of union members’ societies) shared by a hypothetical union-wide policymaker over inflation and output deviations\footnote{Though directly postulated here, it can be derived in a more rigorous way from the individual indirect utility function as in e.g. Woodford 1999.} in the form of the period loss function:

\begin{equation}
L_t^* = \frac{1}{2} \left[ (y_t - y^*)' \Lambda^* (y_t - y^*) + (\pi_t - \pi^*)^2 \right]
\end{equation}

Hence, the loss function will be $W = E_0 \left[ \sum_{t=0}^{T} \delta^t L_t^* \right]$, where $\delta \in (0,1)$ is the discount factor.

Union’s societies care about inflation deviations from a ‘union socially optimal’ level (which is non-zero) and output deviations ($y$ is the column vector of output levels) from the socially desirable level in each country (stacked in the column vector
We assume that, due to distortions in the real economy described above, 
\( y^* > \bar{y}, \bar{y} = 0 \) that is the desirable output is larger than the full-employment one. This usually gives rise to a discretionary inflation bias in the monetary policy-only literature (see e.g. Barro-Gordon 1983, Svensson 1997). \( \Lambda^* \) is the square diagonal matrix of dimension \( n \) consisting of weights attached to output stabilization for each country, 
\[ \Lambda^* = \{ (\lambda_{ij})_{i,j=1,...,n} \mid \lambda_{ij} = 0 \text{ for } i \neq j, \lambda_{ij} = \lambda^*_i \text{ if } i = j \} \]. Suppose further that the fiscal instruments available to the union policymaker are nondistortionary, or there are no deadweight losses of fiscal policy. Then, the centralised policy authority can implement the first best \((y^*, \pi^*)\) in all \( n + 1 \) dimensions, without any need to commit (or if it commits, it is perfectly credible as it has no incentive to cheat when the game is repeated). Intuitively, this happens as there are \( n + 1 \) instruments to achieve \( n + 1 \) targets, so the system can be controlled perfectly (stabilization of shocks is also optimal). We emphasize this in Lemma 1, where the within-stage time sequence is: (i) targets \( \theta^* \equiv (y^*, \pi^*) \) are observed; (ii) private sectors forms inflation expectations according to (3); (iii) shocks \( \omega_t \) are realized; (iv) policy instruments \((f_t, m_t)\) are chosen; (v) macroeconomic outcomes \((y_t, \pi_t)\) are realized.

**Lemma 1.** There exists a unique discretionary equilibrium of the stage game equivalent to the commitment equilibrium where \((y_t, \pi_t) = (y^*, \pi^*)\). Hence, the optimum is implemented by the centralised policymaker choosing \((f_t(\omega), m_t(\omega))\) through the optimal policy mix \((f^*_t(\omega), m^*_t(\omega))\) with:

\[
\begin{align*}
    f^*_t(\omega) &= \Lambda^{-1}y^* + A^{-1}e_t \\
    m^*_t(\omega) &= \pi^* - c'A^{-1}y^* - c'A^{-1}e_t - v_t
\end{align*}
\]

provided \( A \) is non-singular.

**Proof.** Please find Appendix D.1

Note for further use the \( n + 1 \) first order conditions after accounting for expected variables for each instrument:

\[
\begin{align*}
    m : b'\Lambda^*(y_t - y^*) + (\pi_t - \pi^*) &= 0 \\
    f : (A^* + cb')A^*(y_t - y^*) + c(\pi_t - \pi^*) &= 0
\end{align*}
\]

The futility of commitment to policy rules comes from the existence of fiscal instruments usable to address the real distortions (e.g. a production subsidy or a subsidy for firms to reach the optimal level of employment if wages are too high, financed by a nondistortionary tax). In the presence of public debt, this would mean that debt policy is designed optimally such that no benefit of surprise inflation arises. This solves time inconsistency of monetary policy making the unique

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11As social preferences are a function of employment rather, one could think of treating thist as the relevant variable but if a linear relationship between employment and output exists the two specifications are equivalent.

12Note that in practice, however, inflation rates may differ across union members in the short run. This could lead to a change in the result in Lemma 1, i.e. commitment might matter. Albeit an important matter, this is not the focus of this paper, where we are more concerned with monetary-fiscal interactions and fiscal spillovers.
(dynamically consistent and hence subgame perfect) discretionary equilibrium optimal. Hence, note that here there is no argument for delegation of either policy. Stabilization of velocity (aggregate demand) shocks is pursued through monetary policy and stabilization of supply shocks is shared by all authorities with weights depending on the spillover coefficients.

The centralised equilibrium is optimal, but is also picturing a far from real situation, hence we will just regard it as a benchmark. In Bilbie (2001) we presented other arguments for decentralization, related to current institutional situation (i.e. taking decentralization as a constraint for modelling), distortionariness of fiscal instruments, political economy reasons related to the union policymaker being able to implement any desired equilibrium, etc. In the monetary union case, these arguments are enforced by the mere presence of many fiscal authorities with different visions about redistribution, provision of public goods and facing non-synchronized business cycles. Hence, next we study a more realistic situation with decentralization which preserves independence of policy authorities.

