An Evaluation of Forecasting using Leading Indicators

Rebecca A. Emerson and David F. Hendry*
Centre for Economic Forecasting, London Business School,
Nuffield College, Oxford.

December 5, 1994

Abstract

We consider the use of indices of leading indicators in forecasting and macro-economic modelling. The procedures used to select the components and construct the indices are examined since the composition of indicator systems gets altered frequently. Issues of model choice and data-based restrictions are investigated. Possible biases in estimated standard errors are considered, as is cointegration within the indices and between their components and macro-economic variables. The effectiveness of indices ex-post is contrasted with evidence about their ex-ante performance. A framework is proposed for index analysis and applied to selecting indices. The role of leading indicators in macro models is discussed.

Contents

*This research was supported in part by grant R000233447 from the UK Economic and Social Research Council. We are grateful to Mike Clements for helpful comments on an earlier draft.

1 Introduction

There has been a recent revival of interest in using leading indicators of economic activity to forecast a variety of economic time series: see *inter alia* Artis *et al* (1993), Diebold and Rudebusch (1988, 1991a,b), Koch and Rasche (1988), Lahiri and Moore (1991), Samuelson (1987), and Stock and Watson (1989, 1992). This seems to be partly a reaction to recent forecasting failures by macro-econometric systems and partly due to developments in leading-indicator theory. Most work has been done on the US indicator system, perhaps due to the importance placed on leading indicators in the US by their popular press.

An indicator is any variable believed informative about another variable of interest; an index is a weighted average of a set of component indicators. The weights may be fixed as in a Laspreyes index for (say) real gross national product (GNP) or changing as in a Divisia index such as that for retail prices (RPI), and may be based on 'natural' choices such as value shares (like the RPI or FTSE), or ad hoc (as in weighting GNP, inflation and unemployment in an index of 'economic activity').

A leading indicator is any variable whose outcome is known in advance of a related variable that it is desired to forecast; a composite leading index (CLI) is a combination of such variables. For example, the US Commerce Department states that its leading indicator is constructed to lead at cyclical turning points. We will be concerned with the selection of indicators, methods of constructing indices from these (both leading and coincident) and with the theoretical forecasting properties such indicators and indexes may have either in isolation or as a component time series incorporated in another forecasting procedure, such as a vector autoregression (VAR). Our analysis is prompted by the 'real-time' evaluations of the success of leading indicators in forecasting in Diebold and Rudebusch (1991b) and Stock and Watson (1992). These authors show the marked deterioration in practice of post-sample performance relative to the within-sample findings claimed for CLIs. We present an analysis as to why such an outcome might be anticipated in general, and suggest that there are dangers in attempting to include CLIs in other forecasting procedures.

The framework of indicators and indices has two distinct components. In the first, the objective is to construct an index of (say) 'economic activity' or the state of the 'business cycle', based on a variety of components (such as GNP, interest rates, unemployment, inflation etc.) which have no common metric and no natural weights. In the second, the objective is to predict the future state of some measure of the economy, which may be the value of the index constructed at the first stage, but could be GNP. All four possibilities occur (value weighted and ad hoc indices, used to predict or be predicted by themselves or the other type). A 'business cycle' is usually deemed to be a concerted movement of many series over time.

The Harvard A-B-C curves were the earliest construction meant to serve as a prediction system (see Persons, 1924), but fell into disrepute after claims that they failed to predict the 1929 crash and subsequent depression (see Samuelson, 1987). Indicators that occurred in early work included the output of pig-iron and ton-miles of rail shipments, neither of which would be selected today. The dangers in using the largest ex post lagged correlations to forecast are well illustrated by Coen, Gomme and Kendall (1969). The current system of business cycle indicators began with the seminal work of Mitchell and Burns (1938) and Burns and Mitchell (1946). Koopmans (1947) criticism that Burns and Mitchell's work on business cycles and their indicators lacked a micro-economic theory basis often recurs, as does his complaint that the notion of the reference cycle lacked a secure definition, although Vining (1949) offered a robust defence of their approach in terms of model discovery (see Hendry and Morgan, 1994). Neftci (1979) stresses the analytic intractability of the concept of a leading indicator. In practice, the composition of indicator systems gets altered frequently (about every 2.2 years in the US), suggesting that elements do not lead systematically for prolonged periods.

Since the Koopmans–Vining debate, the discussion of leading indicators has considered the two issues of how well existing indicators perform and whether there are there better ones. Although the current US index of leading indicators is a composite index, diffusion indices and first differences of the reference cycle have been used to predict turning points of the business cycle (see Alexander, 1958, and Steckler, 1961). Auerbach (1982), Koch and Rasche (1988), Neftci (1979,1984), Steckler and Schepsman (1973) and Weller (1979), examine leading indicators and the index of leading indicators as quantitative predictors of turning points. Hymans (1973) and Diebold and Rudebusch (1989) examine the CLI ex ante, the former suggesting a spectral approach and the latter, the use of two indices, for upswings and downturns respectively. Stock and Watson (1989) seek to develop a formal probability model for the indicator system then look for the best variables to use as index components and the best index. Their follow-up in 1992 suggests that ex post, the outcome is less than completely satisfactory, a conclusion supported by the results in Diebold and Rudebusch (1991b).

Specifically, the sample selection of a CLI to be ex post the best linear combination for in-sample fit is not an adequate justification for its use in forecasting, and Diebold and Rudebusch show a substantial deterioration in accuracy if the index actually available is used. Possible explanations also include regular revisions in the compositions of indices, so implicitly the relationship between index and forecast target is not constant, and that the final data in-sample may differ in important ways from provisional numbers at the time forecasts are prepared. Stock and Watson (1992) blame the forecasting errors on the fact that the 1990 recession was not a financial one in the US – but that is precisely the danger of leading indicators. As there is no clear basis except extrapolation for CLIs invariably leading, they may suddenly fail to lead. The recent behaviour of unemployment in the UK leading the recovery may be another example (usually it is a 'lagging indicator'): a structural model would seek to account for such changes when the econometric equations experienced predictive failure. Thus, while many of the problems are in common with those facing econometric-model based forecasts, less formal analysis and fewer tests of index-based procedures seem to occur.

Our analysis of indicators and index summaries thereof considers five aspects. First, we consider the theoretical foundations for the indices. Secondly, we discuss the procedures used to select the components. Next, we examine methods of constructing the indices for possible information losses. Issues of cointegration within the indices and between the indicators (especially for the current CLIs in the US and the UK) are discussed. Lastly, we consider whether the indices are ex-post, but not ex-ante, efficient. We then investigate the use of CLIs in macro-economic modelling and forecasting from four angles. First, model choice issues may become important when preselected combinations of variables are added to models. For example, the resulting estimated standard errors may be biased. Next, data-based restrictions analogous to principal components are employed when the indices are used, but tests of these may be feasible. Thirdly, the unstable composition of indices over time may not be correctly reflected by in-sample tests of their value added for ex ante forecasting, particularly when indices are specifically constructed to lead aggregate economic activity at turning points ex post. Finally, issues of the degree of integration of the indicators and of cointegration between the indices and the variables in the macro-economic model are considered. However, we do not consider the prediction of 'turning points'.

The overall objective is to consider the theory of leading indicators and their potential role in general forecasting. The paper proceeds as follows. Section 2 explains the construction of the US and UK indices of leading indicators. Section 3 considers the current analytic basis for CLIs, and notes some potential problems when using leading indices in forecasting. Section 4 develops a more formal framework for constant parameter processes, while 5 considers generalizations, including to non-constant parameters. The penultimate section discusses possible problems with adding leading indicators to economic models for forecasting. The last section concludes.

2 Current UK and US composite leading indicators

In this section, we briefly describe the current CLIs in the UK and US, their components and their method of construction.

2.1 The United Kingdom's index of leading indicators

The present (as of January 1994) cyclical indicators for the economy of the United Kingdom have been in place since July 1993. The Central Statistical Office (CSO) has two leading indicators for the UK economy: one which is constructed to lead by a year or more (the longer-leading indicator), and the other which is intended to have a median lead of six months (the shorter-leading indicator). They are calculated and published monthly in *Economic Trends.*

Using gross domestic product as the aggregate economic activity, the individual indicators are chosen using a computer program developed by the National Bureau of Economic Research (NBER) in the United States. The selection criteria are similar to those used in the US and described in section

[2.2].[1] The current components of the longer-leading indicator $I_{L,t}^{UK}$ are the three-month rate of interest on prime bank bills (inverted) denoted R_{3t}, total dwelling starts for Great Britain, the inverted yield curve ($R_{3t} - R_{bt}$, where R_{bt} is the twenty-year bond rate), the financial surplus/deficit of industrial and commercial companies in 1990 prices, D_t, (deflated by the GDP deflator), and the optimism balance from the Confederation of British Industry (CBI) survey, O_t. The components of the shorter-leading index, $I_{S,t}^{UK}$, are new car registrations, N_t, the balance of new orders from the past four months from the CBI survey, B_t, the consumer confidence index from the EC/Gallup survey, G_t, the change in consumers' outstanding borrowing in 1990 prices, L_t, (deflated by the GDP deflator), and *The Financial Times* Actuaries' 500-share index of common stock prices, F_t. The monthly components of the longer-leading indicator are shown in Fig. 1a–c; the quarterly components in Fig. 2a–d; and the monthly components of the shorter leading index in Fig. 3a–c; the composite indices are shown in Figs. 1d and 3d respectively.

The components in Fig. 1 have similar 'cyclical' patterns, whereas those in Figs. 2 and 3 are much less similar, other than the shorter-leading indicator itself.

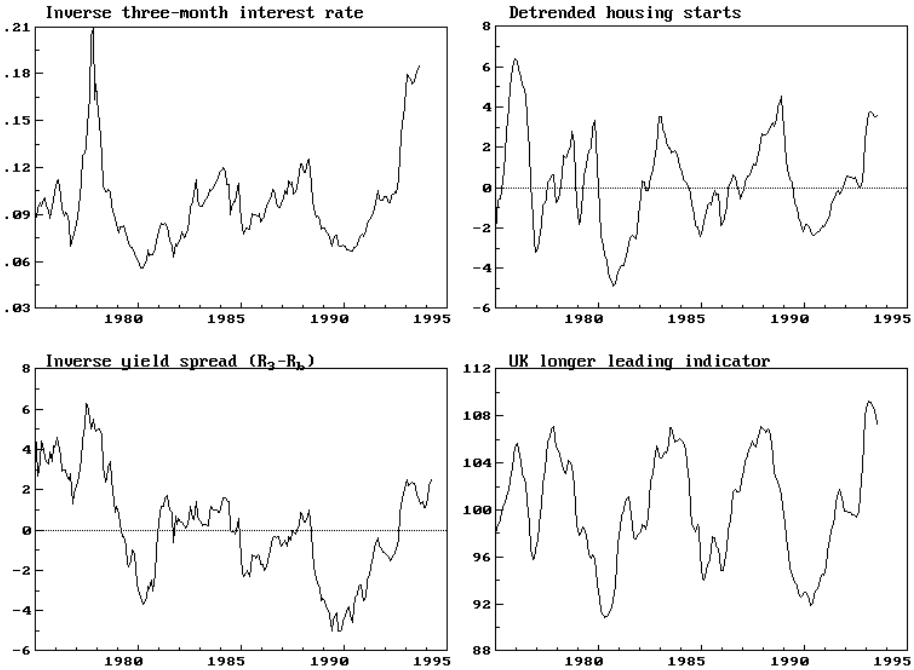

Fig. 1 Time series of UK monthly longer-leading indicators

The composite leading index is constructed from its components as follows.[2] The observed or interpolated individual monthly series are detrended, smoothed by centered moving averages, scaled so that their amplitudes are approximately equal, then weighted, perhaps after 'inversion' so that all weights are positive, such that no single series dominates the index. The composite index takes the value of 100 in the base month.[3] The index is extended from the base month by adding the average proportional change in the components' values, using the formula:

$$I_{m+1}^{UK} = I_m^{UK} + \frac{1}{n} \sum_{r=1}^{n} \left(\frac{A_{r,m+1} - A_{r,m}}{A_{r,m}} \right) \tag{1}$$

where I_m^{UK} is the index in month m, $A_{r,m}$ is the value of the r^{th} component in month m, and n is the total number of components. The longer-leading index is analyzed below.

[1] For the exact list of criteria, see CSO (1975).
[2] For an exact description, see CSO (1983) or Moore (1993).
[3] This is mid-1991 for the longer-leading indicator and mid-1992 for the shorter-leading indicator.

