Questions for Micro: Bargaining and Contracts
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Example of a short question—for Part A of the Micro exam

0) In an optimal contracting setting, an uninformed principal offers a contract to an agent who is privately informed about his cost of producing output. The agent is either type 1 or type 2, where for type 1, the cost of producing output x is just x, while for type 2, the cost of producing x is $2x$. The agent's utility is the difference between his payment and the cost he incurs in producing output. A contract specifies, for each output level x the agent might choose, what payment t he will receive.

a) Contract C_1 specifies that if the agent chooses $x = 2$, then $t = 4$, while if the agent chooses $x = 5$, then $t = 8$. Contract C_2 specifies that if the agent chooses $x = 2$, then $t = 4$, while if the agent chooses $x = 4$, then $t = 5$. If the principal offers contract C_1, what level of output will each type of agent choose? If the principal offers C_2, what level of output will each type of agent choose?

b) What is a direct revelation mechanism (DRM) in this context? What does it mean for a DRM to be truthful?

c) For each contract, present a truthful DRM which will implement the same outcome as the contract.

The remaining questions are all somewhat more difficult than the standard for Part B questions on the Micro exam.

1) This question examines the effect of an increase in a player's risk aversion on his payoff in a bargaining game.

a) Players 1 and 2 bargain over how to divide a dollar. Player 2's utility function for money x is $u_2(x)$, where $u_2(\cdot)$ is increasing and strictly concave and $u_2(0) = 0$. Compare the Nash Bargaining Solutions when 1's utility function is $u_1(x) = x$ and when it is $u_1(x) = g(x)$, where $g(\cdot)$ is strictly increasing, strictly concave, and $g(0) = 0$. In each case, assume that the disagreement payoff is 0 for each player.

b) Now suppose the players bargain over the dollar by making alternating offers, with player 1 making the first offer at $t = 0$ and one period elapsing between offers. Suppose the players have no intrinsic preferences as to when agreement is reached but that, after any offer is rejected, there is a probability $q \in (0, 1)$ that the dollar will disappear. If this happens, each player's payoff is 0. Suppose that player i's utility function for money is $u_i(x)$ and that the players' preferences over lotteries satisfy the expected utility hypothesis. Let $u_2(\cdot)$ satisfy the assumptions made about it in part a). Compare the subgame perfect equilibrium payoffs when $u_1(x) = x$ and $u_1(x) = g(x)$, where $g(\cdot)$ also satisfies the assumptions made in part a).

c) Interpret your results.

2) This question analyzes the pricing strategy of a monopolist (S) selling to a single privately-informed buyer (B). B's utility function is $V(t, x, b) = bx - t$, where x is the quantity purchased, t is the payment B makes to S, and b is B's type. S's cost of producing x is 0 for $x \leq \bar{x}$ and infinite for $x > \bar{x}$. For $x \leq \bar{x}$, S's utility function is $U(t, x) = t$. The timing of the game is:

i) B learns b privately; S believes that $Prob(b = H) = q$ and $Prob(b = L) = 1 - q$.
ii) S proposes a pricing scheme $\{(x_L, t_L), (x_H, t_H)\}$.
iii) B chooses whether to accept or reject.
iv) If B rejects, he gets his reservation utility of 0; if B accepts, he chooses either (x_L, t_L) or (x_H, t_H).

a) Consider the full-information benchmark. If S knew b, what (x, t) would he propose when $b = H$? when $b = L$? Denote these pairs (x_H^*, t_H^*) and (x_L^*, t_L^*) and graph them in (x, t)-space.

b) In the game with hidden information, is the pricing scheme $\{(x_H^*, t_H^*), (x_L^*, t_L^*)\}$ incentive compatible? Why or why not? In what ways will S adjust this pricing scheme to induce truthful revelation?

c) Characterize the family of incentive compatible pricing schemes which give 0 utility to the L-type B and for which the incentive compatibility constraint for the H-type B is binding. Taking as given the fact that the optimal pricing scheme under hidden information must belong to this family, identify the optimal scheme. Does it involve allocative inefficiency? Does the H-type B derive positive surplus? Interpret the dependence of the answers on H, L, and q.