4. Decentralized policymaking

Suppose the $n + 1$ policy authorities do not share the social loss function and are independent and decentralized. Each fiscal authority will care about output in its own country and about the common inflation rate, and the monetary authority cares about inflation deviations and output deviations in each country. Suppose all these targets and policy preference parameters are heterogenous cross-country. Let the period loss functions be given by (where M superscript denotes ‘monetary’ and F ‘fiscal’):

\begin{align*}
L^M_t &= \frac{1}{2} \left[ (y_t - y^M_t)' \Lambda^M (y_t - y^M_t) + (\pi_t - \pi^M_t)^2 \right] \\
L^F_{it} &= \frac{1}{2} \left[ \lambda^F_i (y_{it} - y^F_{it})^2 + (\pi_t - \pi^F_t)^2 \right], \forall i = 1, ..., n
\end{align*}

More specifically, suppose the fiscal instruments available to the fiscal authorities are distortionary or generate deadweight losses. We model this by assuming $y^F_{it}$ is stochastic reflecting systematic biases and that $0 = \bar{y}_i \leq E(y^F_{it}) < y^*_i$ for all $i$.\footnote{The fiscal output target can be equal to the natural rate if deadweight losses are as large as to eliminate welfare gains from increasing output above the full-employment one. Note that we assume targets being stochastic in order to justify non-explicitly modelled distortions with impact on the policymakers’ preferences. This assumption does not change the results in any way.}

This will however mean that the fiscal authorities have incentives to put pressure on the common central bank to target a higher than natural rate or to stimulate output otherwise. We model this by assuming that $y^M$ is stochastic, reflecting for example such ‘political shocks’. Assume that inflation preferred levels $\pi^M_t, \pi^F_t$ are also stochastic, reflecting other incentives of the policymakers. The multi-period loss function will have the same functional form as $W$.

Some motivation for introducing all these relaxations (decentralization, heterogeneity of preferences and distortionariness of fiscal instruments) at the same time is in order. First of all, decentralization of fiscal policies with homogeneity of targets (preferences) is trivially the same as centralization and hence optimal. Furthermore, having done both these relaxations but keeping fiscal instruments non-distortionary would mean that fiscal policy can again be used to address the inflation bias at no
welfare costs (hence the inflation bias will be eliminated). Note also that distortionarity of fiscal instruments is also argument for heterogeneity of output targets and their being different from the union-wide optimum. Hence, we do all three relaxations at the same time.

Heterogeneity of preferences and existence of spillovers make individual policymaking inefficient when compared to the centralised case (that Nash equilibria are inefficient in the presence of externalities is a well-established argument). One possible improvement would be to consider that binding agreements are possible and that authorities decide to cooperate (and commit with respect to the private sector if needed). However, first of all this is not very different from the 'centralised policy' case. Secondly, this policy could only achieve a second-best in the form of a weighted average of the bliss points of policymakers and will again be time-consistent (no need for commitment). Thirdly, once we argued for decentralization, we want to look at the game among authorities in a purely non-cooperative setup\textsuperscript{14}.

4.1. The Coalition Proof Nash Equilibrium. Regarding the game as non-cooperative, i.e. assuming the impossibility of binding agreements conforms our priors about the current institutional situation in Europe. Binding agreements would mean that some policymakers (fiscal or monetary and fiscal) get together before the game is played and sign contracts specifying policies they would follow. This means much more than informal discussions or discretionary adjustments in response to exceptional shocks, it means systematic joint optimization of their loss functions. We argue that this is not realistic in the European setup (e.g., even if a EU Commission member participates in the meetings of the ECB board and the ECB Governor in the ECOFIN, they do not have voting power). Moreover, it is not generally true if independence of authorities is to be desirable and each authority tries to build credibility. Note that this assumption still leaves space for informal discussions or exchange of information (of the sort the Ecogroup is supposed to achieve).

Given the loss functions described in (6) and the structure of the economy described in (1), (2) and (3), we consider the equilibrium of the corresponding policy game. The timing of the stage game is: (i) targets $\theta \equiv (y^F, \pi^F, y^M, \pi^M) \equiv (\theta^F, \theta^M)$ are revealed; (ii) expectations $m_t^e$ and $f_t^e$ are formed; (iii) shocks $\omega_t$ hit the economy; (iv) authorities choose their respective instruments $m_t$, $f_t$ and they do so simultaneously (if not specified otherwise); (v) equilibrium outcomes $(y_t(\theta, \omega), \pi_t(\theta, \omega))$ are realized. Note that realized targets enter in the information set at $t-1$ when expectations are formed.