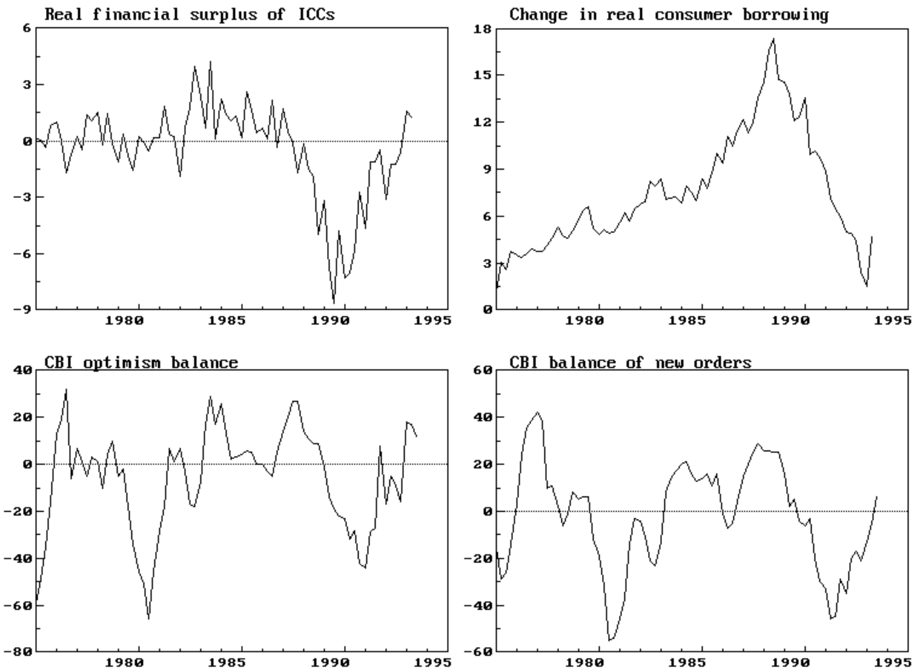

Fig. 2 Time series of UK quarterly leading indicators

2.2 The United States' index of leading indicators

The United States' cyclical indicators have not changed since 1989. The US CLI is calculated monthly by the Bureau of Economic Affairs and published in the Department of Commerce's *Survey of Current Business*. It is intended to lead aggregate economic activity at peaks and troughs on average by a year.[4]

To construct the composite index, the components are first chosen using a computer program developed by the NBER in which a set of indicators is selected based on a scoring system giving points for, 'cyclical timing, economic significance, statistical adequacy, conformity to the business cycle, smoothness, prompt availability (currency), and revisions.' From the list of potential individual indicators (72), a smaller number is chosen, based mainly on timing, but also on the sector of the economy from which they originate, to construct the composite index. For the leading index, only series available at least monthly are used. Currently, the US Bureau of Economic Analysis's composite index of leading indicators has eleven component series: the change in manufacturers' unfilled orders for the durable goods industries, expressed in 1982 dollars and smoothed;[5] change in sensitive materials prices, smoothed; the index of consumer expectations; the average weekly initial claims for unemployment insurance (inverted); vendor performance, expressed as the percentage of companies receiving slower deliveries; the average weekly hours of production of non-supervisory workers for manufacturing industries; manufacturers' new orders for the consumer goods and materials industries, expressed in 1982 dollars; contracts and orders for plant and equipment, expressed in 1982 dollars; broad money measured by M2; index of new private housing units authorized by local building permits; and the index of 500 common stock prices. The individual components and the composite index are pictured in order in Figs. 4–5 , while the diffusion indices based on them are shown in Fig. 6. Finally, Fig. 7 compares the time series of the UK and US leading indicators.

Common 'cyclical' patterns are less in evidence in the US component series, and the resulting CLI is rather different from its UK counterpart. One obviously distinct feature is the trend in the US CLI. Detrending the US CLI improves the visual match somewhat, but the two countries' leading indicators clearly reflect different economic developments. The diffusion indices are much more erratic than the CLIs, and although 'cycles' are visible, these differ considerably from those apparent in the UK's CLIs.

[4] The peaks and troughs of economic activity are designated by the NBER.

[5] The smoothing filter was developed by Statistics Canada, and uses an autoregressive-moving average to minimize the amount of lead time; see Hertzberg and Beckman (1989).

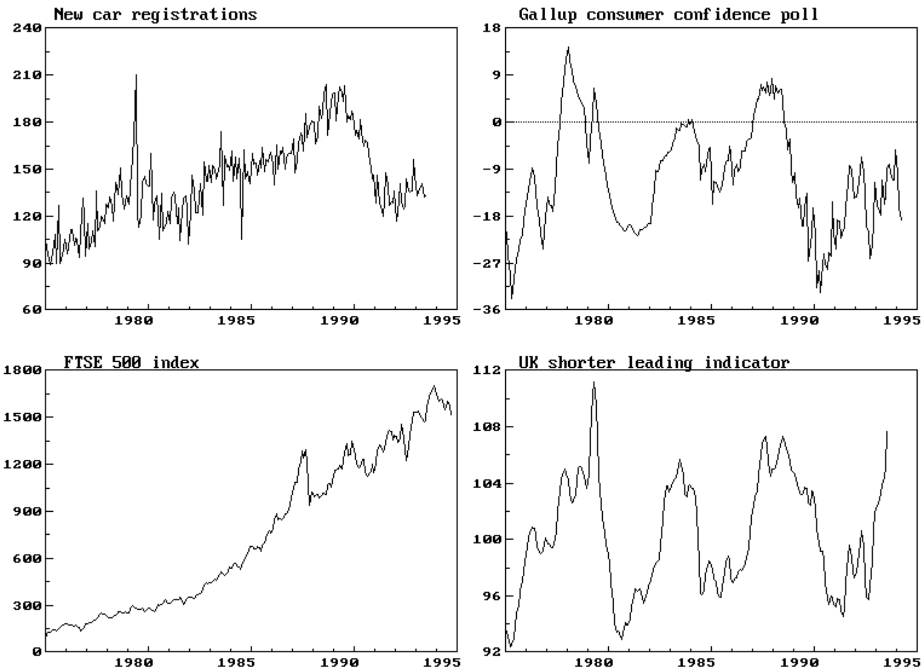

Fig. 3 Time series of UK monthly shorter-leading indicators

The US CLI is constructed from its eleven components in a similar way to that used in the UK and described above.[6] However, the detrending uses month to month percentage changes, standardized by 'long-run averages', then matched to the average of the component indicators. The raw index is produced using the formula:

$$I_{m+1}^{US} = I_m^{US} \frac{(200+c)}{(200-c)}$$

where c is the standardized average change between months m and $m+1$, and I_m^{US} is the value of the index in month m (in the base month, I_m^{US} is set to 100).

The raw index is converted to the final index by setting its trend equal to that of GNP (in January 1989, this target trend was 0.26% per month). As in standardizing the index so that the long-run average is equal to that of the coincident index, setting the trend equal to the trend of GNP aims to make the indices a coherent and consistent system.

3 Analysis of indices of leading indicators

In this section, we examine the potential problems of composite indices of leading indicators.

3.1 Foundations for leading indices

Those who construct leading indices justify them on several grounds. First, it is intended that in a properly constructed index, much of the noise in individual indicators is eliminated by averaging. For example, when data errors are independent, using many series together can strengthen their signal. Alternatively, if one indicator 'fails' on one occasion, then averaging it with other ones need not cause the whole system to fail. If it keeps 'failing', then an indicator will eventually be taken out of the composite index. The main reason usually given is that the components originate from many sectors of the economy; thus, the CLI should work under different causes of cyclical fluctuations:

> How the individual indicators would perform in a particular episode is likely to depend on which presumptive causes of a cyclical reversal are then in operation and how (through which process) they work. Some leading indicators, then, would prove most useful in one set of conditions, others in a different set. To increase the chances of getting true signals and

[6] For a fuller explanation, see Department of the Commerce (1984), pp.65-71.

reduce those of getting false ones, it is advisable to rely on all such potentially useful leading indicators as a group. (Zarnowitz and Boschan, 1977a, p173)

Although such reasons for the current composite indices are sensible, they are mainly reasons for using indices as opposed to individual indicators. The basic assumption is that the indicators do lead systematically, and little evidence is offered to support this claim. Moreover, one cannot be sure when the current cyclical turn originates from one sector of the economy, and that particular sector is represented by one indicator in the index (say), that the movement of that sector will be reflected in the index, as opposed to averaged out. The real tests of an index are how well it performs in predicting turning points, and the frequency with which its composition needs to be revised.

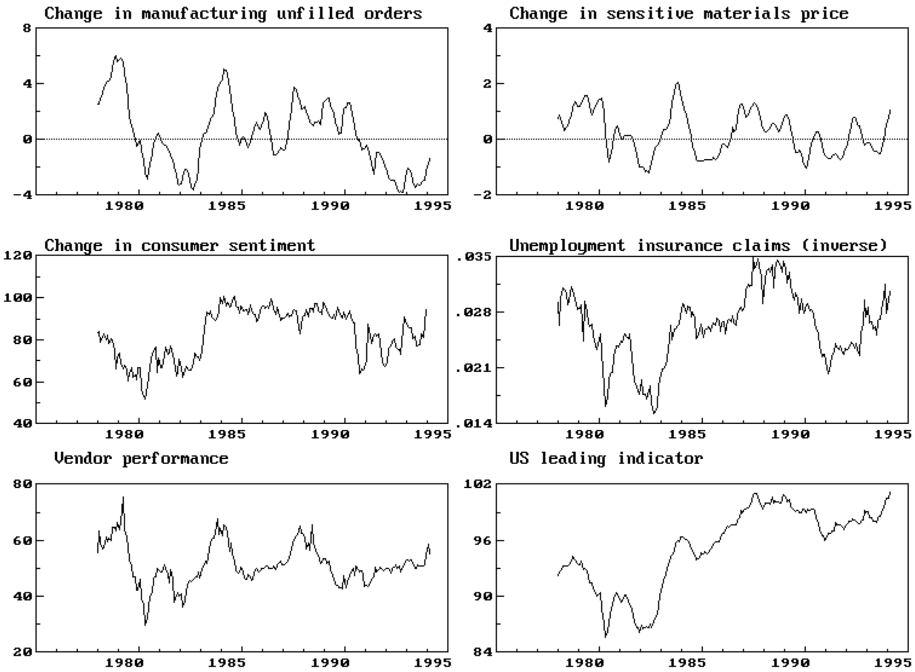

Fig. 4 Time series of US monthly leading indicators

3.2 Selection procedure of the components

The components of the US and UK CLIs are selected using the system of 'scoring' economic variables developed by Moore and Shiskin (1967) for the NBER. This entails giving each economic series points out of 100, based on certain characteristics. Scores are given for six broad categories. A possible maximum 20 points is awarded for each of the categories of economic significance, statistical adequacy, conformity (to the business cycle), and timing. A possible maximum of 10 points is awarded for each of the categories of currency and smoothness.

Although this system of scoring economic variables is implemented with the aim of using the highest scoring variables, in practice, human judgement may be needed to make the final selection of variables. As Zarnowitz and Boschan (1977b) note: 'The selection cannot be guided by the scores alone, since more is required of a good index then of a good single indicator.'

Since the indices of leading indicators are intended to predict turning points in the economy, it is natural to choose those indicators which have predicted turning points. This has been formalized statistically in Neftci (1979) and Stock and Watson (1989) *inter alia*. If indices were constructed by a process of statistical reduction, then those series which formed part of a model could be the components in an index, and the coefficients on the series could be the weights: in effect this is how econometric models are able to forecast. We consider this issue in section 4 below.

Finally, in practice, the potential components of a CLI appear to be chosen by having high bivariate correlations with the variable they are intended to lead. Given the objective of constructing a CLI, this method may have merit and deserves closer analysis since the usual concepts of (say) omitted variable bias become irrelevant. Later sections consider the outcome in terms of multivariate relations, as these are the relevant attribute when econometric modelling, but the robustness to regime shifts and structural change may differ between the two selection methods.