3) A risk-neutral principal, P, contracts with two identical risk-averse agents, A_1 and A_2, each of whom chooses an effort level (e_1 for A_1 and e_2 for A_2) that is unobservable by P. Each agent's wage can be based on a pair of performance indicators, z_1 and z_2, where for $i = 1, 2$, $z_i = e_i + x_i$, and x_1 and x_2 are random shocks, realized after efforts are chosen and normally distributed with mean 0, variance σ^2 and covariance $\rho\sigma^2$. Assume that $0 \leq \rho < 1$. A_i has utility function

$$-\exp\{-r[w_i - \frac{1}{2}e_i^2]\},$$

where w_i is the wage he receives. The certainty equivalent of each agent's reservation utility is 0. P has payoff function $e_1 + e_2 - w_1 - w_2$. P's objective is to choose contracts for the agents to maximize his own expected payoff, subject to satisfying the agents' participation and incentive compatibility contraints.

a) Suppose first that each agent is purely self-interested, choosing effort, given his incentive contract, to maximize the certainty equivalent of his own utility. Let P offer A_i, $i = 1, 2$, a linear contract of the form

$$w_i = \alpha + \beta(z_i + \gamma z_j),$$

where z_i is A_i's own output and z_j the output of the other agent.

i) Given this pair of contracts, what is the certainty equivalent of each agent's expected utility?

ii) For a given effort $\bar{e}$, what value of β induces both agents to choose effort $\bar{e}$?

iii) With α set optimally for P, for any given values of β and γ, what is P's expected payoff?

iv) For any given $\bar{e}$ that P wants to induce, what value of γ maximizes P's expected payoff, as given in **c.**? For any given $\bar{e}$, as the parameter ρ increases, how do P's optimal γ and his maximized expected payoff vary? Interpret your findings.

b) Now suppose the two agents are a devoted married couple who, given any incentive contracts, choose their efforts to maximize the **SUM** of their certainty equivalent utilities. Let P offer A_i, $i = 1, 2$, a linear contract of the form

$$w_i = \alpha + \beta(z_i + \lambda z_j).$$

i) What is the sum of the agents' certainty equivalents? What efforts do the two agents choose? For any given $\bar{e}$ and any given λ, what value of β induces the agents to choose $e_1 = e_2 = \bar{e}$?

ii) With α set optimally for P, for any given values of β and λ, what is P's expected payoff?

iii) For any given common effort level $\bar{e}$ that P wants to induce, what value of λ maximizes P's expected payoff, as given in **b.**? For any given $\bar{e}$, as ρ increases, how do P's optimal λ and his maximized expected payoff vary? Interpret your findings.

c) Now compare the scenarios in part **a)** (self-interested agents) and part **b)** (joint-utility-maximizing agents). Show that, for any $\bar{e} > 0$, when P chooses contract parameters optimally to induce agents to choose $e_1 = e_2 = \bar{e}$, then if $\rho < \frac{1}{2}$, P is strictly better off with joint-utility-maximizing agents, while if $\rho > \frac{1}{2}$, P is strictly better off with self-interested agents. Explain this result.

4) Milgrom and Roberts (*Economics, Organization, and Management*, 1992, Chapter 7, pp. 224-226—these pages are attached) discuss an incentive problem that arose between general and limited partners in oil and gas tax shelters in the U.S. in the early 1980's. Translate their discussion into a simple formal model, defining the relevant economic variables and specifying the preferences, information, and strategies of the players. How might the incentive problem be mitigated?

5) (**Optional**) This question asks you to model self-selection in the insurance market. An individual is subject to a random loss: with probability π_i, she will have income Y_2 only, while with probability $1 - \pi_i$, she will have income Y_1, where $Y_1 > Y_2$. The value of π_i is private information to the individual. She is strictly risk averse, with von Neumann-Morgenstern utility function U, defined on income. Thus, in the absence of insurance, her expected utility is $\pi_i U(Y_2) + (1 - \pi_i)U(Y_1)$.

Many identical, risk-neutral insurance companies compete in offering insurance. They all hold prior beliefs that $Prob(\pi_i = \pi_H) = q$ and $Prob(\pi_i = \pi_L) = 1 - q$, where $\pi_H > \pi_L$. An insurance policy specifies amounts of money, y_2 and y_1, that the individual is left with if she does and does not suffer the loss, respectively. That is, $y_2 = Y_2 - P + B$ and $y_1 = Y_1 - P$, where P is the insurance premium and B is the benefit paid if the loss occurs.

a) In (y_1, y_2) space, graph the indifference curves of the high-risk (π_H) and low-risk (π_L) individuals and the zero-profit lines for firms (for the cases where the customer is i) high-risk, ii) low-risk, and iii) high-risk with probability q).

b) Describe the timing and the nature of the strategies in a screening game in this market.

c) Describe the timing and the nature of the strategies in a signaling game in this market.

d) Briefly describe the analogs, for the insurance market, of the results obtained about the equilibria of the signaling and the screening games in the labor market. Be precise about the equilibrium concepts used.

