Questions for "Contracts, Bargaining, and Theories of the Firm"
Microeconomics Module
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Short Question

A buyer (B) and a seller (S) of widgets bargain over the terms of trade (q, p), where q is the number of widgets produced and traded and p is the total payment from B to S. Given an agreement on terms of trade (q, p), S's payoff is $p - C(q)$, where $C(\cdot)$ is increasing and strictly convex, and B's payoff is $R(q) - p$, where $R(q)$ is increasing and strictly concave.

In the more abstract setting of the Rubinstein bargaining model, players 1 and 2 bargain over how to split a pie of maximum size normalized to 1, and feasible agreements are pairs (x_1, x_2) such that $x_1 \geq 0$, $x_2 \geq 0$, and $x_1 + x_2 \leq 1$. In any period, an efficient agreement is a feasible one in which $x_1 + x_2 = 1$.

a) Letting B be player 1 and S be player 2, show how to convert an arbitrary agreement on terms of trade (q, p) into the (x_1, x_2) notation of the Rubinstein model. Describe the set of (q, p) pairs representing, in any period, the set of efficient agreements.

A pie of the maximum size would be generated by S producing q^* widgets, where q^* solves $R'(q) = C'(q)$. An arbitrary agreement (q, p) maps into

$$x_1 = \frac{R(q) - p}{R(q^*) - C(q^*)} \qquad \text{and} \qquad x_2 = \frac{p - C(q)}{R(q^*) - C(q^*)}.$$

The set of (q, p) pairs representing the set of efficient agreements is $\{(q, p) | q = q^*, C(q^*) \leq p \leq R(q^*)\}$.

Suppose B and S play the following two-period bargaining game. B makes an offer (q_1, p_1) in the first period and, if S rejects it, S makes an offer (q_2, p_2) in the second period; if S's second-period offer is rejected, both players get 0 payoff. Both players discount second-period payoffs using discount factor $\delta < 1$.

b) If the game reaches the second period, what offer (q_2, p_2) does S make in the unique subgame perfect equilibrium (SPE) of any such subgame?

If the game reaches the second period, S offers $(q_2, p_2) = (q^*, R(q^*))$, because this offer both maximizes the size of the pie $(q = q^*)$ and is the largest offer that B is willing to accept (given that if B rejects, he gets payoff 0). This is true whatever offer was made and rejected in the first period.

c) What is B's payoff in the unique SPE of the whole game? Explain your reasoning.

If the game reaches the second period, S will offer $(q_2, p_2) = (q^*, R(q^*))$, which B will accept, yielding S a payoff of $R(q^*) - C(q^*)$. Therefore in the first period, B knows that he must offer S a payoff of at least $\delta(R(q^*) - C(q^*))$ for S to be willing to accept. It is optimal for B to offer S exactly this payoff, and to do so by proposing (q_1, p_1) with $q_1 = q^*$ (since this quantity maximizes the size of the pie). With $q = q^*$, yielding a pie of size $R(q^*) - C(q^*)$, and S getting payoff $\delta(R(q^*) - C(q^*))$, B's payoff is $(1 - \delta)(R(q^*) - C(q^*))$.

Long Question

Under "separate management of projects", headquarters (HQ) uses two different managers to oversee two distinct projects, and HQ makes funding decisions for the two projects **completely separately**. For each project, HQ chooses, on the basis of a noisy indicator z generated by that project's manager, a level of funding w for that project. Assume that $z = e + a + u$, where a is the intrinsic quality of the project, e is the level of effort privately chosen by the manager, at private cost $\frac{1}{2}ce^2$, to make the project look good, and u is a random shock. The random variables a and u are initially unknown to both the manager and HQ and are independently and normally distributed with mean 0 and variances v_a and v_u, respectively. The timing of the game (between HQ and each manager) is:

Stage 1: The manager privately chooses e to maximize $E(w) - \frac{1}{2}ce^2$, where $E(w)$ is the expected level of funding that HQ will allocate to the project.

Stage 2: HQ observes the indicator z and sets w equal to the mean of its posterior belief about a, conditional on z and on its conjecture $\hat{e}$ about the manager's choice of effort.

a) Are a manager's incentives for effort in this setting "explicit" incentives or "implicit" incentives? Explain.

A manager's incentives are implicit, since HQ is not committed as to how it will choose w in response to the level of z.

b) Given HQ's conjecture $\hat{e}$ about a manager's effort, what is a manager's optimal level of effort? What is a manager's equilibrium level of effort? Explain how the equilibrium level of effort depends on v_a/v_u, the "signal-to-noise ratio" in the noisy indicator z.

Hint: Use the statistical fact that, for $y = \theta + \epsilon$, where $\theta \sim N(0, v_\theta)$, $\epsilon \sim N(0, v_\epsilon)$, and θ and ϵ are independent, $E(\theta|y) = [\frac{v_\theta}{v_\theta + v_\epsilon}]y$.

