Transactions with Hidden Action: Part 2

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Incentives for multiple activities

Many jobs involve multiple activities.

Incentive contracts influence not just the *total* amount of effort exerted by an agent but also how an agent allocates effort *among* different activities.

Examples of multiple activities typically performed within a single job:

- producing quantity and producing quality
- producing output and maintaining machines/tools
- short-term and long-term R+D
- teaching and research
- producing output and mentoring younger colleagues

Incentives for multiple activities

Extend the LEN model: let e_i = A's effort on activity i, $i = 1, 2$

- $U(w, e_1, e_2) = -\exp\{-r[w - C(e_1 + e_2)]\}$
- Cost function $= C(e_1 + e_2)$, where $C'(0) = 0$ and $C''(\cdot) \geq 0$
  - e_1 and e_2 are perfect substitutes with respect to A's cost, for ex. e_1 and e_2 may represent time allocations
- measured performance on task i: $z_i = e_i + x_i$
  - $x_i \sim N(0, Var(x_i))$ and x_1, x_2 independent
- linear and additively separable contract:
  $w(z_1, z_2) = \alpha + \beta_1 z_1 + \beta_2 z_2 = \alpha + \beta_1 [e_1 + x_1] + \beta_2 [e_2 + x_2]$

Given $w(z_1, z_2)$, A chooses $e_1, e_2 \geq 0$ to maximize his CE:

$$\max_{e_1, e_2} \quad \alpha + \beta_1 e_1 + \beta_2 e_2 - C(e_1 + e_2) - \frac{1}{2} r \left\{ [\beta_1]^2 \, Var\,(x_1) + [\beta_2]^2 \, Var(x_2) \right\}$$

Claim: Since the marginal cost of increasing e_1 and e_2 is equal,

1. a necessary cond. for A to choose $e_1 > 0$ and $e_2 > 0$ is $\beta_1 = \beta_2$;
2. if $\beta_1 > \beta_2$, then $e_1 > 0$ and $e_2 = 0$;
3. if $\beta_1 < \beta_2$, then $e_1 = 0$ and $e_2 > 0$.

Incentives for multiple activities

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Equal Compensation Principle: Suppose efforts on two activities are perfect substitutes in A's cost function. If A's allocation of effort between these activities cannot be observed by P, then A will devote positive effort to both only if the marginal return to him is the same for each. Otherwise, A devotes no effort to the activity with the lower marginal return.

- *Note:* Equal Compensation Principle holds even if the *total* effort spent by A can be observed by P.

Multiple tasks: implications of Equal Compensation Prin.

Example: Suppose $C(e_1 + e_2) = 0$ if $e_1 + e_2 \leq \overline{e}$ and $C(e_1 + e_2) > 0$ if $e_1 + e_2 > \overline{e}$. Also assume that $Var(x_2) = \infty$ (i.e. performance on task 2 cannot be measured at all), while $Var(x_1) < \infty$. Task 1 can represent short-term R&D (improving an existing technology), while task 2 can represent long-term R&D (inventing a new technology).

- Since $Var(x_2) = \infty$ and A is risk averse, P should set $\beta_2 = 0$.
- Equal Compensation Principle $\Longrightarrow$ if $\beta_1 > 0$, then $e_2 = 0$. Hence, to induce A to devote time to an unmeasured activity, must abandon incentive pay for other activities.
- So the optimal (β_1, β_2) must be either
  1. $\beta_1 = \beta_2 = 0 \Longrightarrow$ A is willing to choose any (e_1, e_2) s.t. $e_1 + e_2 = \overline{e} \Longrightarrow$ low total effort, but no distortion in effort allocation across tasks.
  2. $\beta_1 > 0, \beta_2 = 0 \Longrightarrow e_2 = 0, e_1 > \overline{e} \Longrightarrow$ higher total effort, but distortion in effort allocation
- P's choice between 1. and 2. depends on how e_1, e_2 enter P's benefit function $B(e_1, e_2)$, which hasn't yet been specified.

Multiple tasks: extending Equal Compensation Principle

- For general cost functions $C(e_1, e_2)$, $\frac{\partial C}{\partial e_1} \neq \frac{\partial C}{\partial e_2}$, so $\beta_1 = \beta_2$ is not a necessary condition for $e_1 > 0$ and $e_2 > 0$.