Milgrom, Paul R., and Roberts, D. John, Economics, Organization, and Management, Englewood Cliffs: Prentice-Hall, 1992.

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However, the available data did not provide direct information on all these variables.[4] In such a situation, the next best thing is to identify instruments for the theoretical variables of interest, which are *log*(*r*), *log*(V) and *log*(C″). An *instrument* for a variable is another variable that (1) can be observed, (2) varies directly with the actual variable of interest, and (3) is uncorrelated with the other variables of interest.

To test Equation 7.7, the researchers first estimated $1 - ß$ by dividing the variation in the supplier's profits over time by the variation in their costs. These estimates were then used to tabulate $log(1/ß - 1)$ for the various firms in the sample. The risk aversion r was assumed to be inversely proportional to various measures of the size of the firm, such as the number of the firm's employees. Size variables therefore were used as instruments for $log(r)$ in the equation. The variance V in costs was estimated by determining the trend in costs and then computing the variation in actual costs around the trend over time. In theory, C″ should be inversely proportional to the scope for performance improvement by the agent. The researchers supposed that the scope was proportional to the firm's value added in the production process (in the Kawasaki and McMillan analysis) or to the firm's responsibility under the contract for supplying technology and designing parts and production processes (in the Asanuma and Kikutani analysis). These value-added and responsibility measures were used as instruments for C″ in the actual estimation. With only these instruments for the actual variables of interest, all that could be hoped for is that the signs of the coefficients in the estimated equations would be the same as predicted by the theory: The intensity of incentives ß should be greater for firms with more employees, more value added, and less variability in year-to-year performance. The empirical findings were consistent with these predictions.

The tests we have described represent only weak evidence in support of the theory. The equation whose coefficients were finally estimated was not the exact one predicted by the theory, and the instruments used are not beyond criticism. Moreover, the estimation procedure did not test whether there were other variables affecting actual choices of ß that were not predicted by the theory and, if so, how important those other variables were for understanding incentives. Nevertheless, the evidence obtained is consistent with the theory: Incentive contracts for Japanese suppliers do appear to depend on the considerations identified by the theory in the general way that the theory predicts.

APPLICATION: INCENTIVES IN OIL AND GAS TAX SHELTER PROGRAMS Another study has tested the incentive-intensity principle in the context of the organization of oil and gas tax shelters in the United States in the early 1980s.[5] At that time, many drilling operations were financed by limited partnerships. As you recall from Chapter 6, under the federal tax laws that then prevailed, the partners could often save on taxes if the limited partners paid all the costs of exploring for oil (which were tax deductible when the costs were incurred), whereas the general partner(s) paid the costs of completing wells in which oil was found (which were "capitalized costs" for tax reporting purposes). The general partner and the limited partners would then share any revenues enjoyed when oil was pumped from producing wells.

A problem with this tax-reduction scheme is that it created a difference in in-

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[4] Kawasaki and McMillan used data reported in *MITI's Census of Manufacturers (The Firm Series)* and *Surveys of Industries*. Asanuma and Kikutani limited their attention to Japanese automobile manufacturers, from whom they could obtain somewhat more detailed information.

[5] Mark Wolfson, "Empirical Evidence of Incentive Problems and Their Mitigation in Oil and Gas Tax Shelter Programs," *Principals and Agents: The Structure of Business*, J. Pratt and R. Zeckhauser, eds. (Boston: Harvard Business School Press, 1985), 101–27.

terests between the general partner, who controlled the partnership's activities, and the limited partners because each bore a different kind of expense. If a well were found to have oil, the general partner had to bear 100 percent of the cost of completing the well, but typically received only 25 percent of the oil revenues. Suppose that after the exploration costs have been sunk, a well were found to have only enough oil that the general partner would need to have a 50 percent share of revenues to recover the well-completion costs. Then, it would not be in his or her interest to complete the well, even though the full revenues would more than cover the completion costs.

Several of the prospectuses used by the general partners to attract investors described the problem quite candidly. According to one:

> A situation may arise in which the completion of an initial well (the majority of the costs of which are capitalized costs) on a prospect would be more advantageous to the limited partners than to the general partners. The situation would arise where a completion attempt on an initial well, the majority of the costs of which are paid by the general partners, could apparently result in a marginal well which would return some but not all of the completion cost incurred by the general partners but would return revenue to the limited partners.[6]

The conflict of interest described here is likely to be most severe when many of the wells being drilled are "marginal" prospects. If the well that is found is a gusher, then even the 25 percent of revenues accruing to the general partner would make completion of the well highly profitable. The general partner seen as the agent of the limited partners, therefore, is most likely to be responsive to completion incentives—to have his or her behavior positively affected by explicit incentives—when many of the wells to be completed are marginal ones. No explicit incentives for completing wells are necessary when they are very productive, and giving such incentives would not have much effect on the general partner's behavior. Economic theory predicts that the contracts that are actually used should be responsive to this difference in completion incentives.