Given that there are $n+1$ policy authorities (abstracting from the private sector and nature) the appropriate equilibrium concept to use is not Nash Equilibrium, but Coalition Proof Nash Equilibrium, an equilibrium concept proposed by Bernheim, Whinston and Peleg (1987). The intuition behind this concept makes it appealing for this problem. In multi-player games, in the absence of binding agreements, and allowing for exchange of information one should check not only for incentives of players to deviate individually (which is covered by the Nash Equilibrium) but also for incentives of players to deviate as coalitions, to which the equilibrium

\textsuperscript{14}This is analysed in detail in Bilbiie (2001) for the one-country case and results for a monetary union are available at request.
should be stable (which is not insured by the Nash equilibrium). The corresponding equilibrium is described in Proposition 2.

Proposition 1. There exists an unique Coalition Proof Nash Equilibrium (CPNE) of the decentralized policy game given by:

\[
\begin{align*}
\ell_t^{NE}(\theta, \omega) & = A^{-1}y_t^{NE} + A^{-1} \varepsilon_t \\
\pi_t^{NE}(\theta, \omega) & = \pi_t^{NE} - c' A^{-1} y_t^{NE} - c' A^{-1} \varepsilon_t - v_t
\end{align*}
\]

with the macroeconomic outcomes given by:

\[
\begin{align*}
y_t^{NE}(\theta) & = y^F + \Gamma^{-1} \pi^F - \Gamma^{-1} u \left[ b' \Lambda^M (y^M - y^F) + \pi^M - b' \Lambda^M \Gamma^{-1} \pi^F \right] \\
\pi_t^{NE}(\theta) & = \frac{1}{1 - b' \Lambda^M \Gamma^{-1} u} \left[ b' \Lambda^M (y^M - y^F) + \pi^M - b' \Lambda^M \Gamma^{-1} \pi^F \right]
\end{align*}
\]

where \( u \) is the column unit vector \( u_i = 1 \) for all \( i \) and \( \Gamma \) is the \( n \times n \) diagonal matrix with entries \( \gamma_i = \left( \frac{\omega_i}{\omega} + b_i \right) \lambda^F \), if \( c_i \neq 0 \) and \( \Gamma \) is invertible (i.e. \( \prod_{i=1}^{n} \gamma_i \neq 0 \)).

Proof. Please find Appendix D.2.

When compared to the centralised optimum, the CPNE is obviously inefficient. Making targets of all authorities equal to society’s targets would unsurprisingly ensure implementation of the first best, but this would again be very close to ‘centralised’. The source of inefficiency of the coalition proof Nash equilibrium is, however, coming from disagreement on target levels and spillovers among policymakers rather than merely from credibility problems; i.e., it is still due to the impossibility of binding agreements, but with respect to the other policymakers and not to the private sector. Once binding agreements are possible, a cooperative equilibrium where all fiscal and monetary policymakers minimize a joint loss function (weighted average of all losses in (6)) would achieve a second best\(^{15}\) with no need for commitment. Note that shock stabilization is still ‘optimal’, i.e. the elasticities of instruments with respect to shocks is the same as in the centralised optimum case. The economy is optimally stabilized, but stabilized around sub-optimal macroeconomic outcomes. The source of inefficiency of outcomes can be seen by looking at the first order conditions of the authorities, with the notation explained before:

\[
\begin{align*}
(4.2) \quad b' \Lambda^M (y_t - y^M) + (\pi_t - \pi^M) & = 0 \\
\Gamma (y_t - y^F) + \pi_t u - \pi^F & = 0
\end{align*}
\]

For the monetary authority, all inefficiency comes from the difference between its loss function and society’s (and hence indirectly ignoring the impact on the fiscal authority too). For the fiscal authorities instead, it comes from ignoring the effects of its spillovers on the other fiscal authorities. Note that \( \Gamma \) is diagonal, whereas \( A' + cb' \) (arising in the centralised first order conditions) is not. A comparison of (5) and (7) makes this point clear. This can be more easily seen by looking at

\(^{15}\) A second best equilibrium here would mean a weighted average of bliss points in (6) and would coincide with the first best only incidentally. This case is covered extensively in Bilbiie (2001).
any individual $i$’s fiscal first order condition for each case (centralised, respectively decentralized and non-cooperative):

$$
(a_{ii} + b_i c_i) \lambda_t^* (y_{it} - y_t^*) + \sum_{j=1,j \neq i}^n (a_{ji} + b_j c_j) \lambda_i^*(y_{it} - \gamma_i^*) + c_i (\pi_t - \pi_t^*) = 0
$$

$$
(a_{ii} + b_i c_i) \lambda_t^F (y_{it} - y_t^F) + c_i (\pi_t - \pi_t^F) = 0
$$

Apart from heterogeneity of targets and preference parameters, inefficiency appears due to fiscal authorities ignoring effects of their actions of the other fiscal players (second term in first equation).