- But general point is: When it is easy for A to shift effort across tasks (when efforts are highly substitutable for A), then A's choice of effort on a given task i is very sensitive to *all* of the incentive coefficients (β's) in the contract, not just to β_i.
  - This is called "*the effort substitution problem*".

- And the general implication is: If it is important to P that A devote some effort to all tasks, and performance on some tasks is more easily measured than on others, *optimal incentives on all tasks will typically be "low-powered"*.
  - That is, for a task i where performance is relatively easily measured ($Var(x_i)$ is small), the optimal β_i in the presence of the effort substitution problem will be lower than if task i were the only task.

Multi-task models of incentives: applications

Multi-task models of incentives, and the effort substitution problem they highlight, have implications (explored in Holmstrom and Milgrom, JLEO, 1991) for

1. incentive contracts: which performance indicators should be used and how intensively should they be weighted?
2. job design: how flexibly or rigidly should jobs be defined?
3. asset ownership: should a worker be an independent contractor, owning the assets he works with (tools, reputation, etc.) or an employee, not owning the assets?

Is there a tradeoff between risk and incentives?

Prendergast (JEL 1999; JPE 2002) surveys empirical research on relationship between degree of performance-related pay and measures of uncertainty:

- For compensation of executives, evidence is mixed; for compensation of non-executives, little evidence of a negative relationship.
- For sharecroppers, studies find a *positive* relationship btw. measures of risk and fraction of output received by sharecroppers.
- For managers of retail stores, studies find that in risky environments, managers are more likely to be franchisees, with strong output-based compensation, rather than employees, with weak performance incentives.

Is there a tradeoff between risk and incentives?

The theoretical model predicting a *negative* relationship between the riskiness of the environment and the strength of performance-related pay assumed:

1. P **knows what tasks** A should be performing and how hard A should be working at each.
2. P's only instrument for providing incentives is **explicit** performance-related pay.
3. P's only concern is to provide A with incentives for **effort**.
4. P's and A's are **exogenously matched**.

Relaxing the assumptions above can help explain why a positive relationship might be observed in some settings between "uncertainty" and strength of performance-related pay.

Is there a tradeoff between risk and incentives?

1. "Uncertainty" reflects how well P understands the best approach for A to take (Prendergast, 2002; Raith, 2008).
   - In well-understood environments, P knows what A should be doing, and provides incentives by monitoring A's inputs—e.g. routine tasks, minor product improvements
   - In unfamiliar or unstable environments, P delegates choice of approach to A and provides incentives by using strong performance-related pay—e.g. radical innovation
   - More precisely,
     i) the more uncertain P is about how A should do his job, the more P delegates authority to A;
     ii) the more P delegates authority to A, the greater the optimal use of incentive pay;
     iii) evidence favoring a tradeoff between insurance and incentives should be stronger when controlling for degree of delegation of authority to A.

     - DeVaro and Kurtulus (2010) find empirical support for these three hypotheses.

Is there a tradeoff between risk and incentives?

2. Greater randomness in environment $\Rightarrow$ career concerns (implicit incentives) less effective in inducing effort $\Rightarrow$ more need for explicit performance-related pay (Prendergast, 2000)

3. P may care *both* about workers' incentives for effort and about supervisors' incentives for truthfully evaluating workers' performance (evaluations used for *sorting*) (Prendergast, 2002)
   - strong performance-related pay for workers $\Rightarrow$ strong incentives for supervisors to manipulate evaluations acc. to private biases
   - in very uncertain environments, supervisors' evaluations less valuable even if truthful $\Rightarrow$ this drawback of strong incentives matters less

Is there a tradeoff between risk and incentives?

4. Endogenous matching of employees (differing in risk aversion) to firms (differing in riskiness of environment)—Ackerberg and Botticini, 2002
   - Compared to more risk-averse employees, less risk-averse employees are **both** more willing to work at riskier firms and, at any given firm, more willing to accept strong performance-related pay.
   - This creates a new force towards *positive* correlation between riskiness and strength of incentives, though the force towards *negative* correlation (for a *given* A and P) remains.
   - *Example:* With 2 As ($r_1 = 0$ and $r_2 > 0$) and 2 Ps ($V_1 = 0$ and $V_2 = 0$), total surplus higher if risk-neutral A works for risky P, facing strong incentive pay, and risk-averse A works for riskless P, facing weak incentive pay.