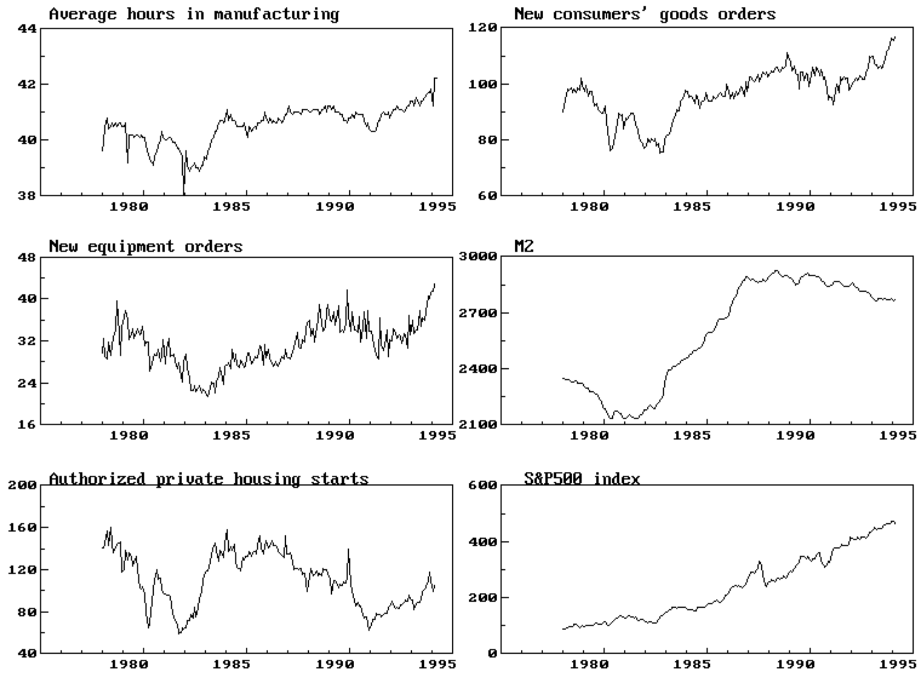

Fig. 5 Time series of US monthly leading indicators

3.3 Constructing leading indices

The construction process for CLIs does not draw upon the time-series properties of the components beyond the aspect of systematically leading. For example, data dynamics and cointegration seem potentially relevant attributes (the latter is considered in the next section). Since aggregate economic activity is autoregressive, it is partly forecastable, although few CLIs seem to make use of this attribute. In addition to excluding lags of aggregate economic activity from leading indicators, there is little justification for only using contemporaneous values of the components. Thus, CLIs as currently constructed appear to impose strong restrictions on models relative to other economic forecasting methods. This issue is analyzed in section 4.2

The assembly procedure used by the US and the UK for their composite leading indicators described above makes the index essentially a restriction on the vector of components. We consider this idea analytically in the next section. The method used to construct the indices has implications for other issues than just their forecastability. For example, the lack of formalization currently precludes confidence intervals being constructed by statistical agencies.

The trend-adjustment procedure of the US index does not seem to add information to the leading index which would help prediction of aggregate economic turning points. Since the method for computing the trend uses averages of the data 'for the initial and terminal specific cycles of a time series', the trend changes every time the data are revised. It would be interesting to examine whether turning points are different in the trend-adjusted series from the 'raw' index.

Alternatively, one could think of the index after the trend-adjustment procedure as being a combination of conditional and unconditional forecasts. The 'raw index' would be the conditional forecast, while the long-run trend would be the unconditional forecast. Such forecast combinations are discussed in greater detail in Clements and Hendry (1994), where in the specific case of an AR(1) process, they show that it is possible to reduce mean-squared forecast error by combining the conditional and unconditional forecasts. For their particular example, they show that the mix of the forecasting methods changes with the forecast horizon. Thus, there might be some virtue in the 'trend adjustment' playing such a role, but it remains to be demonstrated that the current method improves the conditional forecast.

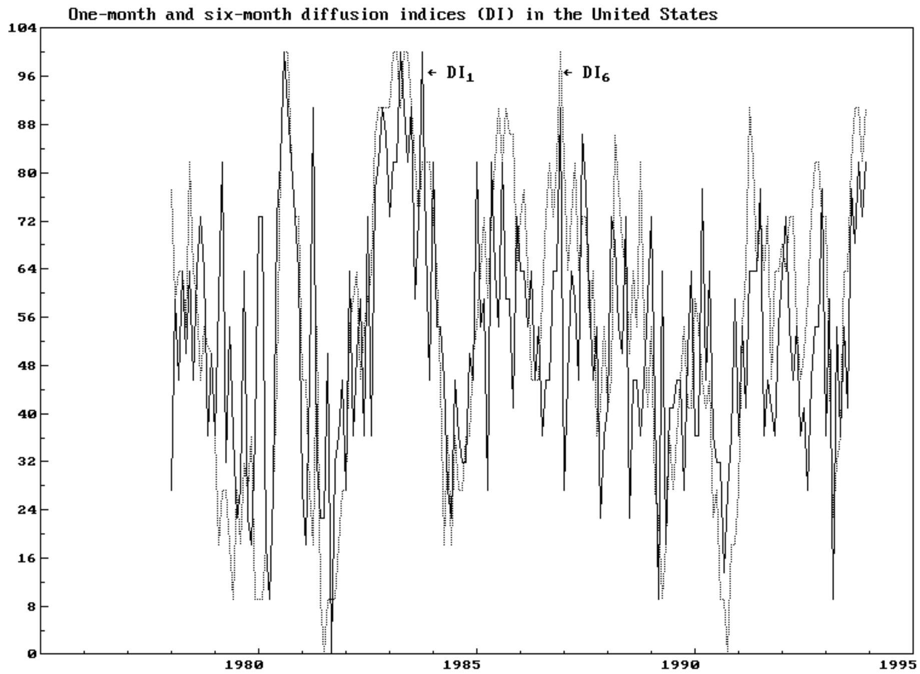

Fig. 6 US one-month and six-month diffusion indicators

3.4 Ex-post and ex-ante efficiency of the indices

Because of the method of component selection and index construction, the indices are ex-post efficient, but need not be ex-ante efficient for predicting turning points. In terms of their selection criteria, the indices are optimal when they are first introduced, since they are based on realized data. Thus, they are ex-post efficient, subject to possible mistakes in selection, compilation etc. However, the composition of the indices is altered frequently due to prediction deterioration. For example, Diebold and Rudebusch (1991a,b) demonstrate that the current and past US CLIs are not ex-ante efficient due to this difficulty. Also, Stock and Watson (1989,1992) demonstrated that some alternative CLIs constructed for real-time forecasting by a more formal probabilistic approach were also ex-post inefficient. Both of these cases were discussed in the first section.

The work by Diebold and Rudebusch (1989,1991a,b) and Stock and Watson (1989,1992), and the dynamic nature of the changes made to the US and UK CLIs, point to the fact that the data generation process is not constant. That is, the relationships between aggregate economic activity, and within the set of constituent variables, are not constant over time. For example, the latest US recession was not preceded by a large downturn in monetary variables and thus CLIs which relied heavily on that relationship had problems picking up the downturn (see Stock and Watson, 1992).

A non-constant process poses problems for all forecasting devices at times when the parameters alter. The primary advantage of an econometric system is to explain why breaks occur, and progressively move towards more robust (deeper) parameterizations than those which directly characterize the phenomenological level. The converse difficulty for leading-indicator based methods is that apparent breaks can occur for reasons that would not necessarily affect an econometric model. For example, the demand for M2 depends on relative rates of return, and hence alters for portfolio reasons as well as transactions changes: the demand function could be stable, yet the correlation with GNP unstable.

3.5 Issues of integration and cointegration

Present methods of constructing the indices do not take formal account of the orders of integration of the components (e.g. whether aggregate economic activity is $\mathsf{I}(1)$ or $\mathsf{I}(0)$), whether subsets of the components are cointegrated, or whether the components are cointegrated with aggregate economic activity.[7]

[7] See Nelson and Plosser (1982) and Campbell and Mankiw (1987) for discussions of the empirical frequency of unit roots and whether (e.g.) US GNP has a unit root. We define all stationary series to be mutually cointegrated.

Whether or not a variable has a unit root should influence how any model involving it is constructed, and how the target process is forecast. Both the US and the UK indices are mainly constructed from differenced data. However, the US CLI and the UK shorter and longer-leading indicators contain series which are integrated of order one. In section 6 we consider whether the information contained in the UK components is efficiently used. Empirically, the hypothesis of a unit root in the US CLI cannot be rejected (with or without a linear trend, for any lag up to 13), whereas for several lags, both the UK indices allow the unit-root hypothesis to be rejected. Our framework explicitly allows for integrated-cointegrated time series.

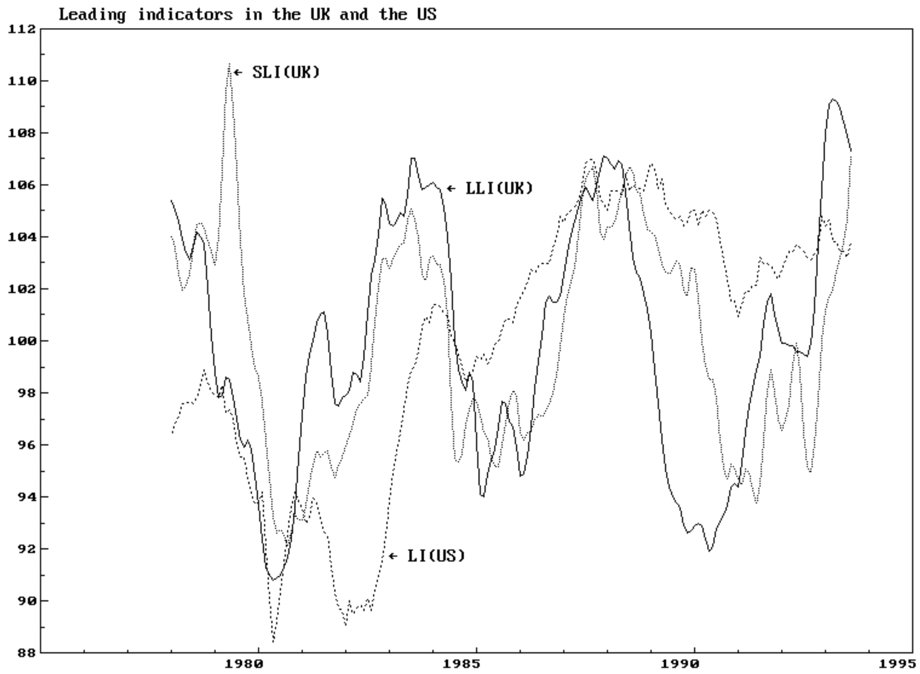

Fig. 7 Comparison of UK and US leading indicators

A less obvious point is that if a CLI is cointegrated with the levels of a set of variables such as consumption and income (c and y), then it cannot be cointegrated with $\mathsf{I}(0)$ linear combinations of these (e.g. $s = y - c$) when c and y are $\mathsf{I}(1)$, and s is $\mathsf{I}(0)$. Thus, care is required in specifying precisely what role the CLI is intended to play.

4 A constant-parameter framework for index analysis

In this section, we consider constant parameters in a simple DGP: section 5 considers some generalizations, including parameter changes. Let $\mathbf{x}_t$ denote a vector of n observable economic magnitudes, $\mathbf{i}_t$ an $r \times 1$ vector of components of the index (the individual leading indicators), g_t the leading index and $a_t = \boldsymbol{\lambda}'\mathbf{x}_t$ the index of economic activity, where $\boldsymbol{\lambda}$ is an $n \times 1$ vector of coefficients. We assume that the index weights are fixed for simplicity, so:

$$g_t = \boldsymbol{\phi}'\mathbf{i}_t \tag{2}$$

where $\boldsymbol{\phi}$ is an $r \times 1$ vector of weights. Finally, $\mathbf{i}_t$ is a subset of $\mathbf{x}_t$ given by $\mathbf{i}_t = \mathbf{S}\mathbf{x}_t$ where $\mathbf{S}$ is $r \times n$ and time units are determined by the highest frequency involved in the $\mathbf{i}_t$.