The bad (not surprising) news is that the Nash Equilibrium is inefficient. The good (maybe surprising) news is that it is coalition proof\footnote{And Perfectly Coalition Proof (see Appendix B) in the repeated game.}, i.e. it is insensitive to incentives of players to deviate by forming credible coalitions, not only to individual incentives. This means that allowing exchange of information among fiscal authorities and the common central bank (which conforms our view about the current arrangement in Europe) is harmless. The importance of coalition proofiness increases when we consider solutions to inefficiency of the Nash Equilibrium.

5. Delegation as a coordination device

It is a usual result in policy games or in any games in the presence of externalities that the Nash Equilibrium is inefficient when binding agreements are not possible. It is also by now mainstream to look at mechanisms that insure implementation of a ‘better’ or even optimal equilibrium in the non-cooperative game. One stream of literature looks at reputation or trigger mechanisms when the game is repeated—we abstract from these arguments here. Another looks at institutional design, i.e. delegation as a solution to inefficiencies and our approach could be roughly judged as belonging here\footnote{For repeated games mechanisms, see e.g. Canzoneri and Henderson (1991) and Ghosh and Masson (1994). For institutional design see Persson and Tabellini (1995).}, although to the best of our knowledge this specific problem has not been studied with these tools up to now\footnote{Beetsma and Bovenberg (2001) in contemporaneous and unrelated work make a similar argument for debt targets.}\footnote{Delegation to Rogoff-conservative authorities is not considered here as it could not lead to implementation of the first best but would at best achieve an intermediary equilibrium, without influencing schock stabilisation, however (as it usually happens in the credibility literature - see Lockwood et al 1995).}. We are looking for those institutional mechanisms that would insure that the centralised equilibrium is implemented when policymaking is decentralized and non-cooperative (and discretionary, for that matter). In order to preserve intuition and clarity we deal with implementation of the centralised first best, but this is only for expository purposes. This has straightforward implications for the much-debated design of monetary and fiscal regimes in Europe, implications we will make explicit later. Here, as the ultimate goal is policy-relevance, we focus on monetary policy being delegated by an inflation target and fiscal policy by excessive deficit penalties (linear contracts)\footnote{Delegation to Rogoff-conservative authorities is not considered here as it could not lead to implementation of the first best but would at best achieve an intermediary equilibrium, without influencing schock stabilisation, however (as it usually happens in the credibility literature - see Lockwood et al 1995).}. We provide a more detailed treatment of alternative delegation schemes in Bilbiie (2001) for the one-country case, for contracts, targets, implementation of different equilibria or for different timings of the game. The
main result is stated here in Proposition 3, where the timing of the stage game is the same as before, however an additional ‘delegation stage’ (say stage (0)) appears.

**Proposition 2.** The first best centralised equilibrium in Lemma 1 can be implemented in the decentralized non-cooperative policy game in a coalition-proof manner and with discretionary policymaking. This is done by a principal delegating all policies to instrument-independent authorities at stage (0), before (i), i.e. modifying their loss functions in the form: inflation target \( \pi \) for the common central bank and deficit contracts \( T_{it}(f_{it}, f) \) for each fiscal authority \( i \)

\[
L_{it}^{MD}(\cdot, \pi) = \frac{1}{2} \left( (y_t - y^M)' \Lambda^M (y_t - y^M) + (\pi_t - \pi)^2 \right)
\]

\[
L_{it}^{FD}(\cdot, T_{it}(\cdot)) = \frac{1}{2} \left( \lambda^F (y_{it} - y^F)^2 + (\pi_t - \pi^F)^2 \right) + T_{it}(f_{it}, f), \forall i = 1, \ldots, n
\]

Supposing further that \( T_{it}(f_{it}, f) \) is linear\(^{20}\) of the form \( T_{it}(f_{it}, f) = \pi_t \times (f_{it} - f) \), where \( \pi_t \) is the marginal penalty/reward, the unique state-independent target and contracts implementing the centralised optimum are:

\[
\hat{\pi}(\theta, \theta^*) = \pi^* + b' \Lambda^M (y^* - y^M)
\]

\[
\tau_i(\theta, \theta^*) = (a_{it} + b_i c_i) \lambda^F (y^F_i - y^F_i) + c_i (\pi^F_i - \pi^F_i), \forall i = 1, \ldots, n
\]

where \( \theta \) and \( \theta^* \) are as before short-hand notations for the target vectors and \( f \) is left at the discretion of the principal.

**Proof.** Please find Appendix D.3

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\( ^{20} \)Technically, we need linearity of contracts to insure uniqueness and state-independence. However, we will argue later that linear contracts present also practical advantages in terms of implementability.
targets $y^*$, the optimal inflation target would be instead $\pi^* - b^\prime \Lambda^M y^*$ to eliminate the inflation bias arising from discretionarity ($b^\prime \Lambda^M y^*$). Such a bias does not arise here, so the target is just $\pi^*$.