Explicit vs. implicit incentives

Explicit incentives: provided by explicit *contractual commitment* by principal; these have been the focus so far

Implicit incentives: arise when principal(s) has some *ex post discretion* how to respond to agent's performance; agent has *implicit incentives* to change performance to influence principal's discretionary decisions

- implicit incentives are necessarily *intertemporal* incentives

Contexts in which implicit incentives may be important:

- effort incentives for employees and managers
- incentives for firms to provide high-quality products
- incentives for cost reduction by regulated firms
- incentives for appropriate behavior by elected politicians or civil servants

Implicit incentives

Implicit incentives may arise via different mechanisms:

1. infinitely-repeated interaction can induce 'cooperative' behavior
2. learning about agent's intrinsic characteristics (e.g. ability)
   a) hidden information version: signalling
   b) hidden action version: signal-jamming

We'll study two important examples of implicit incentives: *career concerns* and *ratchet effect*

- both can be modeled as type 2a) or type 2b)
- here, model as type 2b)

Signal-jamming model of career concerns (based on Holmström, 1999)

- Players: one risk-neutral manager; many identical risk-neutral firms
- Two periods, $t = 1, 2$
- Publicly observable output $z_t = e_t + a + u_t$

  e_t = effort; privately chosen by agent (hidden action)

  a = ability; initially unknown to everyone (no hidden information)

  u_t = random shock
- Beliefs about the random variables, shared by all players:
  $a \sim N(0, Var(a))$; $u_t \sim N(0, Var(u))$; u_t iid and independent of a
- A firm hiring manager in pd t gets payoff $z_t - w_t$; otherwise, gets 0.
- Manager maximizes $E[w_1 - C(e_1) + \delta(w_2 - C(e_2))]$.
- In each pd, socially efficient e_t^* maxes $Ez_t - C(e_t)$, so $1 = C'(e_t^*)$.

Signal-jamming model of career concerns

Assume **no explicit incentive contracts**, so w_t cannot depend on z_t.

- Thus $w_1 = \overline{w}_1$.
- Firms' offers of w_2 can depend on (publicly observed) z_1, but not on z_2.
- Thus, manager has no incentive for effort in period 2, so sets $e_2 = 0$.

Conditions determining equilibrium level of e_1:

1. Manager chooses e_1 to max. $E[\overline{w}_1 - C(e_1) + \delta(w_2 - C(0))]$, given how he expects w_2 to depend on $z_1 = e_1 + a + u_1$.
2. Each firm chooses w_2 optimally, given observed z_1, given that it anticipates $e_2 = 0$, given wages it expects other firms to offer, and given its conjecture $\widehat{e}_1$ about e_1.
3. Each firm's conjecture $\widehat{e}_1$ is correct.

Signal-jamming model of career concerns: eqm analysis

- Competition btw. firms to hire manager in pd 2 yields $w_2 = E(a \mid z_1)$.
- Define τ, which measures the informativeness of output about ability:

$$\tau \equiv \frac{Var(a)}{Var(z)} = \frac{Var(a)}{Var(a) + Var(u)} = \frac{Cov(a, z)}{Var(z)} \in (0, 1).$$

- Then $E(a \mid z_1) = \tau(z_1 - E(z_1)) = \tau(z_1 - \widehat{e}_1)$.
  - Follows from OLS formulae!
- Anticipating how w_2 will be determined, manager chooses e_1 to maximize

$$E[\overline{w}_1 - C(e_1) + \delta(\tau(z_1 - \widehat{e}_1) - C(0))] = E[\overline{w}_1 - C(e_1) + \delta(\tau(e_1 + a + u_1 - \widehat{e}_1) - C(0))].$$

- Since manager's choice of e_1 is *privately observed*, manager when choosing e_1 takes firms' conjecture $\widehat{e}_1$ as *fixed*.
- So manager's FOC for e_1 is $\delta\tau = C'(e_1)$.
  - This FOC happens to be independent of $\widehat{e}_1$.
- **Hence, the equilibrium level of e_1, $\widetilde{e}_1$, satisfies $\delta\tau = C'(\widetilde{e}_1)$.**
- Since firms' conjecture $\widehat{e}_1$ must be correct in eqm, $\widehat{e}_1 = \widetilde{e}_1$.