For analytical tractability, we take the data generation process (DGP) to be the first-order $\mathsf{I}(1)$ vector autoregression:

$$\mathbf{x}_t = \boldsymbol{\tau} + \boldsymbol{\Upsilon}\mathbf{x}_{t-1} + \mathbf{u}_t \tag{3}$$

where $\mathbf{u}_t \sim \mathsf{IN}[\mathbf{0}, \boldsymbol{\Omega}]$ and $\boldsymbol{\Upsilon}$ has no eigenvalues outside the unit circle: longer lags complicate, but do not alter the principles of, the analysis. Under the hypothesis that there are p cointegrating combinations of $\mathbf{x}_t$:

$$\boldsymbol{\Upsilon} = \mathbf{I} + \boldsymbol{\alpha}\boldsymbol{\beta}' \tag{4}$$

where $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are $n \times p$ matrices of rank p, such that $\boldsymbol{\beta}'\mathbf{x}_t$ is $\mathsf{I}(0)$ (see Johansen and Juselius (1990) and Banerjee *et al.* (1993) for an exposition). Letting $\boldsymbol{\tau} = \boldsymbol{\gamma} - \boldsymbol{\alpha\mu}$, reformulate (3) in $\mathsf{I}(0)$ space as:

$$\Delta\mathbf{x}_t = \boldsymbol{\gamma} + \boldsymbol{\alpha}\left(\boldsymbol{\beta}'\mathbf{x}_{t-1} - \boldsymbol{\mu}\right) + \mathbf{u}_t, \tag{5}$$

where $\mathsf{E}[\Delta\mathbf{x}_t] = \boldsymbol{\gamma}$ and $\mathsf{E}[\boldsymbol{\beta}'\mathbf{x}_t] = \boldsymbol{\mu}$. Below, we also consider the issues that arise when $(\boldsymbol{\tau}, \boldsymbol{\Upsilon}, \boldsymbol{\Omega})$ are not constant over time.

4.1 Activity index

We are not concerned with the mechanics of how an activity index might be selected, which was discussed above, but with the properties of such indices, given $\boldsymbol{\lambda}$. Then a_t is a reparameterization and reduction of $\mathbf{x}_t$ defined from (3) by:

$$a_t = \boldsymbol{\lambda}'\mathbf{x}_t = \boldsymbol{\lambda}'\boldsymbol{\tau} + \boldsymbol{\lambda}'\boldsymbol{\Upsilon}\mathbf{x}_{t-1} + \boldsymbol{\lambda}'\mathbf{u}_t = \mu_0 + \mu_1 a_{t-1} + \eta_t \tag{6}$$

where $\mu_0 = \boldsymbol{\lambda}'\boldsymbol{\tau}$, and μ_1 is determined by minimizing the expected sum of squares of $\{\eta_t\}$ in:

$$\eta_t = \boldsymbol{\lambda}'\left(\boldsymbol{\Upsilon} - \mu_1\mathbf{I}_n\right)\mathbf{x}_{t-1} + \boldsymbol{\lambda}'\mathbf{u}_t = \boldsymbol{\lambda}'\left([1-\mu_1]\,\mathbf{I}_n + \boldsymbol{\alpha\beta}'\right)\mathbf{x}_{t-1} + \boldsymbol{\lambda}'\mathbf{u}_t. \tag{7}$$

Thus, the index has a first-order autoregressive component, but in general η_t will be autocorrelated as it contains linear combinations of $\mathbf{x}_{t-1}$.

More fundamentally, we must determine when $\{a_t\}$ is $\mathsf{I}(0)$ or $\mathsf{I}(1)$. First, when the weights are a linear combination of (a subset of) the cointegrating vectors, so $\boldsymbol{\lambda} = \boldsymbol{\beta}\mathbf{k}$ where $\mathbf{k}$ is $p \times 1$, then:

$$\boldsymbol{\lambda}'\left([1-\mu_1]\,\mathbf{I}_n + \boldsymbol{\alpha\beta}'\right) = \mathbf{k}'\boldsymbol{\beta}'\left([1-\mu_1]\,\mathbf{I}_n + \boldsymbol{\alpha\beta}'\right) = \mathbf{k}'\left([1-\mu_1]\,\mathbf{I}_p + \boldsymbol{\beta}'\boldsymbol{\alpha}\right)\boldsymbol{\beta}' \tag{8}$$

and hence $\{\eta_t\}$ will be $\mathsf{I}(0)$ irrespective of the value of μ_1. Secondly, when $\boldsymbol{\lambda} \neq \boldsymbol{\beta}\mathbf{k}$, then $\{\eta_t\}$ will be stationary only if $\mu_1 = 1$, in which case the activity index will be a random walk with drift (plus a potentially autocorrelated error). In words, either the items in the index cointegrate using the weights $\boldsymbol{\lambda}$ so $\{a_t\}$ is stationary, or the index will be $\mathsf{I}(1)$ so its autoregressive representation must have a unit root. This result is unaffected by continually resetting $a_t = 100$ in a base period.

Such an outcome places tight restrictions on admissible combinations of levels of variables that can be used in any stationary index: otherwise only the change in the index will be $\mathsf{I}(0)$. Moreover, when stationarity is desired, this analysis suggests using recent multivariate cointegration approaches to determine the activity index. Detrending a_t (removing a linear deterministic trend) will not suffice to remove the unit root, which induces a stochastic – not a deterministic – trend: see Phillips and Durlauf (1986). However, differencing a_t is sufficient when the DGP is $\mathsf{I}(1)$ given the representation in (5), but will be unnecessary when $\boldsymbol{\lambda} = \boldsymbol{\beta}\mathbf{k}$.

The covariance of the activity index with any element $x_{it} = \mathbf{j}_i'\mathbf{x}_t$ of $\mathbf{x}_t$ is given by (ignoring means):

$$\mathsf{E}[a_t x_{it}] = \mathsf{E}[\boldsymbol{\lambda}'\mathbf{x}_t\mathbf{x}_t'\mathbf{j}_i].$$

This is well defined only when $\boldsymbol{\lambda} = \boldsymbol{\beta}\mathbf{k}$ and x_{it} is stationary; or when a_t and x_{it} are both $\mathsf{I}(1)$ but cointegrate. When $\mathbf{x}_t$ is $\mathsf{I}(1)$, the index is $\mathsf{I}(0)$, and its covariance with Δx_{it} is of concern, then:

$$\mathsf{E}[a_t\Delta x_{it}] = \mathbf{k}'\mathsf{E}[\boldsymbol{\beta}'\mathbf{x}_t\Delta\mathbf{x}_t']\,\mathbf{j}_i = \mathbf{k}'\left(\mathsf{E}\left[\boldsymbol{\beta}'\mathbf{x}_t\mathbf{x}_{t-1}'\boldsymbol{\beta\alpha}'\right] + \boldsymbol{\beta}'\boldsymbol{\Omega}\right)\mathbf{j}_i = \mathbf{k}'\mathbf{R}\boldsymbol{\alpha}_i + \mathbf{k}'\boldsymbol{\beta}'\boldsymbol{\omega}_i,$$

where $\mathbf{R} = \mathsf{E}\left[\boldsymbol{\beta}'\mathbf{x}_t\mathbf{x}_{t-1}'\boldsymbol{\beta}\right]$ is the first-order autocovariance matrix of the cointegrating vectors, and $\boldsymbol{\omega}_i$ is the i^{th} column of $\boldsymbol{\Omega}$.

4.2 Leading index

Next, from (3), since $\mathbf{i}_t = \mathbf{S}\mathbf{x}_t$:

$$\mathbf{i}_t = \mathbf{S}\mathbf{x}_t = \mathbf{S}\boldsymbol{\tau} + \mathbf{S}\boldsymbol{\Upsilon}\mathbf{x}_{t-1} + \mathbf{S}\mathbf{u}_t = \boldsymbol{\gamma} + \boldsymbol{\Gamma}\mathbf{i}_{t-1} + \mathbf{w}_t \tag{9}$$

where $\boldsymbol{\gamma} = \mathbf{S}\boldsymbol{\tau}$, and $\boldsymbol{\Gamma}$ derives from 'minimizing' the expected matrix sum of squares of $\{\mathbf{w}_t\}$ in:

$$\mathbf{w}_t = \left([\mathbf{I}_r - \boldsymbol{\Gamma}]\,\mathbf{S} + \mathbf{S}\boldsymbol{\alpha\beta}'\right)\mathbf{x}_{t-1} + \mathbf{S}\mathbf{u}_t. \tag{10}$$

The choice of the indicators could retain or lose cointegration, and could have a VAR or a VARMA representation accordingly. For simplicity when required, we assume that a good choice of $\mathbf{S}$ has been made ($\mathbf{S\Upsilon} = \mathbf{\Gamma S}$), and so (9) is the VAR:

$$\mathbf{i}_t = \boldsymbol{\gamma} + \mathbf{\Gamma i}_{t-1} + \mathbf{w}_t \tag{11}$$

where $\mathbf{w}_t \sim \mathsf{IN}\left[\mathbf{0}, \mathbf{\Sigma}\right]$ with $\mathbf{\Sigma} = \mathbf{S\Omega S}'$.

Further:

$$g_t = \boldsymbol{\phi}'\mathbf{i}_t = \boldsymbol{\phi}'\mathbf{S}\mathbf{x}_t = \boldsymbol{\varphi}'\mathbf{x}_t \tag{12}$$

and as a reparameterization and reduction of $\mathbf{x}_t$, a similar analysis to that for a_t applies with the added constraint that g_t should lead either a_t or the relevant components of $\mathbf{x}_t$. In particular, when a stationary CLI is desired, $\boldsymbol{\varphi} = \boldsymbol{\beta}\mathbf{h}$ is required. From the Granger representation theorem (see e.g. Engle and Granger, 1987), cointegration entails Granger (1969) causality in at least one direction, so some predictability must result from the use of cointegrating combinations. Indeed, when $\boldsymbol{\varphi} = \boldsymbol{\beta}\mathbf{h}$, for $\Delta\mathbf{x}_t$:

$$\mathsf{E}\left[g_{t-1}\Delta x_{it}\right] = \mathbf{h}'\mathsf{E}\left[\boldsymbol{\beta}'\mathbf{x}_{t-1}\Delta\mathbf{x}_t'\right]\mathbf{j}_i = \mathbf{h}'\mathsf{E}\left[\boldsymbol{\beta}'\mathbf{x}_{t-1}\mathbf{x}_{t-1}'\boldsymbol{\beta}\boldsymbol{\alpha}'\right]\mathbf{j}_i = \mathbf{h}'\mathbf{V}\boldsymbol{\alpha}_i,$$

where $\mathbf{V} = \mathsf{E}\left[\boldsymbol{\beta}'\mathbf{x}_{t-1}\mathbf{x}_{t-1}'\boldsymbol{\beta}\right]$ is the variance matrix of the cointegrating vectors. This is well defined, but the resulting correlation could take any value in the interval $(-1, 1)$. Moreover, the covariance need not be the best that can be achieved.

When $\mathbf{i}_t$ satisfies the VAR in (11), we first consider the conditions under which g_t is a sufficient summary of the component indices. Then we analyze its role relative the original VAR (3), and consider its ability to predict future variables or indexes. Let $\mathbf{Q}$ be an $r \times r$ non-singular matrix such that:

$$\mathbf{Q} = \begin{pmatrix} \boldsymbol{\phi}' \\ \mathbf{Q}^* \end{pmatrix} \quad \text{and so} \quad \mathbf{Qi}_t = \begin{pmatrix} g_t \\ \mathbf{i}_t^* \end{pmatrix},$$

where $\mathbf{Q}^*$ is $(r-1) \times r$. From (11) and (12):

$$\mathbf{Qi}_t = \mathbf{Q}\boldsymbol{\gamma} + \mathbf{Q\Gamma i}_{t-1} + \mathbf{Qw}_t = \boldsymbol{\gamma}^* + \mathbf{CQi}_{t-1} + \mathbf{Qw}_t \tag{13}$$

where $\mathbf{Q\Gamma Q}^{-1} = \mathbf{C}$. Then:

$$\begin{pmatrix} g_t \\ \mathbf{i}_t^* \end{pmatrix} = \boldsymbol{\gamma}^* + \begin{pmatrix} c_{11} & \mathbf{c}_{12}' \\ \mathbf{c}_{21} & \mathbf{c}_{22} \end{pmatrix} \begin{pmatrix} g_{t-1} \\ \mathbf{i}_{t-1}^* \end{pmatrix} + \mathbf{w}_t^*. \tag{14}$$

Thus, g_t is a sufficient summary of $\mathbf{i}_t$ for prediction if $\mathbf{c}_{12} = \mathbf{0}$ and $\mathbf{c}_{22} = \mathbf{0}$, and otherwise, $\mathbf{i}_{t-1}^*$ also contains relevant information for predicting $\mathbf{i}_t$. These are testable conditions. Conversely, necessary conditions for g_t to predict elements of $\mathbf{i}_t$ are that $c_{11} \neq 0$ and/or $\mathbf{c}_{21} \neq \mathbf{0}$. Generalizations of this approach will apply below.