The excessive deficit marginal contract is also a function of targets but only of the respective country’s targets and parameters. First, ignoring the inflation term, if we consider that fiscal policy has an overall positive effect on output ($a_{i,t} + b_t c_t > 0$) and it faces deadweight losses ($y_t^F < y_t^F$) the contract is in fact a reward ($\tau_t < 0$): its role would just be to compensate deadweight losses that restrain fiscal expansion. This happens when fiscal policy has no impact on inflation. When we look at the second term, however, we see that it could compensate the first if the impact of fiscal policy on inflation is positive enough and the fiscal authority has a too high inflation target. In this case the fiscal welfare cost of deadweight losses dominates that of increasing inflation, so there is a structural deficit bias compared to the centralised optimum. Other relationship among targets and structural coefficients would imply different interpretations. Note that the reference value $f_i$ remains at discretion of the principal - a natural choice would be to set it equal to $f_i^*$, the optimal deficit in each country.

One condition we impose for this delegation scheme to work is that the same principal delegates all policies (at the European level this could be the European Parliament, or more specifically the Monetary and Economic Affairs Committee of the EP). This is not usually recognized in the related multiple-policymakers literature (see, e.g. Persson and Tabellini 1995) but we argue it is essential for implementation based on the results in Bilbiie (2000). There is shown that absent this restriction (i.e. when having one principal for each policymaker) principals would not have the right incentives in order to delegate with the ‘optimal’ parameters (optimal contracts would not be subgame perfect in the delegation game) and hence the coordination problem is just relocated one stage earlier (at the delegation stage where principals play and choose delegation parameters).

We argued in the previous section that coalition proofness of the Nash Equilibrium was ‘good news’. In light of these results, we argue it is ‘even better news’. It insures that once delegation has taken place to make sure the first best is achieved, policymakers would not have incentives to credibly deviate as coalitions either. Note that if the original equilibrium were not coalition-proof, one could think about a delegation scheme that insured elimination of ‘collusion incentives’. Put differently, even if a coalition proof equilibrium did not exist in the original game, appropriately designed delegation could insure existence in the game with delegation e.g. by imposing costs of collusion that would make it suboptimal in the first place. Such costs would be futile here as there are no incentives to collude.

Another important question here is related to the timing of the game: in reality, as fiscal policy is less flexible, one might consider a situation where the fiscal authorities move first, i.e. they act as Stackelberg leaders taking the monetary reaction function as a constraint. A Stackelberg equilibrium would of course be inefficient (although part of externalities are internalized by leadership) and a next step is to see how to design optimal delegation such that the first best is achieved in the Stackelberg game. This is potentially important as not recognizing timing might lead to the ‘wrong’ delegation parameters. The authority that moves first would have an incentive to ‘cheat’ given that it has a first-mover advantage and observes the penalty of the follower. Subgame perfection would be thus lost, so
contract design should take into account this incentive. We do not present a formal
treatment here as we studied this case at length in the one-country model in Bilbiie
(2001). The intuition is still the same as in the simultaneous-move case, only the
parameters are different.

6. Conclusions and policy implications

A monetary union with spillovers of one fiscal policy on common monetary pol-
icy and the other fiscal authorities raises new issues for policy institutional design.
An example of this is the case of the European Monetary Union, where once these
spillovers have been recognized the necessity to have not only an independent Cen-
tral Bank targeting price stability but also some fiscal constraints seemed obvious.
This lead to the clear mandate of the ECB to target price stability and to fiscal
rules of the form embedded in the Treaty of Maastricht and developed by the adop-
tion of the Stability and Growth Pact. The argument of this paper is that while
these restrictions are indeed necessary they are also far from being sufficient. Once
necessity has been accepted, we argue that next step should deal with optimal
design.

Hence, we propose an institutional design scheme that would achieve the desired
equilibrium in terms of inflation and output (or employment). We start, naturally,
by defining what a first best is in this model, finding that a centralised authority that
(i) chooses all policy instruments; (ii) shares the social loss function and (iii) has at
its disposal non-distortionary taxes can achieve the social optimum with no need
to commit as there is no incentive to cheat the private sector. This is apparently
against the dynamic inconsistency literature but is fundamentally consistent with it
as the use of the non-distortionary fiscal instrument attacks the economic distortion
making the natural rate of output too low at it its source\footnote{This is just an example; an equivalent policy would be designing optimal labour market policies or a public debt policy that does not create benefits of surprise inflation. See Alesina and Tabellini (1987) or Svensson (1997) on this.}. Clearly, restrictions
(i)-(iii) are too strong - this case just pictures an idealistic situation and we provide
arguments for decentralization. Hence, we study a world in which policy has been
decentralized and policy authorities’ loss functions differ from society’s (similarity in
a reduced-dimensional space would just be a particular case, whereas full similarity
would essentially be the centralised case).