Signal-jamming model of career concerns: eqm analysis

- **Equilibrium $\widetilde{e}_1$ satisfies $\delta\tau = C'(\widetilde{e}_1)$.**
- Socially efficient e_1^* satisfies $1 = C'(e_1^*)$.
- So if $0 < \delta \leq 1$, then since $\tau \in (0, 1)$, we conclude that $0 < \widetilde{e}_1 < e_1^*$.
- Thus, implicit incentives (career concerns) generate a positive, but socially suboptimal, level of period-1 effort.
- Implicit incentives are
  - increasing in δ (manager's concern with future wages)
  - increasing in τ (informativeness of output about ability)

Career concerns: extensions and applications

a) With more than 2 periods, the implicit incentives generated by career concerns would diminish over time as the uncertainty about ability diminished $\Longrightarrow$ expect explicit incentives to be stronger later in managers' careers (Gibbons and Murphy, 1992).

b) In the presence of implicit incentives, comparative performance evaluation can have unexpected, and undesirable, consequences (Meyer and Vickers, 1997).

c) Career concerns *may* give managers implicit incentives to do *inefficient* things, e.g.
   - maximize short-term profit at the expense of long-term profit (Stein, 1989)
   - follow the herd and ignore own information (Scharfstein and Stein, 1990)
   - forego investments that are profitable in expectation but subject their own reputations to risk (Holmström, 1999)

A hidden-action model of the "ratchet effect"

Employee (A) works for employer (P) for 2 pds; contracts last only 1 pd.

- Suppose A performs the *same* task in both periods, and performance in period 1 provides information about A's ability, or difficulty of task, or environment, and this info is useful to P for evaluating A in period 2: $Cov(z_1, z_2) > 0$.
- At start of period 2, P will want to use this information:
  - good period-1 performance (z_1) will induce P to judge A's period-2 performance (z_2) against higher standard ("ratcheting up" of standards)
  - higher z_1 implies that for any given z_2, P will attribute less to A's effort and more to ability/ease of task/environment, so that for any given z_2, w_2 will be less.
- For A,therefore, increased 1st-period performance (z_1) has 2 effects
  - increased 1st-period reward (w_1)
  - decreased 2nd-period reward (w_2) – **the ratchet effect**
- The ratchet effect is an *implicit disincentive* for period-1 effort: it arises because P designs the period-2 contract *in response to* period-1 performance.
- Applications:
  - workers paid through piece rates
  - regulated firms
  - managers in centrally planned economies

A hidden-action model of the "ratchet effect"

- Ex ante, the two periods are identical: $C_1(e) = C_2(e) \equiv C(e)$ and $B_1(e) = B_2(e) \equiv B(e)$.
- Measured performance in pd t=$z_t = e_t + x_t$, $t = 1, 2$, where e_t is A's effort in period t, and random shocks x_1 and x_2 are identically normally dist. with $Ex_t = 0$ and $Cov(x_1, x_2) > 0$.
- Suppose P, at start of each period, offers A a one-period contract, and A chooses whether to accept or reject.
- Let A's utility function be $-\exp\{-r[w_1 + w_2 - C(e_1) - C(e_2)]\}$
  - ACE becomes $E(w_1 + w_2) - C(e_1) - C(e_2) - \frac{1}{2} rVar(w_1 + w_2)$
  - let A's reservation utility in each pd, in CE terms, be 0
- Main questions:
  1. What determines the strength of the ratchet effect?
  2. Under optimal sequence of contracts, how will A's effort vary over time?
  3. Would P be better off if could commit to a two-period contract?

Sequence of 1-period contracts: 2nd period

- Sequence of one-period contracts: $w_t = \alpha_t + \beta_t z_t, \quad t = 1, 2$
  - α_2 and β_2 can depend on z_1
- Define $V_2 \equiv Var\,(z_2|z_1)$ —this is independent of z_1, given normal dists.
- In pd 2, using result from 1-pd model, P's choice of $(\beta_2^*,\ e_2^*)$ satisfies $\beta_2^* = B'(e_2^*)/\left[1 + rV_2C''(e_2^*)\right]$ and $\beta_2^* = C'(e_2^*)$
  - z_1 more informative about $x_2 \Longrightarrow$ higher $Cov(x_1, x_2) \Longrightarrow$ lower V_2, higher β_2^*, and higher e_2^*.
- In pd 2, P chooses α_2^*, given $(\beta_2^*,\ e_2^*)$, so A's participation constraint binds, given the observed value of z_1:

$$0 = ACE_2 = E(w_2|z_1) - C(e_2) - \frac{1}{2}rVar(w_2|z_1),$$

so

$$\alpha_2^* = -\beta_2 E(z_2|z_1) + k_1 = -\beta_2 \frac{Cov(x_1, x_2)}{Var(x_1)} z_1 + k_2,$$

where k_1 and k_2 are constants independent of z_1.