Next, without loss of generality, $\mathbf{x}_t$ can be reparameterized in terms of the (non-overlapping) items for the putative CLI, the remaining indicators, the activity index and the rest of the (perhaps combinations of) $\mathbf{x}_t$ as:

$$\mathbf{Px}_t = \begin{pmatrix} \mathbf{QS} \\ \boldsymbol{\lambda}' \\ \mathbf{\Psi} \end{pmatrix} \mathbf{x}_t = \begin{pmatrix} \mathbf{Qi}_t \\ a_t \\ \mathbf{\Psi x}_t \end{pmatrix} = \begin{pmatrix} \boldsymbol{\varphi}' \\ \mathbf{Q}^*\mathbf{S} \\ \boldsymbol{\lambda}' \\ \mathbf{\Psi} \end{pmatrix} \mathbf{x}_t = \begin{pmatrix} g_t \\ \mathbf{i}_t^* \\ a_t \\ \mathbf{\Psi x}_t \end{pmatrix} \tag{15}$$

where $\mathbf{P}$ is an $n \times n$ non-singular matrix such that $|\mathbf{P}| = 1$, and $\mathbf{\Psi}$ is $(n-r-1) \times n$. From (3), $\mathbf{x}_t$ follows a cointegrated autoregressive process, so substituting (3) into (15) yields:

$$\mathbf{Px}_t = \mathbf{P}\boldsymbol{\tau} + \mathbf{P\Upsilon x}_{t-1} + \mathbf{Pu}_t = \mathbf{P}\boldsymbol{\tau} + \left(\mathbf{P\Upsilon P}^{-1}\right)\mathbf{Px}_{t-1} + \mathbf{Pu}_t = \boldsymbol{\tau}^* + \mathbf{\Xi Px}_{t-1} + \mathbf{v}_t \tag{16}$$

where $\mathbf{P\Upsilon P}^{-1} = \mathbf{I} + \mathbf{P}\boldsymbol{\alpha\beta}'\mathbf{P}^{-1}$ so that $\mathbf{\Xi} - \mathbf{I} = \mathbf{P}\boldsymbol{\alpha\beta}'\mathbf{P}^{-1} = \boldsymbol{\alpha}^*\boldsymbol{\beta}^{*\prime} = \mathbf{\Theta}$ where $\boldsymbol{\beta}^* = \mathbf{P}^{-1\prime}\boldsymbol{\beta}$. Consequently, in levels:

$$\begin{pmatrix} g_t \\ \mathbf{i}_t^* \\ a_t \\ \mathbf{\Psi x}_t \end{pmatrix} = \begin{pmatrix} \tau_1^* \\ \boldsymbol{\tau}_2^* \\ \tau_3^* \\ \boldsymbol{\tau}_4^* \end{pmatrix} + \begin{pmatrix} \xi_{11} & \boldsymbol{\xi}_{12}' & \xi_{13} & \boldsymbol{\xi}_{14}' \\ \xi_{21} & \boldsymbol{\xi}_{22} & \xi_{23} & \boldsymbol{\xi}_{24} \\ \xi_{31} & \boldsymbol{\xi}_{32}' & \xi_{33} & \boldsymbol{\xi}_{34}' \\ \boldsymbol{\xi}_{41} & \boldsymbol{\xi}_{42} & \boldsymbol{\xi}_{43} & \boldsymbol{\xi}_{44} \end{pmatrix} \begin{pmatrix} g_{t-1} \\ \mathbf{i}_{t-1}^* \\ a_{t-1} \\ \mathbf{\Psi x}_{t-1} \end{pmatrix} + \begin{pmatrix} v_{1t} \\ \mathbf{v}_{2t} \\ v_{3t} \\ \mathbf{v}_{4t} \end{pmatrix} \tag{17}$$

and hence in an $\mathsf{I}(0)$ form:

$$\begin{pmatrix} \Delta g_t \\ \Delta \mathbf{i}_t^* \\ \Delta a_t \\ \mathbf{\Psi}\Delta \mathbf{x}_t \end{pmatrix} = \begin{pmatrix} \tau_1^* \\ \boldsymbol{\tau}_2^* \\ \tau_3^* \\ \boldsymbol{\tau}_4^* \end{pmatrix} + \begin{pmatrix} \theta_{11} g_{t-1} + \boldsymbol{\theta}_{12}' \mathbf{i}_{t-1}^* + \theta_{13} a_{t-1} + \boldsymbol{\theta}_{14}' \mathbf{\Psi} \mathbf{x}_{t-1} \\ \boldsymbol{\theta}_{21} g_{t-1} + \boldsymbol{\theta}_{22} \mathbf{i}_{t-1}^* + \boldsymbol{\theta}_{23} a_{t-1} + \boldsymbol{\theta}_{24} \mathbf{\Psi} \mathbf{x}_{t-1} \\ \theta_{31} g_{t-1} + \boldsymbol{\theta}_{32}' \mathbf{i}_{t-1}^* + \theta_{33} a_{t-1} + \boldsymbol{\theta}_{34}' \mathbf{\Psi} \mathbf{x}_{t-1} \\ \boldsymbol{\theta}_{41} g_{t-1} + \boldsymbol{\theta}_{42} \mathbf{i}_{t-1}^* + \boldsymbol{\theta}_{43} a_{t-1} + \boldsymbol{\theta}_{44} \mathbf{\Psi} \mathbf{x}_{t-1} \end{pmatrix} + \begin{pmatrix} v_{1t} \\ \mathbf{v}_{2t} \\ v_{3t} \\ \mathbf{v}_{4t} \end{pmatrix}. \tag{18}$$

The forecasting objective may be to predict future values of levels or changes in a_t or elements of $\mathbf{x}_t$ (or functions thereof), including $\mathbf{i}_t$. Any or all of $\mathbf{x}_{t-1}$, or the linear combinations $\mathbf{i}_{t-1}^*$, a_{t-1} or g_{t-1} could be used for forecasting. We consider the various possibilities in turn, focusing on predicting changes, from which levels can be obtained by integration. For one-step forecasts, the results are invariant but it matters which functions are selected for longer horizons (see Clements and Hendry, 1993).

From (18), the index g_t is potentially a CLI for predicting future values of elements derivable from $\mathbf{\Psi}\Delta\mathbf{x}_t$ when the relevant elements of $\boldsymbol{\theta}_{41} \neq \mathbf{0}$. The CLI is a sufficient statistic for future $\mathbf{\Psi}\Delta\mathbf{x}_t$ when $\boldsymbol{\theta}_{42} = \mathbf{0}$, $\boldsymbol{\theta}_{43} = \mathbf{0}$ and $\boldsymbol{\theta}_{44}\mathbf{\Psi} = \mathbf{0}$ as well, which hypotheses are testable. However, when $\boldsymbol{\theta}_{42} \neq \mathbf{0}$, $\boldsymbol{\theta}_{43} = \mathbf{0}$ and/or $\boldsymbol{\theta}_{44}\mathbf{\Psi} \neq \mathbf{0}$, the omitted variables may attenuate or enhance the marginal predictive value of g_{t-1} when it alone is used.

Similar comments apply to forecasting Δa_t: g_t is potentially a CLI when $\theta_{31} \neq 0$. Using only the index g_{t-1} instead of the entire vector $\mathbf{x}_{t-1}$ involves no loss of information for predicting future Δa_t when $\boldsymbol{\theta}_{32} = \mathbf{0}$, $\theta_{33} = 0$ and $\boldsymbol{\theta}_{34}'\mathbf{\Psi} = \mathbf{0}'$ which again are testable assumptions given a specification of a_t and g_t. In general, strong exclusion restrictions are imposed by regarding a CLI as a sufficient predictor. While there certainly exist grounds for wishing to use parsimonious models (see Box and Jenkins, 1972, and Hendry and Clements, 1993), the methods for selecting CLIs and their component indicators described above suggest that the resulting reductions may lose information.

Suppose that the DGP is given by (3), all of the elements of $\mathbf{x}_t$ are stationary, but $\boldsymbol{\xi}_{32} \neq \mathbf{0}$, $\xi_{33} \neq 0$ and $\boldsymbol{\xi}_{34}'\mathbf{\Psi} \neq \mathbf{0}'$, yet the modeller estimates:

$$a_t = d_0 + d_1 g_{t-1} + e_t \tag{19}$$

and uses it to forecast future values of the activity index; we consider a one-step ahead forecast. We assume that the modeller knows the value of $\mathbf{d} = (d_0 : d_1)'$, and let $\widetilde{a_{T+1}}$ be the conditional expectation forecast of a_{T+1} given by:

$$\widetilde{a_{T+1}} = \mathsf{E}\left[a_{T+1} \mid g_T\right] = d_0 + d_1 g_T. \tag{20}$$

However, the optimal conditional forecast of $\mathbf{x}_{T+1}$is:

$$\mathsf{E}\left[\mathbf{x}_{T+1} \mid \mathbf{x}_T\right] = \boldsymbol{\tau} + \mathbf{\Upsilon}\mathbf{x}_T \tag{21}$$

and thus of a_{T+1} is:

$$\mathsf{E}\left[a_{T+1} \mid \mathbf{x}_T\right] = \tau_3^* + \xi_{31} g_T + \boldsymbol{\xi}_{32}' \mathbf{i}_T^* + \xi_{33} a_T + \boldsymbol{\xi}_{34}' \mathbf{\Psi} \mathbf{x}_T. \tag{22}$$

The forecast error from using (20) with known parameters is:

$$e_{T+1} = a_{T+1} - \widetilde{a_{T+1}} = \tau_3^* + \xi_{31} g_T + \boldsymbol{\xi}_{32}' \mathbf{i}_T^* + \xi_{33} a_T + \boldsymbol{\xi}_{34}' \mathbf{\Psi} \mathbf{x}_T - d_0 - d_1 g_T + v_{3t} \tag{23}$$

and therefore the conditional expected forecast error is:

$$\mathsf{E}\left[e_{T+1} \mid \mathbf{x}_T\right] = (\tau_3^* - d_0) + (\xi_{31} - d_1)\, g_T + \boldsymbol{\xi}_{32}' \mathbf{i}_T^* + \xi_{33} a_T + \boldsymbol{\xi}_{34}' \mathbf{\Psi} \mathbf{x}_T \tag{24}$$

which is non-zero unless considerable (chance) cancellation occurs. The resulting biases are due to the restrictions on how the component indices enter the CLI, the absence of lagged values of the activity index, and the omission of useful predictors $\mathbf{\Psi}\mathbf{x}_T$ together with the consequential biased coefficients in (19).

4.3 Selecting a CLI

An alternative application of this framework is to select the 'optimal' composite leading indicator for forecasting a_t. Since $a_t = \boldsymbol{\lambda}'\mathbf{x}_t$, consider the linear transformation of $\mathbf{x}_{t-1}$ from (3) that maximises predictability. From (6), this is:

$$a_t = \mu_0 + \boldsymbol{\psi}'\mathbf{x}_{t-1} + e_t = \mu_0 + \ell_{t-1} + e_t^* \tag{25}$$

where $\boldsymbol{\psi} = \boldsymbol{\Upsilon}'\boldsymbol{\lambda}$ and $\ell_{t-1} = \boldsymbol{\psi}'\mathbf{x}_{t-1}$ is defined to be the 'best' leading indicator as it leaves an innovation error $\{e_t^*\}$:

$$\mathsf{E}\left[e_t^* \mid \mathbf{x}_{t-1}\right] = \mathbf{0} \quad \text{with} \quad \mathsf{E}\left[e_t^{*2} \mid \mathbf{x}_{t-1}\right] = \boldsymbol{\lambda}'\boldsymbol{\Omega}\boldsymbol{\lambda}. \tag{26}$$

This formulation highlights that the best weights are $\boldsymbol{\psi}$, but leaves the usual selection problems of which variables to include and how to estimate the weights. Moreover, it reveals that howsoever the weights are selected, their sampling uncertainty should be included in measures of forecast uncertainty associated with any CLI method.