Here we show in the absence of binding agreements (i.e. in a non-cooperative
setup) the Nash equilibrium of the decentralized policy game will be coalition proof:
policymakers have no credible collusion incentives, i.e. no incentives to deviate from
the ‘Nash’ policies as coalitions. The equilibrium is, however, also inefficient due
to non-internalization of spillovers between the policy authorities. Preserving the
decentralized non-cooperative setup we look at optimal delegation (institutional
design) that would insure implementation of the social optimum. We find that
an optimal inflation target for the Common Central Bank and optimally designed
linear excessive deficit contracts for each fiscal authority do the job and solve for
the optimal delegation parameters.
An implication of our analysis is that the design of fiscal rules in Europe should be rethought. The scheme we propose specifies linear penalties for excessive deficits (and rewards for surpluses), but marginal penalties are a decision variable of the principal and have to be optimally designed to eliminate the inefficiencies arising from decentralized policymaking. We provide an intuitive interpretation of these marginal penalties. The delegation scheme we propose has some desirable properties. First, both the inflation target and the deficit contracts are state-independent (not contingent upon realization of shocks), which enhances their credibility and makes implementation easier. Secondly, they would implement the centralised optimum in a coalition proof manner - given delegation, the equilibrium is immune to incentives of authorities to collude. That could mean in an European context that informal discussions of the sort the Eurogroup is supposed to encourage would not provide incentives to deviate. If the initial equilibrium were not coalition proof, contracts could still be designed to eliminate collusion incentives. Thirdly, while the linearity of contracts is needed for technical reasons, we argue that it is also desirable for practical purposes as non-linear contracts might be hard both to understand and implement.

Importantly, this delegation scheme also preserves independence of policymakers (which is more important in a monetary union given heterogeneity of fiscal goals). We have in mind for fiscal policy a similar delegation scheme to the usual inflation-targeting regime and would consist of: (i) the fiscal authority would be assigned the appropriate loss function by the principal (the European Parliament or the relevant committee of the EP) specifying the penalties for excessive deficits and the target level of the deficit; (ii) it would be given instrument-independence (as opposed to goal independence) to minimize the loss function; (iii) it will be held accountable by the principal to whom it would have to report periodically. Efficient practical implementation of such a regime (equivalent to the 'inflation forecast targeting' paradigm) still requires a great amount of research into the way fiscal policy impacts on the economy. Moreover, enforcement of the contracts would hardly, if at all, be insured by the penalties themselves. What is more important for its disciplining effect is credibility of the government with respect to other governments, financial markets and its ultimate 'principal', the voting public in a manner similar to that of the functioning of an inflation contract (see Beetsma 2000 for a related argument). Note also that we need an unique principal imposing all the delegation schemes. Leaving delegation responsibilities at each country’s level would result in inefficient delegation as there are no incentives in equilibrium to delegate in an optimal way (the only way to ensure optimal delegation would be cooperation at the delegation stage - this is shown in Bilbiie 2000).

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22Another practical implication is providing support for Svensson’s (1999) arguments in favour of the ECB adopting a clear inflation targeting regime.
23Alternative delegation forms can be considered: deficit targets, output contracts or targets, but we have chosen deficit contracts for practical relevance. These and other cases are considered in Bilbiie (2001) for a closed economy.
24For a critique of state-dependent delegation see e.g. McCallum 1995. For a proposal of state-contingent penalties see Beetsma and Jensen 2000.
25See Bernheim et al (1987) for an illustrative example of coalition proofness whereby players meet in one room and 'discuss' their strategies.
Fiscal constraints in Europe have been widely criticized (see e.g. Buiter et al 1993). One argument is that fiscal spillovers are either not existing or quantitatively unimportant. The contracts we propose, however, do not depend on only fiscal spillovers but also on interaction of monetary and fiscal policy. Secondly, it is argued that fiscal constraints on deficit could be binding when a country is in recession. Beetsma and Jensen (2000) solve this by proposing state-contingent sanctions. However, the actual payment of the fine would take place after some lengthy procedure after the excessive deficit has been observed, involving decision by the Ecofin council. Additionally, sanctions are also waived under special circumstances. Another objection is related to the ‘blunt’ design of such rules concerning the overall fiscal stance and not specifying anything about the means, leaving room for ‘creative accounting’ of governments (either raise taxes instead of cut spending, or cut investment instead of consumption). Some of these issues are taken up in a separate paper (Bilbeie and Stasavage 2003).

Finally, note an important limitation of our paper, namely the absence of debt. Introducing debt would mean introducing a dynamic component in the structural model. This would mean that the model cannot be solved period-by-period but by dynamic programming, as the stage Nash equilibrium and the subgame perfect (time consistent) equilibrium would be no longer equivalent. We have chosen to trade off this enrichment for simplicity, running the additional cost of not being able to model a debt-based contract or target for which there is increasing support. This is an important potential development.

