[Short Question for Bargaining, Contracts and Theory of the Firm]

A risk-neutral principal P contracts with an agent A for the production of output. A's cost of producing output level x is $\frac{x^2}{2s}$, where s (A's type) is privately observed by A before contracting. P's revenue from output x is px. P believes s is drawn from a distribution with density f on support $[\underline{s}, \overline{s}]$, where $\underline{s} > 0$.

Assume P must offer A a contract of the form $t = \theta x$, where t is the payment from P to A and the value of θ can be chosen by P. After the contract is offered, A (knowing s) chooses wheter to accept or reject it. If A rejects the contract, both parties receive 0 payoff. If A accepts the contract, then A chooses a level of output x, and P pays A the amount $t = \theta x$; A's payoff is $t - \frac{x^2}{2s}$, and P's payoff is $px - t$.

1. What is the allocatively efficient output for A of type s?

   The allocatively efficient output maximizes revenue minus cost, $px - \frac{x^2}{2s}$. The FOC is $p = \frac{x}{s}$, so the allocatively efficient output is $x = sp$.

2. Explain in words what the incentive compatibility and participation constraints mean, in this setting, for A of type s.

   The incentive compatibility constraint requires that, given the contract, A chooses output x to maximize his payoff, given his type s. The participation constraint requires that the payoff of A of type s from accepting the contract and choosing x optimally must exceed 0, the payoff A gets from rejecting the contract.

3. Given a choice of θ by P, what level of output x will A of type s choose, and how will A's rent vary with s?

   A of type s chooses x to maximize $\theta x - \frac{x^2}{2s}$, yielding FOC $\theta = \frac{x}{s}$, so $x = s\theta$. A's rent is the difference between his maximized payoff under the contract and his payoff from rejecting the contract: $s\theta^2 - \frac{s^2\theta^2}{2s} - 0 = \frac{s\theta^2}{2}$. Both the output and the rent of A are increasing in s. Since the lowest type of A is $\underline{s} > 0$, all types of agent earn positive rent for any $\theta > 0$, so in part 4. below we can ignore A's participation constraint.

4. Derive P's optimal choice of θ. Explain why it does not depend on the density function of s.

   P chooses θ to maximize her expected profit, $E_s[px - \theta x]$, subject to A's incentive compatibility constraint. P's objective involves the expectation over A's type s. From

part 3., A will choose $x = s\theta$, so P chooses θ to maximize $E_s[p\theta s - \theta^2 s]$. P's payoff is proportional to s, so even if in this problem P could choose θ as a function of s, her optimal θ would be independent of s. Hence, the θ that maximizes P's expected payoff is independent of the density of s. The optimal θ solves the FOC $ps - 2\theta s = 0$, so the optimal θ is $\frac{p}{2}$ for all s. (Economically, P is a monopsonistic buyer of output, and the supply curve for output, $x(\theta; s)$, rotates outwards as s increases, leaving the monopsonists's optimal price per unit θ unchanged. But I'm not expecting this interpretation to be provided.)

5. How does the output produced by A of type s under the optimal contract compare to the allocatively efficient output? Explain your answer.

   Given the optimal θ, A of type s chooses output $x = \frac{sp}{2}$, which is strictly less than the allocatively efficient output $x = sp$. Even though P is here constrained to use a linear contract, she faces the familiar tradeoff between her two objectives of allocative efficiency and rent extraction: increasing θ for $\theta < p$ induces each type of A to produce an output closer to the allocatively efficient level but at the same time gives each type of agent a higher rent. (Interpreting P as a monopsonist, P's optimal price reflects the familiar downward monopsonistic distortion.)