We suspect that one difficulty is that different 'indicators' are relevant at different points in time, and which will be relevant is as hard to predict as the original problem of forecasting the target outcomes. To visualise the difficulty of using CLIs, as presently implemented, partition $\boldsymbol{\beta}$ into $\boldsymbol{\beta}_1$ and $\boldsymbol{\beta}_2$, where $\boldsymbol{\beta}_1'\mathbf{x}_{t-1} - \boldsymbol{\mu}_1$ is non-zero over the selection period, but zero over the forecast horizon, and $\boldsymbol{\beta}_2'\mathbf{x}_{t-1} - \boldsymbol{\mu}_2$ behaves in the opposite manner, so is not selected as part of the CLI within sample, although both $\boldsymbol{\beta}_1$ and $\boldsymbol{\beta}_2$ are far from zero. Then the forecasts could be badly out – in the simplest case we have:

$$\widehat{\Delta\mathbf{x}_{T+1}} = \boldsymbol{\gamma} + \boldsymbol{\alpha}\left(\boldsymbol{\beta}_1'\mathbf{x}_T - \boldsymbol{\mu}_1\right) \simeq \boldsymbol{\gamma} \tag{27}$$

whereas:

$$\Delta\mathbf{x}_{T+1} \simeq \boldsymbol{\gamma} + \boldsymbol{\alpha}\left(\boldsymbol{\beta}_2'\mathbf{x}_T - \boldsymbol{\mu}_2\right) + \mathbf{u}_{T+1} \tag{28}$$

resulting in the forecast error:

$$\Delta\mathbf{x}_{T+1} - \widehat{\Delta\mathbf{x}_{T+1}} \simeq \boldsymbol{\alpha}\left(\boldsymbol{\beta}_2'\mathbf{x}_T - \boldsymbol{\mu}_2\right) + \mathbf{u}_{T+1}. \tag{29}$$

Interestingly, cointegration analysis is more likely to have detected the near zero in-sample combinations $(\boldsymbol{\beta}_2'\mathbf{x}_{t-1})$ as cointegrated.

5 Generalizations

5.1 Non-constant parameters

The previous section established the framework, and most of its implications generalize to the empirically realistic case of a process with non-constant parameters. Consider the DGP as given instead by:

$$\mathbf{x}_t = \boldsymbol{\tau}_t + \boldsymbol{\Gamma}_t\mathbf{x}_{t-1} + \mathbf{w}_t. \tag{30}$$

The algebra closely follows that already sketched with the original parameters indexed by time. If a modeller estimates (19), the forecast will be sub-optimal because of the earlier reasons, augmented by the parameter non-constancy. Parameter non-constancy within-sample is testable, but present methods of constructing CLIs do not appear to check this key aspect. We discussed the general issue of the impact of non-constancy on modelling and forecasting above, and reiterate our view that unless correlations have a more substantive basis than that they were observed over a past period, they offer little prospect of delivering a reliable forecasting method. A DGP such as that in (30) explains why the composition of CLIs may need regular revision, but by eschewing attempts to understand the evolution of the $(\boldsymbol{\tau}_t : \boldsymbol{\Gamma}_t)$ in the space of the $\{\mathbf{x}_t\}$, will not induce a progressive accumulation of knowledge. Parameter non-constancy also induces predictive failure in econometric equations, which offer no more nor less protection against it at any point in time, but should provide a framework for consolidating and improving understanding to reduce likely future failures. Clements and Hendry (1994a) consider the effects of structural breaks on econometric models in more detail.

5.2 Longer lags

Allowing more lags in the DGP complicates the algebra without adding greatly to the analysis. For example, extend the DGP to:

$$\Delta\mathbf{x}_t = \boldsymbol{\tau} + \boldsymbol{\alpha}\left(\boldsymbol{\beta}'\mathbf{x}_{t-1}\right) + \mathbf{A}\Delta\mathbf{x}_{t-1} + \mathbf{u}_t. \tag{31}$$

This points up the drawbacks of using only contemporaneous variables in a CLI, since lagged changes also have predictive power when $\mathbf{A} \neq \mathbf{0}$. Indeed, if cointegration was weak, only lagged changes would matter. Conversely, as considered above, if lagged effects were small, cointegrated combinations offer the only predictive possibility. Overall, therefore, an analysis which does not a priori restrict the extent of cointegration, the degree of differencing, and the lag length seems preferable.

5.3 Non-linearities

Similarly, non-linearities again complicate the analysis, and suggest that either clever functional-form transforms will be needed, or that linear weights will not be constant. Several analyses have considered mapping the formulation to one of probabilities of turning points (see e.g. Stock and Watson, 1989), but the underlying issues of constancy of predictive power are not altered by doing so, as stressed by Stock and Watson (1992).

6 An empirical analysis of the UK longer-leading indicator

We first investigate the UK longer-leading indicator, as a prelude to studying its role in VAR modelling. Since the econometric model dataset is only available quarterly, we converted the three component monthly indicators and the CLI to quarterly data by selecting the last monthly outcome of each quarter. The resulting time series closely resemble the corresponding monthly variables, and as shown in (32), 87 per cent of the variance of the quarterly change in the CLI is explained by the quarterly change in its components. More precisely, over the available quarterly sample 1975(2)–1993(2), regressing $\Delta I_{L,t}^{UK}$ on its synthetic components yields:

$$\widehat{\Delta I_{L,t}^{UK}} = -\underset{(0.14)}{0.75}\ \Delta R_{3t} + \underset{(0.07)}{0.64}\ \Delta S_t - \underset{(0.18)}{0.62}\ \Delta\left(R_{bt} - R_{3t}\right) + \underset{(0.06)}{0.17}\ \Delta D_t - \underset{(0.01)}{0.01}\ \Delta O_t \tag{32}$$
$$\mathsf{R}^2 = 0.87,\ \hat{\sigma} = 0.84,\ DW = 1.58.$$

This is far from an identity as defined by (1) for monthly data, but nevertheless explains much of the variance of the changes in the index. The main 'explanatory' variables all have the correct sign and are highly significant, other than ΔO_t.

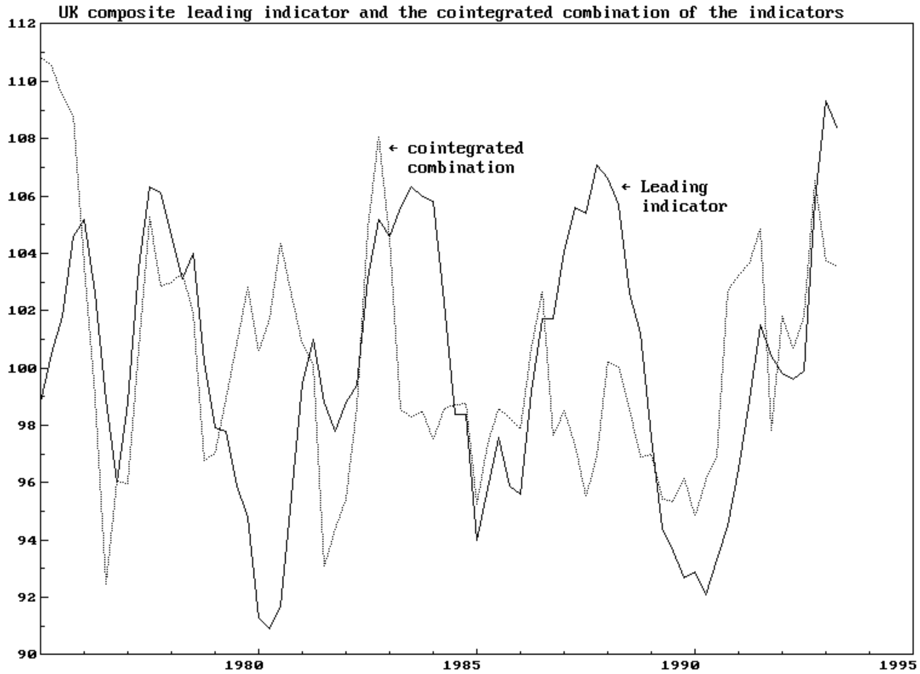

Fig. 8 Cointegrating combination of indicators and UK longer-leading index

Next, we consider the analysis in section 4.2. The VAR for the component indicators is a five-dimensional system for ΔR_{3t}, ΔS_t, $\Delta\left(R_{bt} - R_{3t}\right)$, ΔD_t, and ΔO_t. This was estimated by multivariate least squares in levels, using 2 lags on every variable plus an intercept, and tested for cointegration using the Johansen (1988) approach in PcFiml 8 (see Doornik and Hendry, 1994). Table 1 shows the outcome for the cointegration test statistics for 1975(4)–1993(2).

Table 1: Cointegration analysis

r	1	2	3	4	5
μ	0.36	0.32	0.14	0.08	0.04
Max	27.1	$23.6^{\dagger}$	9.4	5.3	2.6
Tr	$68.0^{\dagger}$	$40.9^{\dagger}$	17.3	7.9	2.6

† The unadjusted test is significant at 5%.

Strictly, no test is significant, but that may be due to the small sample size: two of the eigenvalues are quite large at over 0.3. Next, Table 2 shows the two eigenvectors corresponding to the two largest eigenvalues. The vector estimated in (32) does not lie in the cointegration space, as assuming there are two cointegration vectors, $\chi^2(3) = 16.9$.

Table 2: Cointegration vectors

$\hat{\boldsymbol{\beta}}'$	R_3	S	$R_b - R_3$	D	O
1	1	−1.01	0.02	−0.68	0.25
2	0.79	1	−0.08	0.18	0.05

The feedback coefficients for the first two eigenvectors are shown in Table 3.

Table 3: Feedback coefficients

$\hat{\boldsymbol{\alpha}}$	1	2
R_3	−0.02	0.11
S	0.03	−0.28
$R_b - R_3$	−0.01	0.13
D	−0.03	−0.14
O	−1.80	−1.08

Figure 8 shows the time series of the 'cointegrating combination' and the UK longer-leading CLI: they are not highly correlated, and often not in phase.

Finally, we consider the restrictions placed on the leading indicators by their combination in the CLI, $I_{L,t}^{UK}$, as in (14). If the five-dimensional VAR is modelled parsimoniously, it becomes nearly diagonal, so no single vector with all of its components non-zero could capture all the available information for predicting the component indicators. Replacing R_3 by $I_{L,t}^{UK}$, the test of $\mathbf{c}_{12} = \mathbf{0}$ accepts, so no additional variables help predict the CLI. However, $\mathbf{c}_{22} = \mathbf{0}$ is strongly rejected, with seven coefficients having t-values in excess of 2.7 in absolute value, so the CLI is not fully informative. Conversely, both $c_{11} = 0$ and $\mathbf{c}_{21} = \mathbf{0}$ are strongly rejected so the CLI does capture some of the dynamic behaviour.

7 The use of leading indicators in macro-economic models

In this section, we consider the use of indices, or leading indicators, in macro-economic modelling and forecasting. When modellers use an index, instead of all of the components separately, they are assuming that the way in which the index enters the model matches the way the components also enter the model. Testing the significance of an index involves testing the joint hypothesis of the components entering the model and having the same relationship in the model as in the index. Thus, the insignificance of the index does not preclude that some (or all) of the components of the index belong in the model, albeit with different coefficients than those in the index.

We use a simple example to illustrate the general point, examining the role of g_t in a VAR for a subset of variables $\mathbf{y}_t = \mathbf{F\Psi x}_t$ none of which are the indicators $\mathbf{i}_t$ or the activity index. From the DGP:

$$\mathbf{x}_t = \boldsymbol{\tau} + \boldsymbol{\Upsilon}\mathbf{x}_{t-1} + \mathbf{u}_t \tag{33}$$

where $g_t = \boldsymbol{\varphi}'\mathbf{x}_t$, using $\mathbf{y}_t = \mathbf{F\Psi x}_t$ then:

$$\mathbf{y}_t = \mathbf{F\Psi}\boldsymbol{\tau} + \mathbf{F\Psi\Upsilon x}_{t-1} + \mathbf{F\Psi u}_t = \mathbf{b} + \mathbf{B y}_{t-1} + (\mathbf{F\Psi\Upsilon} - \mathbf{BF\Psi})\,\mathbf{x}_{t-1} + \mathbf{F\Psi u}_t. \tag{34}$$

We denote the postulated initial model by:

$$\mathbf{y}_t = \mathbf{b} + \mathbf{B}\mathbf{y}_{t-1} + \zeta_t \tag{35}$$

where $\boldsymbol{\zeta}_t = (\mathbf{F\Psi\Upsilon} - \mathbf{BF\Psi})\,\mathbf{x}_{t-1} + \mathbf{F\Psi u}_t$. From (35):

$$\mathsf{E}\left[\mathbf{y}_t \mid \mathbf{x}_{t-1}\right] = \mathbf{b} + \mathbf{B}\mathbf{y}_{t-1} + (\mathbf{F\Psi\Upsilon} - \mathbf{BF\Psi})\,\mathbf{x}_{t-1} \tag{36}$$

so, in general, information will be lost by modelling only the subset $\mathbf{y}_t$.