Appendix A. Proof of Lemma 1

Proof. Equivalence of commitment and discretionary equilibria in this model can be shown in more than one way. We choose to solve for the commitment equilibrium and show that the rational expectations constraint (i.e. system of n + 1 constraints) does not bind26. Thus, the centralised authority commits at stage (0), before (i), to state-contingent rules solving:

\[
\min_{\pi_t, \Theta_t} E[\int L_t^*(y_t, \pi_t, \Theta_t)] \text{ s.t. } (1), (2), (3)
\]

The Lagrangian of this problem is, where \(\rho_{t-1}^m, \rho_{t-1}^f\) are the multiplier and respectively n-vector of multipliers of the respective constraints in (3):

\[
\mathcal{L}_t = E[L_t^*] - \rho_{t-1}^m [m_t^e - E_{t-1}(m_t)] - \left(\rho_{t-1}^f\right)' f_t^e
\]

\[
= \int [L_t^* + \rho_{t-1}^m \pi_t + \left(\rho_{t-1}^f\right)' f_t^e | d\Phi(\omega) - \rho_{t-1}^m m_t^e - \left(\rho_{t-1}^f\right)' f_t^e
\]

26Note that since the stage game is identical at all t and there is no persistence in any variable and no intertemporal link, the equilibrium of the stage game is the same as the equilibrium of the repeated game - minimising W boils down to minimising \(L_t^*\) every period. The latter game would have other equilibria (in particular for infinite repetitions), but we abstract from reputation or trigger mechanisms that would insures their implementation.
The first order conditions are, for any realization of shocks $\omega_t$, for each control respectively:

$$
\frac{\partial \mathcal{L}_t^*}{\partial m_{t}} = b^t A^*(y_t - y^*) + (\pi_t - \pi^*) + \rho_{t-1}^m = 0 \tag{A2}
$$

$$
\frac{\partial \mathcal{L}_t^*}{\partial f_{t}} = E_t \{-b^t A^*(y_t - y^*)\} - \rho_{t-1}^f = 0
$$

$$
\nabla_t E_t \mathcal{L}_t^* = (A^t + cb^t) A^*(y_t - y^*) + c(\pi_t - \pi^*) + \rho_{t-1}^f = 0
$$

$$
\nabla_t f_{t} \mathcal{L}_t^* = E_t \{-cb^t A^*(y_t - y^*)\} - \rho_{t-1}^f = 0
$$

where $\nabla_t \mathcal{L}$ is the vector of derivatives of $\mathcal{L}$ with respect to each instrument in the vector $f$. Eliminating the multipliers from first two and last two equations and rearranging results in:

$$
b^t A^*(y_t - Af^e) + \pi_t - \pi^* = 0 \tag{A3}
$$

$$
A^t A^*(y_t - y^*) + cb^t A^*(y_t - Af^e) + c(\pi_t - \pi^*) = 0
$$

Taking expectations at $t-1$ of this system one obtains (provided $A$ is invertible):

$$
\pi^e = \pi^*, y^e = y^*
$$

$$
m^e = \pi^* - c A^{-1} y^*
$$

$$
f^e = A^{-1} y^*
$$

Substituting back in the A2 system we get the optimal policy rules and $(y^*, \pi^*)$ as the equilibrium of the game as formulated in Lemma 1. The second order conditions are trivially satisfied given the elliptic level curves of the loss functions (i.e. loss functions are strictly quasiconcave) and the linear constraints, so the equilibrium is unique-we do not present the formal argument due to space constraints.

To prove that this equilibrium is the same as the discretionary one, one could solve for the latter or, alternatively, prove that the Lagrange multipliers are zero in the commitment equilibrium. We choose the second method. For example, from the first order conditions with respect to $m^e$ and $f^e$ in A1 after substituting for expected variables we obtain:

$$
\rho_{t-1}^m = -b^t A^*(AA^{-1}y^*-y^*) = 0
$$

$$
\rho_{t-1}^f = -cb^t A^*(AA^{-1}y_t-y^*) = 0
$$

This shows that the value of relaxing all (any of) the rational expectations constraints is zero. 

\hfill \Box

Appendix B. Sketch of Proof - Proposition 2

**Sketch of Proof and Intuition.** To look for the Coalition Proof Nash Equilibrium we proceed as follows: (a) we look for the Nash Equilibrium (Equilibria); (b) we prove it is coalition proof (or we look for the one that is coalition proof among the Nash Equilibria).