- **Hence pd-2 contract offer is** $w_2 = \alpha_2^* + \beta_2 z_2 = k_1 + \beta_2\left[z_2 - E(z_2|z_1)\right]$
  - A's pd-2 performance z_2 is judged against the standard $E(z_2|z_1)$

Sequence of 1-period contracts: 1st period

- At start of period 1, A anticipates total compensation

$$w_1 + w_2 = \alpha_1 + \beta_1 z_1 + k_2 + \beta_2^* \left[z_2 - \frac{Cov(x_1, x_2)}{Var(x_1)} z_1 \right]$$
$$= [\alpha_1 + k_2] + \left[\beta_1 - \beta_2^* \frac{Cov(x_1, x_2)}{Var(x_1)} \right] z_1 + \beta_2^* z_2$$

- Hence, net marginal return to e_1 is $\beta_1 - \beta_2^* \frac{Cov(x_1,x_2)}{Var\,(x_1)}$, whereas it would be β_1 in a 1-period model.
- **Ratchet effect:** For any given β_1, A's 1st-period incentives are weaker in the 2-period model than in a 1-period model. Ratchet effect measured by

$$\beta_2^* \frac{Cov(x_1, x_2)}{Var(x_1)}.$$

- P's optimal choice of β_1 takes the ratchet effect into account. Nevertheless, **P's optimal sequence of contracts induces $e_1^* < e_2^*$.**
  - One reason why $e_1^* < e_2^*$ is that $Var(z_1) > Var\,(z_2|z_1)$.
  - Another reason is that the ratchet effect makes it more costly, in terms of risk imposed on A, to provide any given level of 1st-period incentives.

Commitment and long-term contracts

- Suppose, instead, P offers A a *binding two-period* contract.
- A cares about *total compensation* $w_1 + w_2 = \alpha + \beta_1^E z_1 + \beta_2^E z_2$.
- P solves

$$\max_{(\beta_1^E, e_1), (\beta_2^E, e_2)} B(e_1) + B(e_2) - C(e_1) - C(e_2) - \frac{1}{2} r Var(\beta_1^E z_1 + \beta_2^E z_2)$$
$$\text{s.t.} \quad \beta_1^E = C'(e_1) \text{ and } \beta_2^E = C'(e_2)$$

  - This max. problem is symmetric wrt periods 1 and 2. Hence if the solution is unique, it satisfies $\beta_1^E = \beta_2^E$ and $e_1 = e_2$. Assume uniqueness.
- The (symmetric) solution can be implemented by choosing $w_t = \alpha + \beta z_t$
  - P offers the **same** wage contract in both periods, **committing** not to use 1st-period performance in determining 2nd-period pay.
- P must do at least as well if can commit to a 2-pd contract than if can write only 1-pd contracts. And, optimal 2-pd contract is *different* from optimal sequence of 1-pd contracts: former induces $e_1 = e_2$ while latter induces $e_1 < e_2$.
- **Hence, P is strictly better off if he can commit to a two-period contract: the ratchet effect lowers P's payoff, as well as social surplus, from contracting.**