Consider adding g_{t-1} to (35) when $(\mathbf{F\Psi\Upsilon} - \mathbf{BF\Psi}) \neq \mathbf{0}$:

$$\mathbf{y}_t = \mathbf{b}^* + \mathbf{B}^*\mathbf{y}_{t-1} + \boldsymbol{\delta} g_{t-1} + \zeta_t^* \tag{37}$$

and testing $\boldsymbol{\delta} = \mathbf{0}$. From (34) and (17):

$$\mathbf{y}_t = \mathbf{b} + \mathbf{B}\mathbf{y}_{t-1} + (\mathbf{F\Psi\Upsilon} - \mathbf{BF\Psi})\,\mathbf{P}^{-1}\boldsymbol{\Xi}\begin{pmatrix} g_{t-1} \\ \mathbf{i}^*_{t-1} \\ a_{t-1} \\ \boldsymbol{\Psi}\mathbf{x}_{t-1} \end{pmatrix} + \mathbf{F\Psi u}_t \tag{38}$$

so the implicit restrictions for reducing (38) to (37) are similar to those discussed above. The consequences of invalid reduction include biased coefficients and standard errors, inefficient estimation due to the omitted variables, and potential loss of invariance to changes elsewhere in the economy.

The restrictions imposed by the index on the data used in their construction are rarely tested. The indices are constructed (by a governmental statistical agency) outside of the economic model where they are later used (by other economists). This implicitly reduces the parameter space of the model. Using an index in a model is similar to principal components, and may lead to the standard errors being downward biased when used in an economic model.[8] The issue now is that the vector $\boldsymbol{\varphi}$ defining g_t is data-based by selecting the historically high correlations with $\mathbf{x}_{t-1}$, but $\boldsymbol{\varphi}$ is taken as known when g_t is used in the model. The standard errors are usually underestimated in such a setting, so the forecast performance may be less good than anticipated. This difficulty is exacerbated by the fact that either the components or the method of construction of the indices change with almost every business cycle. Thus, although the entire time series is altered to be like the new index once the changes are made, the index is not optimal for each sub-period. Finally, the issue of cointegration between the components of the index and the components of the model arises again, and can be analyzed as in section 4.

8 The role of the UK longer-leading indicator in a monetary system

To illustrate the role of a leading indicator in a linear dynamic system, we now consider the four-equation monetary model analyzed by Hendry and Mizon (1993) and Hendry and Doornik (1994), and also investigated by Boswijk (1992), Ericsson, Campos and Tran (1990) and Johansen (1992) *inter alia.* The data-set comprises M, Y, P, and R_n which are respectively nominal M_1, real total final expenditure (TFE) at 1985 prices, the TFE deflator and the (learning adjusted) differential between the three-month local authority interest rate (R_a) and the M_1 retail sight-deposit interest rate (R_m, the rate on interest-bearing checking accounts at commercial banks, introduced in 1984(3)). R_n is a measure of the opportunity cost of holding M_1 (see Hendry and Ericsson (1991) for details). Money and expenditure are in £million, the deflator is unity in 1985, and the interest rate is annual, in fractions. Lower case letters denote logs of the corresponding capitals. The data are quarterly, seasonally adjusted for 1963(1)–1989(2), but after allowing for the leading-indicator sample period, estimation is usually over 1976(1)–1989(2). We adopted the specification in Hendry and Doornik (1994) and so analyzed $m - p$, y, Δp, and R_n, adding $I^{UK}_{L,t}$ to create a five-variable VAR. A lag length of two was selected given the small sample.

Figure 9 shows the time-series graph of $I^{UK}_{L,t}$ and the two cointegration vectors (denoted by c_{1t} and c_{2t}) defined by Hendry and Doornik (1994) as:

$$c_{1t} = (m_t - p_t) - y_t + 7.0\Delta p_t + 7.0R_{nt} \quad \text{and} \quad c_{2t} = y_t - 0.0063t - 3.4\Delta p_t + 1.8R_{nt}. \tag{39}$$

[8] See Judge *et al.* (1985) pp. 909–12 for a discussion of principal components regression.

The first is interpreted as the excess demand for money (deviation from the long-run money-demand equation) and the second the excess demand for goods. The overall pattern of behaviour is similar, and the correlations of $I_{L,t}^{UK}$ with c_{1t} and c_{2t} are both about 0.55.

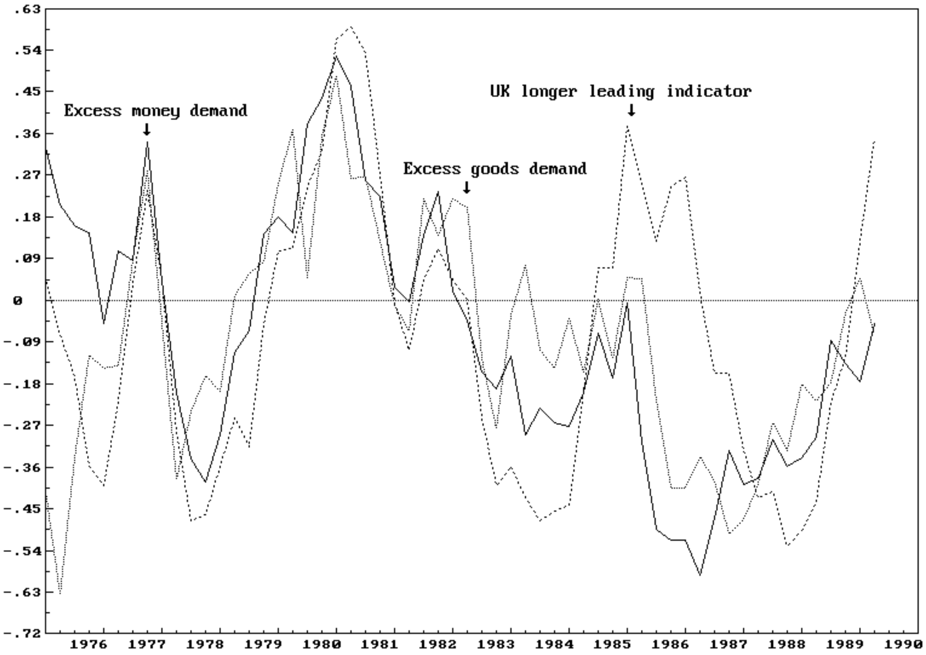

Fig. 9 UK longer-leading index and cointegration vectors

Following a similar reduction sequence to Hendry and Doornik (1994) leads to the parsimonious representation shown in Table 4.

Table 4: FIML estimates with $I_{L,t}^{UK}$

$$
\begin{aligned}
\Delta (m-p)_t &= -\underset{(0.41)}{1.18}\ \Delta^2 p_t - \underset{(0.37)}{0.74}\ \Delta R_{nt} - \underset{(0.007)}{0.085}\ c_{1t-1} + \underset{(0.002)}{0.005} \\
\Delta y_t &= \underset{(0.007)}{0.064}\ D_{out} - \underset{(0.023)}{0.17}\ c_{2t-1} + \underset{(0.001)}{.0068} \\
\Delta^2 p_t &= \underset{(0.005)}{0.011}\ D_{oil} + \underset{(0.028)}{0.12}\ c_{2t-1} - \underset{(0.009)}{0.031}\ I_{L,t-1}^{UK} - \underset{(0.005)}{0.024}\ c_{1t-1} \\
\Delta R_{nt} &= \underset{(0.10)}{0.27}\ \Delta R_{nt-1} - \underset{(0.036)}{0.069}\ c_{2t-1} \\
I_{L,t}^{UK} &= \underset{(0.09)}{1.33}\ I_{L,t-1}^{UK} - \underset{(0.09)}{0.45}\ I_{L,t-2}^{UK} + \underset{(0.005)}{0.012}
\end{aligned}
$$

In Table 4, D_{out} and D_{oil} are dummy variables for output and oil-price shocks.

The likelihood-ratio test of the over-identifying restrictions on the original VAR yields $\chi^2(39) = 39.9$ which is insignificant. Thus, the reduction is valid, and leads to a somewhat different specification. In particular, the leading indicator is not significant in the income equation, although 'leading' income is one of the main rationales for the CLI. However, it is significant in the inflation equation, and induces the additional significance of c_{1t-1}, thereby violating weak exogeneity of $\Delta^2 p_t$ for a conditional money-demand model. The parameter estimates in the money-demand equation become less well determined, but the impact of c_{2t-1} on both Δy_t and $\Delta^2 p_t$ is now stronger.

Table 5 records the residual standard deviations which are generally smaller than in Hendry and Doornik (1994). Thus, there is some information in the longer-leading index that explains the previous error in the inflation equation, but otherwise does not greatly influence the system. Unfortunately, the sample is too short to sustain unrestricted modelling of the component indicators as well as the monetary data, so we are unable to determine which individual indicator variables contribute most.

Table 5: Residual standard deviations

$$\begin{bmatrix} \Delta(m-p)_t & \Delta y_t & \Delta^2 p_t & \Delta R_{nt} & I^{UK}_{L,t} \\ 1.30\% & 0.78\% & 0.63\% & 0.016 & 0.0020 \end{bmatrix}$$

If the cointegrating combination index is used in place of the longer-leading CLI, no reduction is significant from a VECM to a model thereof, and the final selection is close to that found using the CLI. The significance of the excess demand vector in the inflation equation is further emphasised, but most other coefficients are about the same.

If we allow both the cointegrating combination index and the CLI to be elements of a VAR, only the former enters significantly as a level, forcing the CLI to insignificance.

A dramatically different answer ensues if the economic cointegration vectors (c_{1t} and c_{2t}) are deleted, and only the two measures of 'composite leading indicators' are used. Now the longer-leading CLI lagged is significant in every equation and the cointegrating combination version is irrelevant. This may reflect the method of selecting the CLI to lead within sample. However, it seems as if a DVAR can benefit from adding a CLI, possibly as a proxy for the omitted cointegration vectors, even if a VECM cannot be much improved. Of course, the over-identification test for dropping c_{1t} and c_{2t} is highly significant against the original cointegrated VAR, and a valid reduction is hard to obtain.

Such results are far from encouraging for the use of CLIs in DVARs. Ex post, an apparent improvement is seen, but appears to derive from proxying the cointegration vectors. Ex ante, however, regime shifts in the CLIs will cause the DVAR with CLIs to lose its relative robustness (see Clements and Hendry, 1994a). Thus, the augmented DVAR may get the worst of both worlds, and will usually perform worse than a pure DVAR for a shift in the long-run equilibrium.

9 Conclusion

The US Department of Commerce indicator system is altered sufficiently frequently to suggest that historical leading indicators do not in practice systematically lead for long. It is hard to be sure precisely what the indices aim to track (a 'reference' or business cycle or 'aggregate economic activity' or 'co-movements in a broad range of macroeconomic aggregates'). Issues of cointegration need to be tackled when constructing the index and choosing the component series. Several of the papers discussed above provide an excellent starting point for these tasks.

The UK indicator system is also revised frequently, although it is clear what the indicators are intended to track. However, the amplitude adjustment adopted leaves room for ad hoc judgment.

Although there is much work that can and will be done on the topic in the near future, four hurdles remain when trying to construct a leading index. First, the causes of business cycles change over time. Secondly, the relationships between economic variables change over time: both the size and timing of relationships change. Thirdly, due to both data revisions and the method of dating business-cycle turning points, that a turn has occurred will not become apparent until some time after the event, and turns more than a few years old can have their dating changed.

The empirical evidence confirms the theoretical analysis: information is lost by restricting the way the component indicators enter the CLI, and the weights do not lie in the cointegration space. Nevertheless, the UK longer-leading indicator appears to be stationary, and helps explain some of the variance of inflation in a VECM. It is significant in every equation of a DVAR, so acts as a proxy for the omitted cointegration effects, but is a poor proxy in as much as the test of that reduction is highly significant.

Lastly, 'aggregate economic activity' or 'business cycles' are not sufficiently precise concepts to provide a rigorous basis for evaluating the performance of leading indicators. On the positive side, the selection of indicators could be made more formal. This would involve testing for information losses due to reduction, including cointegration analyses, parametric restrictions and parameter non-constancy. Such a step would also induce a greater convergence between leading-indicator approaches and econometric models.

References

Alexander, S.S. (1958), 'Rate of Change Approaches to Forecasting-Diffusion Indexes and First Difference', *Economic Journal*, 68, 288-301.

Artis, M.J., R.C. Bladen-Hovell, D.R. Osborn, G.W. Smith and W. Zhang (1993), 'Turning Point Prediction in the UK: Preliminary Results Using CSO Leading Indicators', mimeo, University of Manchester.

Auerbach, A.J. (1982), 'The Index of Leading Indicators: Measurement Without Theory Thirty-five Years Later', *The Review of Economics and Statistics*, 64, 589-595.