(a) to solve for the Nash Equilibrium, note that the problem of each authority can be formulated as
\[
\begin{align*}
\min_{M_t} L^M_t, \min_{F_t} L^F_t \quad \text{for all } i, \text{ s.t. (1) (i.e. to each relevant constraint) and (2).}
\end{align*}
\]
However, the authorities no longer internalize the effect of their decisions on expectations (nor on the other policymakers). Also note that looking at the Nash Equilibrium of the stage game is sufficient given that the repeated game will have the same Subgame Perfect (and hence time consistent) equilibrium given that the structure of the stage games is the same. Importantly, we abstract from reputational or trigger mechanisms that would sustain other equilibria as time consistent in the repeated game. The first order conditions are, at every \( t \):
\[
\begin{align*}
\frac{\partial L^M_t}{\partial m_t} &= b^t \Lambda^M_t (y_t - y^M_t) + (\pi_t - \pi^M_t) = 0 \quad \text{(B1)} \\
\frac{\partial L^F_t}{\partial f_{it}} &= (a_{it} + b_i c_i) \lambda^F_t (y_{it} - y^F_t) + c_i (\pi_t - \pi^F_t) = 0, \forall i = 1...n
\end{align*}
\]

We can write the \( n \) fiscal conditions concentrated constructing the diagonal matrix \( \Gamma \) with entries \( \gamma_i \), where
\[
\gamma_i = \left( \frac{a_{ii} + b_i}{\pi_t} \right) \lambda^F_t
\]
and using the \( u \) unit vector like:
\[
\begin{align*}
\Gamma \left( y_t - y^F_t \right) + \pi_t u - \pi^F_t &= 0 \quad \text{(B2)} \\
b^t \Lambda^M_t \left( y_t - y^M_t \right) + (\pi_t - \pi^M_t) &= 0
\end{align*}
\]
Taking expectations of this (using (1) and (2) and the game sequence) results in:
\[
\begin{align*}
\Gamma \left( A^F_t - y^F_t \right) + \pi^*_t u - \pi^F_t &= 0 \quad \text{(B3)} \\
b^t \Lambda^M \left( A^F_t - y^M_t \right) + (\pi^*_t - \pi^M_t) &= 0
\end{align*}
\]
Solving for expected variables and substituting back in B2 using again (1) and (2) the policy instruments are determined as in Proposition 1, as well as the outcomes.

The equilibrium is unique as loss functions are strictly quasi-concave (level curves are ellipses) and the constraints are linear. It is also a minimum as second order conditions are trivially satisfied.

(b) The fact that the Nash Equilibrium is unique considerably simplifies our task of finding Coalition Proofness. Intuitively, an equilibrium is Coalition Proof if it is Pareto Optimal in the set of self-enforcing agreements. Self-enforceability implies that in this equilibrium no coalition can benefit from deviating in a self-enforcing way. The way one checks for this is recursive: one looks at all component games induced on the original game by a sub-coalition playing the conjectured candidate equilibrium. If the equilibrium in these component induced games is the same as the original equilibrium, then we are done since the initial candidate was unique, hence it will be trivially efficient in the set of self-enforcing equilibria.

We argue that in the decentralized policy game described in text the unique Nash Equilibrium we found is Coalition Proof. The intuition is the same as in the Cournot \( n \)-firms game considered by Bernheim and Whinston (1987). Fixing policies (i.e. equilibrium policies) for a subset of players a game is induced upon the rest of players. This component game (for any subset of players) has the same structure as the initial policy game, more specifically a unique Nash equilibrium - hence, for all subsets of players this equilibrium is self-enforcing, there are no incentives for collusion (credible collusion). Trivially, given that it is unique, it will also be efficient in the self-enforcing agreements set. Moreover, in the repeated game
and given the stage subgame is always the same, the equilibrium will be *Perfectly Coalition Proof* (see Bernheim et al 1987 for details)-intuition is the same as for subgame perfectness of the unique Nash Equilibrium. This is enough to make our point - a rigorous proof would require induction but we omit it here.

□

Appendix C. Proof of Proposition 3

PROOF. Suppose delegation took place in the stage game at stage (0), each authority being assigned the delegated loss function presented in text. Consider the decentralized non-cooperative policy game whereby the two authorities solve (with discretion) minm_{i} L^{MD}_{i}, minf_{i} L^{FD}_{i} for all i, s.t. (1) (i.e. to each relevant constraint) and (2) but not to (3) (they do not internalize effects of policies on expectations). The first order conditions are, for the monetary and fiscal authorities, respectively:

\[
\frac{\partial L^{MD}_{i}}{\partial m_{i}} = b^{i} \Lambda^{M} (y_{i} - y^{M}) + (\pi_{i} - \pi) = 0 \\
\frac{\partial L^{FD}_{i}}{\partial f_{i}} = (a_{ii} + b_{i}c_{i}) \lambda^{F} (y_{it} - y_{i}^{F}) + c_{i}(\pi_{i} - \pi^{F}) + \tau_{i} = 0, \forall i = 1...n
\]

These determine the Coalition Proof Nash Equilibrium for this game (where coalition proofness results by the same argument as before). Comparing the system C1 with the first order conditions (5) - at the centralised optimum - one determines the values of \(\pi\) and \(\tau_{i}\) directly as in text using Lemma 1 as the values that lead to elimination of the inefficiency terms in the original Coalition Proof Nash Equilibrium.

□

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