To test this theory, the researcher divided drilling programs into three types: exploratory programs, developmental programs, and balanced programs. *Exploratory drilling programs* were ones in which wells were drilled in new areas, where the greatest likelihood was that no oil would be found but any wells that were found were unlikely to be marginal. In these programs, the conflict between the general and limited partners' interests in completing wells was likely to be small, and the general partners' completion decision was likely to be little affected by any special contractual incentives. Some 96 percent of the money invested in these exploratory drilling programs in the sample was in contracts that were designed to minimize taxes, with no special allowances to improve the general partners' completion incentives. *Developmental drilling programs* were ones in which all drilling occurred in an area that had been previously explored and where oil was known to be present, but where no more major finds were expected. Many developmental wells turn out to be marginal wells, so we should expect that the general partner would have been quite responsive to incentives to complete these wells. The researcher found that only 23 percent of the money invested in these programs was in contracts that provided no completion incentives. For *balanced drilling programs*, which contained a mix of exploratory and developmental wells, the corresponding figure was 37 percent.

This evidence provides a useful test of one aspect of the incentive-intensity

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[6] *Prospectus of the Hilliard Fund* (1982), 22, as quoted by Wolfson.

principle. The impact of any given monetary incentive on the agent's behavior varies with circumstances, and the principle predicts that incentives will be more intense and more often incorporated into contracts when the agent's responsiveness to them is high. The evidence in this case generally confirms the prediction of the principal-agent model: Incentives are provided when they are likely to make a difference.

The Monitoring Intensity Principle

So far, we have assumed that the measurement of performance is outside the scope of the model; that is, the variance V with which efforts are measured has been treated as outside the employer's control (other than through the determination of γ). Often, however, it is possible for an employer to improve measurement by devoting resources to that objective. For example, in a factory, the number of workers per supervisor could be reduced to allow closer monitoring, or more quality-control tests could be made. For service workers, customers could be interviewed to learn whether they were satisfied with the service. In a telephone ordering or service operation, call-counting and timing equipment could be installed or supervisors could listen in on incoming calls to see how well they are handled. All of these things are costly, but all improve the employer's information about how employees are performing.

To investigate how much should be spent on monitoring, suppose that the variance of the performance measure can be controlled at a cost. Let $M(V)$ be the minimum amount that must be spent on monitoring needed to achieve an error variance as low as V. It is generally costly to reduce the error variance, so we suppose that M is a *decreasing* function—settling for a larger V entails lower monitoring costs. We also suppose that the marginal cost of variance reduction is a rising function, that is, $M'(V)$ is increasing. Rewriting Equation 7.4b to include the cost of the resources that are spent on measurement, we have:

$$\text{Total Certain Equivalent} = P(e) - C(e) - \tfrac{1}{2}rV\beta^2 - M(V) \qquad (7.8)$$

The relationship between e and β is still determined by the incentive constraint Equation 7.5, which is unaffected by the introduction of costly measurement. We may therefore hold e and β fixed and choose V to maximize the expression in Equation 7.4b. Taking the derivative of Equation 7.8 with respect to V leads to:

$$-\tfrac{1}{2}r\beta^2 - M'(V) = 0 \qquad (7.9)$$

According to this equation, the marginal cost of reducing V, which is $-M'(V)$—a positive number—must be equal to $\frac{1}{2}r\beta^2$ at the efficient solution.

> **The Monitoring Intensity Principle:** Comparing two situations, one with β set high and another with β set lower, we find that V is set lower and more resources are spent on measurement when β is higher: When the plan is to make the agent's pay very sensitive to performance, it will pay to measure that performance carefully.

The determination of V is illustrated in Figure 7.3. The downward sloping curve gives the marginal cost of reducing the variance with which performance is measured. Because the risk premium is $\frac{1}{2}\, r\beta^2V$, the marginal cost of variance changes is depicted in the figure by a solid line at level $\frac{1}{2}r\beta^2$. When the incentive intensity is reduced from β to $\bar{\beta}$, the chosen level of V increases: Fewer resources are spent on measurement.

There may appear to be some circularity in our several observations. In the incentive-intensity principle, we claim that β should tend to be set large when V is low. In the last paragraph, we claim that firms should try to reduce V when β is large.