Baba, M., N. Nomura and S. Tahara (1983), 'Major Features of New Business Cycle Indicators in Japan', 605-620 in K.H. Oppenlaender and G. Poser (eds.) *Leading Indicators and Business Cycle Surveys, Papers presented at the* 16^{th} *CIRET Conference Proceedings.*

Banerjee, A., Dolado, J.J., Galbraith, J.W. and Hendry, D.F. (1993), *Co-integration, Error Correction and the Econometric Analysis of Non-Stationary Data.* Oxford: Oxford University Press.

Beckman, B.A. and T.R. Tapscott (1989), 'Composite Indexes of Leading, Coincident, and Lagging Indicators', *Survey of Current Business*, 67, 24-28.

Bladen-Hovell, R.C. and W. Zhang (1992), 'A BVAR Model for the UK Economy: A Forecast Comparison with LBS and NI Models,' mimeo, University of Manchester.

Boswijk, H.P. (1992), *Cointegration, Identification and Exogeneity.* Amsterdam: Tinbergen Institute Research Series, Vol. 37.

Burns, A.F. and W.C. Mitchell (1946), *Measuring Business Cycles*, National Bureau of Economic Research.

Campbell, J.Y. and N.G. Mankiw (1987), 'Are Output Fluctuations Transitory?' *Quarterly Journal of Economics*, vol.102, pp. 857-880.

Central Statistical Office (1975), 'Cyclical Indicators for the United Kingdom Economy', *Economic Trends*, 257, 95-99.

Central Statistical Office (1976), 'Changes to the Cyclical Indicator System', *Economic Trends*, 271, 67-69.

Central Statistical Office (1979), 'Cyclical Indicators for the UK Economy,' *Economic Trends*, No.3, pp.70-80.

Central Statistical Office (1980), 'Cyclical Indicators: some Developments and an Assessment of Performance,' *Economic Trends*, No.3, pp.82-87.

Central Statistical Office (1983), 'Output Measures: Calculation and Interpretation of the Cyclical Indicators of the U.K. Economy,' Occasional Paper No.16 (Revised), October.

Central Statistical Office (1989), 'Table B,' *Economic Trends*, 433, 82.

Chow, G.C. and G.H. Moore (1972), 'An Econometric Model of Business Cycles', 739-781 in *Econometric Models of Cyclical Behavior, Volume II*, Columbia University Press.

Clements, M.P. and D.F. Hendry (1993), 'Forecasting in Cointegrated Systems,' mimeo, University of Oxford.

Clements, M.P. and Hendry, D.F. (1994), 'Towards a Theory of Economic Forecasting', in C. Hargreaves (ed.), *Non-stationary Time-Series Analyses and Cointegration*, Oxford: Oxford University Press.

Clements, M.P. and Hendry, D.F. (1994a), 'Intercept Corrections and Structural Change', unpublished paper, Institute of Economics and Statistics, Oxford.

Coen, P.G., E.D. Gomme and M.G. Kendall (1969), 'Lagged Relationships in Economic Forecasting', *Journal of the Royal Statistical Society,* A, 132, 133-63.

Diebold, F.X. and G.D. Rudebusch (1989), 'Scoring the Leading Indicators', *Journal of Business*, 62, 369-391.

Diebold, F.X. and G.D. Rudebusch (1991a), 'Turning Point Prediction with the Composite Leading Index: An Ex Ante Analysis', 231-56 in K. Lahiri and G.H. Moore (eds.) *op. cit.*

Diebold, F.X. and G.D. Rudebusch (1991b), 'Forecasting Output with the Composite Leading Index: A Real-Time Analysis', *Journal of the American Statistical Association*, 86, 603-10.

Doornik, J.A. and Hendry, D.F. (1994), *PcFiml version 8: An Interactive Econometric Modelling System*, London: International Thomson Publishing.

Engle, R.F. and Granger, C.W.J. (1987), 'Cointegration and Error Correction: representation, estimation and testing', *Econometrica*, **55**, 251-76.

Ericsson N.R., Campos, J., and Tran, H-A . (1990), 'PC-GIVE and David Hendry's Econometric Methodology', *Revista De Econometria*, **10**, 7-117.

Granger, C.W.J. (1969), 'Investigating Causal Relations by Econometric Models and Cross-Spectral Methods', *Econometrica*, **37**, 424-38.

Hendry, D.F. and M.P. Clements (1993), 'On Model Selection when Forecasting,' mimeo, University of Oxford.

Hendry, D.F. and J.A. Doornik (1994), 'Modelling Linear Dynamic Econometric Systems', in *Scottish Journal of Political Economy*, **41**.

Hendry, D.F. and Ericsson, N.R. (1991b), 'Modeling the Demand for Narrow Money in the United Kingdom and the United States', *European Economic Review*, **35**, 833-86.

Hendry, D.F. and G.E. Mizon (1993), 'Evaluating Dynamic Econometric Models by Encompassing the VAR', in P.C.B. Phillips (ed.), *Models, Methods and Applications of Econometrics*. Oxford: Basil Blackwell.

Hendry, D.F. and M.S. Morgan (1994), *The Foundations of Econometric Analysis*, Cambridge University Press.

Hertzberg, M.P. and B.A. Beckman (1989), 'Business Cycle Indicators: Revised Composite Indexes', *Business Conditions Digest*, January, 97-102.

Hymans, S. (1973), 'On the Use of Leading Indicators to Predict Cyclical Turning Points', *Brookings Papers and Economic Activity*, 2, 339-384.

Johansen, S. (1988), 'Statistical Analysis of Cointegration Vectors', *Journal of Economic Dynamics and Control*, **12**, 231-54.

Johansen, S. (1992), 'Testing Weak Exogeneity and the Order of Cointegration in UK Money Demand', *Journal of Policy Modeling*, **14**, 313-34.

Johansen, S. and K. Juselius (1990), 'Maximum Likelihood Estimation and Inference on Cointegration – With Applications to the Demand for Money', *Oxford Bulletin of Economics and Statistics*, 52, 169-210.

Judge, G.G., Griffiths, W.E., Hill, R.C., Lütkepohl, H. and Lee, T-C. (1985), *The Theory and Practice of Econometrics*, New York: Wiley.

Koch, P.D. and R.H. Rasche (1988), 'An Examination of the Commerce Department Leading-Indicator Approach', *Journal of Business & Economic Statistics*, 6, 167-187.

Koopmans, T.C. (1947), 'Measurement without Theory', *The Review of Economics and Statistics*, 29, 161-172.

Lahiri, K. and G.H. Moore (eds.) (1991), *Leading Economic Indicators: New Approaches and Forecasting Records*, Cambridge University Press.

Lesage, J.P. (1992), 'Scoring the Leading Indicators: A Bayesian Turning Points Approach,' *Journal of Forecasting*, vol. 11, pp. 35-46.

Marsland, J. and M. Weale (1992), 'The Leading Indicator in a VAR Model of the UK,' mimeo, Downing College and Clare College, Cambridge.

Mitchell, W.C. and A.F. Burns (1938), *Statistical Indicators of Cyclical Revivals*, National Bureau of Economic Research.

Moore, B. (1993), 'A Review of CSO Cyclical Indicators,' *Economic Trends*, No. 477, pp. 99-107.

Moore, G.H. (1950), *Statistical Indicators of Cyclical Revivals and Recessions*, National Bureau of Economic Research, New York.

Moore, G.H., ed. (1961), *Business Cycle Indicators*, 2 vols, National Bureau of Economic Research, New York.

Moore, G.H.(1969), 'Generating Leading Indicators from Lagging Indicators,' *Western Economic Jour-*

nal, vol. 7, no.2, pp.135-44.

Moore, G.H. and J. Shiskin (1967), *Indicators of Business Expansions and Contractions*, National Bureau of Economic Research Occasional Paper 103, National Bureau of Economic Research, New York.

Morgenstern, O. (1959), 'Security Markets and Foreign Capital Issues', 507-561 in *International Financial Transactions and Business Cycles.*

Neftci, S.N. (1979), 'Lead-Lag Relations, Exogeneity and Prediction of Economic Time Series', *Econometrica*, 47, 101-113.

Neftci, S.N. (1982), 'Optimal Prediction of Cyclical Downturns,' *Journal of Economic Dynamics and Control*, vol. 4, 225-241.

Neftci, S.N. (1984), 'Are Economic Time Series Asymmetric over the Business Cycle?' *Journal of Political Economy*, 92, 307-328.

Nelson, C.R. and Plosser, C.I. (1982), 'Trends and random walks in macroeconomic time series: some evidence and implications', *Journal of Monetary Economics*, **10**, 139-62.

Niemira, M.P. (1991), 'Using Composite Leading Indicators of Consumption to Forecast Sales and to Signal Turning Points in the Stock Market', in K. Lahiri and G.H. Moore (eds.) *op. cit.* Press.

Osterwald-Lenum, M. (1990), 'Recalculated and Extended Tables of the Asymptotic Distribution of Some Important Maximum Likelihood Cointegration Test Statistics', *Oxford Bulletin of Economics and Statistics.*

Pain, N. (1992), 'The UK Economy,' *National Institute Economic Review*, May, pp. 8-25.

Persons, W. M. (1924), *The Problem of Business Forecasting.* Pollak Foundation for Economic Research Publication, 6. London: Pitman.

Phillips, P.C.B. (1991), 'Optimal Inference in Cointegrated Systems,' *Econometrica*, vol. 59, no.2, pp. 283-306.

Phillips, P.C.B. and S.N. Durlauf (1986) 'Multiple Time Series Regression with Integrated Processes', *Review of Economic Studies*, **53**, 473-95.

Samuelson, P.A. (1987), 'Paradise Lost and Refound: The Harvard ABC Barometers', *Journal of Portfolio Management*, 4, 4-9.

Shiskin, J. (1961), 'Signals of Recession and Recovery: An Experiment with Monthly Reporting,' National Bureau of Economic Research Occasional Paper 77, New York.

Shiskin, J. (1967), 'Reverse Trend Adjustment of Leading Indicators', *The Review of Economics and Statistics*, 49, 45-49.

Shiskin, J. and G.H. Moore (1968), 'Composite Indexes of Leading, Coinciding and Lagging Indicators, 1948-1967,' A Supplement to National Bureau Report 1, National Bureau of Economic Research, New York.

Steckler, H.O. (1961), 'Diffusion Index and First Difference Forecasting', *The Review of Economics and Statistics*, 43, 201-208.

Steckler, H.O. and M. Schepsman (1973), 'Forecasting with an Index of Leading Series', *Journal of the American Statistical Association*, June, 291-296.

Stock, J.H. and M.W. Watson (1989), 'New Indexes of Coincident and Leading Economic Indicators', *NBER Macroeconomics Annual 1989*, 351-409.

Stock, J.H. and M.W. Watson (1992), 'A Procedure for Predicting Recessions with Leading Indicators: Econometric Issues and Recent Experience', *NBER Working Paper,* 4014.

Toda, H.Y. and P.C.B. Phillips (1993), 'Vector Autoregressions and Causality,' *Econometrica*, 61, no.6, 1367-1394

U.S. Department of Commerce (1969), *Business Conditions Digest*, November, pp.i-iii, 1-8, 107

U.S. Department of Commerce (1977), 'Composite Indexes: A Brief Explanation and the Method of Construction,' *Handbook of Cyclical Indicators*, pp.73-76.

U.S. Department of Commerce (1979), *Business Conditions Digest*, March, pp. iii and iv.

U.S. Department of Commerce (1983), *Business Conditions Digest*, February, pp. iii-iv, 108-109.

U.S. Department of Commerce (1984), *Business Conditions Digest*, March, pp.107-109.

U.S. Department of Commerce (1984), *Handbook of Cyclical Indicators.*

Vining, R. (1949), 'Koopmans on the Choice of Variables to be Studied and of Methods of Measurement', *The Review of Economics and Statistics*, 31, 77-86.

Weller, B.R. (1979), 'Usefulness of the Newly Revised Composite Index of Leading Indicators as a Quantitative Predictor', *Journal of Macroeconomics*, 1, 141-147.

Zarnowitz, V. and C. Boschan (1977a), 'Cyclical Indicators: An Evaluation and New Leading Indexes', *Handbook of Cyclical Indicators*, 170-183.

Zarnowitz, V. and C. Boschan (1977b), 'New Composite Indexes of Coincident and Lagging Indicators', *Handbook of Cyclical Indicators*, 185-198.

Zarnowitz, V. and P. Braun (1992), 'Major Macroeconomic Variables and Leading Indicators Some Estimates of Their Interrelations, 1886-1982,' NBER Working Paper #2812.