The ratchet effect and hidden information

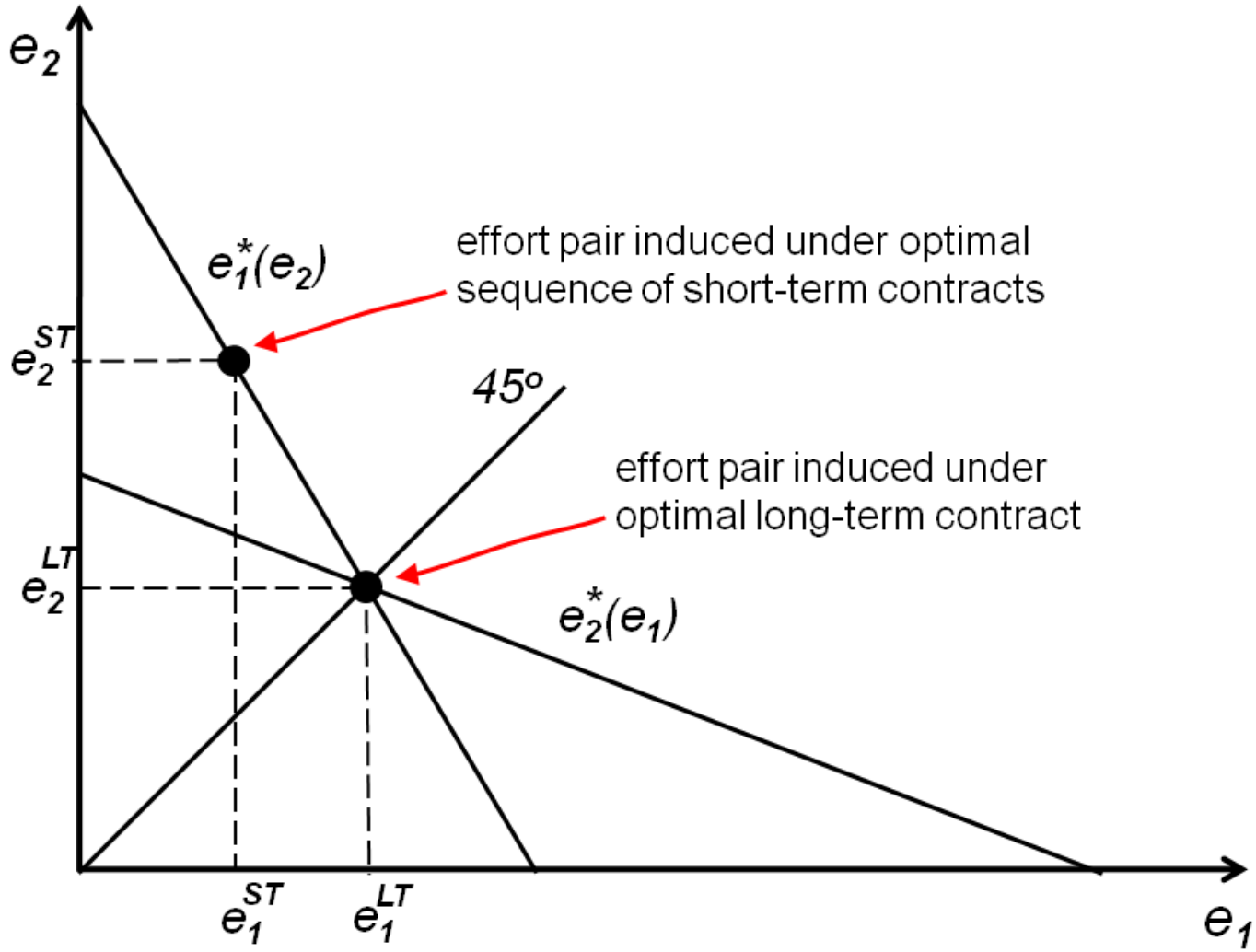
Example: regulation over 2 periods, where firm's privately-observed marginal cost $c \in \{c_L, c_H\}$ remains *constant* over time:

1. Suppose regulator can commit to a policy for only 1 period at a time.
   - In period 2, if 1st-period policy induced types c_H and c_L to separate, regulator knows c and will set $p_2 = c$.
   - So type c_L anticipates being "exploited" in pd 2 if, in pd 1, he reveals $c = c_L$—*the ratchet effect weakens incentives to reveal hidden info.*
   - Because of the ratchet effect, the optimal policy in a 1-period relationship is not optimal in 1st period of a 2-period relationship, if the regulator cannot commit for more than 1 period.

2. Suppose, instead, that regulator *can* commit, at start of period 1, to policies for both periods.
   - It is optimal for regulator to commit to use the *same* policy in both periods, and this common policy is the optimal one for a one-period relationship.

Thus, just as with hidden action, inability to commit to long-term contracts is costly: ratchet effect lowers regulator's payoff.