The model in this part is isomorphic to the two-period version of Holmstrom's career concerns model which I presented in my lectures. At Stage 2, given its conjecture that

$e = \hat{e}$, HQ believes that $z - \hat{e} = a + u$. (In equilibrium this conjecture will be correct.) Define $y \equiv z - \hat{e}$. HQ, believing that $y = a + u$, computes the posterior mean of a, given y, using the formula given: $E(a|y) = [\frac{v_a}{v_a + v_u}]y$. Hence HQ at Stage 2 sets $w = [\frac{v_a}{v_a + v_u}](z - \hat{e})$. Anticipating this behavior by HQ, a manager at Stage 1 chooses e to maximize

$$E(w) - \frac{1}{2}ce^2 = E\left[\left(\frac{v_a}{v_a + v_u}\right)(z - \hat{e})\right] - \frac{1}{2}ce^2 = \left(\frac{v_a}{v_a + v_u}\right)E(a + e + u - \hat{e}) - \frac{1}{2}ce^2,$$

so the optimal e solves the FOC $\left(\frac{v_a}{v_a + v_u}\right) = ce$. This FOC says that a manager's optimal level of effort does not depend on $\hat{e}$; hence the effort that solves this FOC **is** the equilibrium level of effort.

Under "joint management of projects", a single manager oversees both projects, generating two noisy indicators, $z_1 = a_1 + e_1 + u_1$ and $z_2 = a_2 + e_2 + u_2$, where e_1 and e_2 are efforts privately chosen by the single manager, at private cost $\frac{1}{2}c(e_1)^2 + \frac{1}{2}c(e_2)^2$, to make the projects look good, a_j is the intrinsic quality of project j, $j = 1, 2$, and u_j is a random shock affecting z_j, $j = 1, 2$. The manager and HQ share a common prior belief that the random variables a_1, a_2, u_1, and u_2 are normally distributed with mean 0; that a_1 and a_2 are identically distributed with variance v_a and correlation coefficient $\eta \geq 0$; that u_1 and u_2 are identically distributed with variance v_u and correlation coefficient $\rho \geq 0$; and that for all $j, k \in \{1, 2\}$, a_j and u_k are independent. The timing of the game under "joint management of projects" is:

Stage 1: The manager privately chooses e_1 and e_2 to maximize

$$E(w_1) + E(w_2) - \frac{1}{2}c(e_1)^2 - \frac{1}{2}c(e_2)^2,$$

where $E(w_j)$ is the expected level of funding HQ will allocate to project $j = 1, 2$.

Stage 2: HQ observes the indicators z_1 and z_2 and, for each project $j = 1, 2$, sets w_j equal to the mean of its posterior belief about a_j, conditional on z_1, z_2 and on its conjectures $\hat{e}_1$ and $\hat{e}_2$ about the manager's choice of efforts.

c) Under "joint management of projects", what will be the manager's equilibrium levels of e_1 and e_2?

Hint: Note that the model is symmetric with respect to the two projects, and once again use the statistical fact stated after part **b)**.

HQ sets $w_1 + w_2 = E(a_1 + a_2|z_1, z_2)$, and given the symmetry with respect to the two projects, $E(a_1 + a_2|z_1, z_2) = E(a_1 + a_2|z_1 + z_2)$. Define $y_j = z_j - \hat{e}_j$. In equilibrium, HQ's

conjectures about e_1 and e_2 will be correct, implying that $y_j = a_j + u_j$. HQ, believing that $y_j = a_j + u_j$, computes the posterior mean of $a_1 + a_2$, given $y_1 + y_2$, using the same statistical fact as used above:

$$E(a_1{+}a_2|y_1{+}y_2) = \left(\frac{var(a_1+a_2)}{var(a_1+a_2)+var(u_1+u_2)}\right)(y_1{+}y_2) = \left(\frac{v_a(1+\eta)}{v_a(1+\eta)+v_u(1+\rho)}\right)(y_1{+}y_2).$$

Anticipating this behavior by HQ, the manager at Stage 1 chooses e_1 and e_2 to maximize

$$E(w_1{+}w_2){-}\frac{1}{2}c(e_1)^2{-}\frac{1}{2}c(e_2)^2 = \left(\frac{v_a(1+\eta)}{v_a(1+\eta)+v_u(1+\rho)}\right)E(z_1{-}\hat{e}_1{+}z_2{-}\hat{e}_2){-}\frac{1}{2}c(e_1)^2{-}\frac{1}{2}c(e_2)^2,$$

so for $j = 1, 2$, the FOC for e_j is $[\frac{v_a(1+\eta)}{v_a(1+\eta)+v_u(1+\rho)}] = ce_j$. As in part **b)**, these FOCs do not depend on $\hat{e}_1$ or $\hat{e}_2$, so the efforts that solve these FOCs are the equilibrium levels.

d) Derive an inequality that determines whether the equilibrium efforts on each project are higher or lower under "joint management" than under "separate management". Provide some intution for your result.

Simple algebra shows that $[\frac{v_a(1+\eta)}{v_a(1+\eta)+v_u(1+\rho)}] > (<)\ [\frac{v_a}{v_a+v_u}]$ according to whether $\eta > (<)\rho$. Hence equilibrium effort on each project is higher (lower) under "joint management" than under "separate management" if $\eta > \rho$ ($\eta < \rho$). In part **b)**, a is the "signal" and u is the "noise"; in part **c)**, $a_1 + a_2$ is the "signal" and $u_1 + u_2$ is the "noise". If $\eta > \rho$ ($\eta < \rho$), then moving from separate to joint management raises (lowers) the signal to noise ratio, and hence raises (lowers) the weight HQ places on each z in updating its beliefs about project qualities; this in turn means that equilibrium incentives for effort (signal-jamming) are higher (lower). Students may find it helpful to discuss special cases, e.g. $\eta > 0, \rho = 0$ and $\eta = 0, \rho > 0$.