Ratchet effect with hidden action: supplementary notes



Ratchet effect with hidden action: supplementary notes

ST: outcome with a sequence of one-period ('short-term') contracts
LT: outcome with a 2-period ('long-term') contract

- Let $B_t(e) = e$, $C_t(e) = \frac{1}{2}e^2$, $w_t = \alpha_t + \beta_t z_t$, $t = 1, 2$
- With 2-period contract, P chooses (e_1, β_1), (e_2, β_2) to maximize

$$
\begin{aligned}
& e_1 + e_2 - \frac{1}{2}e_1^2 - \frac{1}{2}e_2^2 - \frac{1}{2}r\mathrm{Var}(w_1 + w_2) \\
& \qquad \text{s.t. } e_1 = \beta_1, e_2 = \beta_2 \\
& \Rightarrow \max_{e_1, e_2} e_1 + e_2 - \frac{1}{2}e_1^2 - \frac{1}{2}e_2^2 - \frac{1}{2}r\left(e_1^2 V + e_2^2 V + 2e_1 e_2 C\right),
\end{aligned}
$$

where $V \equiv \mathrm{Var}(x_1) = \mathrm{Var}(x_2)$, $C \equiv \mathrm{Cov}(x_1, x_2)$.

- First-order conds. (can check that these determine a unique max.):

$$
\begin{aligned}
\mathrm{FOC}(e_1) : e_1 &= \frac{1 - rCe_2}{1 + rV} \equiv e_1^*(e_2) \\
\mathrm{FOC}(e_2) : e_2 &= \frac{1 - rCe_1}{1 + rV} \equiv e_2^*(e_1)
\end{aligned}
$$

- These functions are graphed on the previous slide.

Ratchet effect with hidden action: supplementary notes

With a sequence of 1-period contracts, P chooses (e_2, β_2) at the start of period 2 to maximize

$$e_2 - \frac{1}{2}e_2^2 - \frac{1}{2}r\text{Var}(w_2|z_1) \text{ s.t. } e_2 = \beta_2$$
$$\Rightarrow \max_{e_2} e_2 - \frac{1}{2}e_2^2 - \frac{1}{2}re_2^2\text{Var}(z_2|z_1),$$

where $\text{Var}(z_2|z_1) = V - \frac{C^2}{V} \leq V$.

First-order condition:

$$\text{FOC}(e_2) : e_2^{ST} = \frac{1}{1 + r\text{Var}(z_2|z_1)}$$

Note that $e_2^{ST} \geq e_2^*(0)$ since $\text{Var}(z_2|z_1) \leq V$, and $e_2^{ST} > e_2^*(0)$ as long as $\text{Cov}(x_1, x_2) \neq 0$.

Also, $e_2^{ST} = \beta_2^{ST}$.

Ratchet effect with hidden action: supplementary notes

The main notes show that in period 1, A chooses $e_1 = \beta_1 - \frac{C}{V}\beta_2^{ST}$. So P chooses (β_1, e_1) to maximize

$$e_1 + e_2^{ST} - \frac{1}{2}e_1^2 - \frac{1}{2}(e_2^{ST})^2 - \frac{1}{2}r\text{Var}(w_1 + w_2)$$
$$\text{s.t. } e_1 = \beta_1 - \frac{C}{V}\beta_2^{ST},$$

where $e_2^{ST} = \beta_2^{ST} = \frac{1}{1+r\text{Var}(z_2|z_1)}$,

$$\begin{aligned}\text{Var}(w_1 + w_2) &= \text{Var}\left((\beta_1 - \frac{C}{V}\beta_2^{ST})z_1 + \beta_2^{ST} z_2\right) \\ &= \hat{\beta}_1^2 V + (\beta_2^{ST})^2 V + 2\hat{\beta}_1 \beta_2^{ST} C,\end{aligned}$$

and $\hat{\beta}_1 \equiv \beta_1 - \frac{C}{V}\beta_2^{ST}$.

Ratchet effect with hidden action: supplementary notes

Hence P's problem in period 1 can be written as

$$\max_{e_1} e_1 - \frac{1}{2}e_1^2 - \frac{1}{2}r(\hat{\beta}_1^2 V + (\beta_2^{ST})^2 V + 2\hat{\beta}_1 \beta_2^{ST} C) \text{ s.t. } e_1 = \hat{\beta}_1.$$

The first-order condition for this problem yields

$$e_1 = \frac{1 - rC\beta_2^{ST}}{1 + rV} = \frac{1 - rCe_2^{ST}}{1 + rV} = e_1^*(e_2^{ST}).$$

Note: With a sequence of 1-period contracts, optimal e_1 is the same *function* of e_2 as with a 2-period contract. But in the two models, the optimal values of e_2 differ, hence so do the optimal values of e_1.