Signalling Hidden Information

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Signalling hidden information

In signalling models, the privately-informed party moves first, choosing a signal that may be informative about its type; then the uninformed party (or parties) responds.

This class of hidden-information model is applicable to:

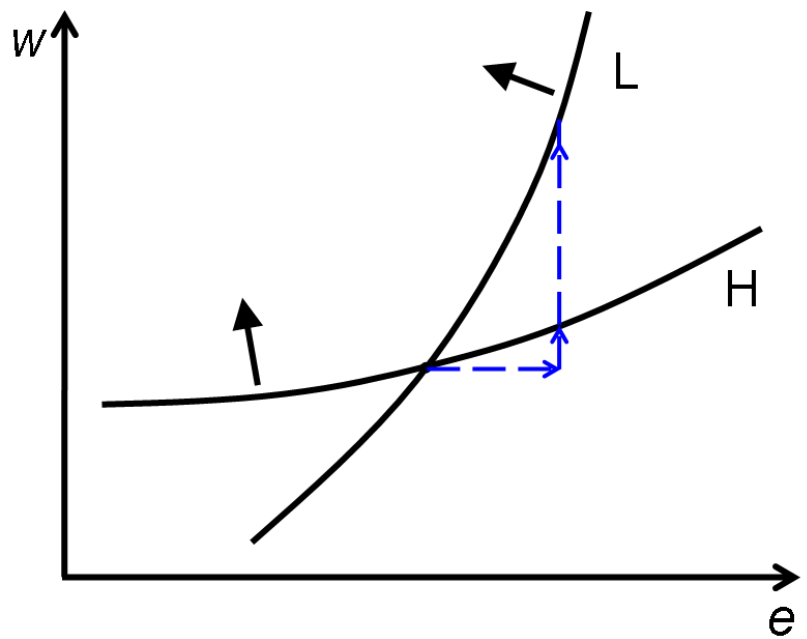
- labor markets: workers use education to signal ability to firms
- product markets:
  - sellers use warranties to signal their product's quality to buyers
  - incumbent firm uses limit-pricing to signal its cost to deter a potential entrant
- capital markets: entrepreneurs choose capital structure (debt/equity) to signal their project's quality to investors

Analysis of signalling models is essentially the same whether there is one or many equally uninformed parties—typically, uninformed parties all choose same response, so it is as if there is a single uninformed party.

Education as a signal of ability in the labor market

Here, we'll study Spence's (1973) model of education as a signal of ability in the labor market, as formalized by Cho and Kreps (1987). Exposition follows Gibbons (1992).

- Players: worker (privately informed about ability); 2 (or more) identical firms
- Worker preferences: $U(w, e, n) = w - C(n, e)$, where $C(n, e) =$ (psychic) cost to worker of ability n of obtaining education e
  - Assume $C_e > 0$, $C_{ee} > 0$, $C_n < 0$, and, crucially, $C_{en} < 0$



Education as a signal of ability in the labor market

- Worker preferences: $U(w, e, n) = w - C(n, e)$, where $C_{en} < 0$
- Firm preferences: profit $= y(e, n) - w$ if employ worker and 0 otherwise; $y_e \geq 0$ and, crucially, $y_n > 0$
  - NB: ability of worker enters not only worker's payoff but also firm's payoff
- Timing of the game:
  1. Worker privately observes n; firms believe that $Pr(n = H) = q$ and $Pr(n = L) = 1 - q$.
  2. Worker chooses a level of education $e \geq 0$.
  3. Firms observe worker's education (but observe nothing about ability directly) and then simultaneously make wage offers.
  4. Worker accepts higher of the two offers, flipping a coin in case of a tie.
- Strategies: worker: $e(n)$; each firm i: $w_i(e)$
- Beliefs: Assume firms hold same belief about n, given e, and let $\mu(n|e) =$ posterior probability assigned by firms to worker having ability n, given worker chose e

Education as a signal of ability in the labor market

Questions:

1. Will individuals with privately-observed abilities self-select in equilibrium by choosing different levels of education, and thereby credibly reveal their abilities to employers?
2. Will education choices be inefficient relative to full-information situation? If so, what form will inefficiencies take?
3. How will wages vary with education in equilibrium?
4. How would credit constraints affect equilibrium education and wages, and degree of inequality?
5. To what extent does empirically observed variation of wages with education reflect education's role as a signal vs. education's role in enhancing productivity?

Perfect Bayesian Equilibrium: definition

Definition

A *Perfect Bayesian Equilibrium* (PBE) in the signalling game is a strategy for the worker $e^*(n)$, strategies for each firm $w_1^*(e)$ and $w_2^*(e)$, and a common belief for the firms $\mu(n|e)$ that satisfy the four requirements:

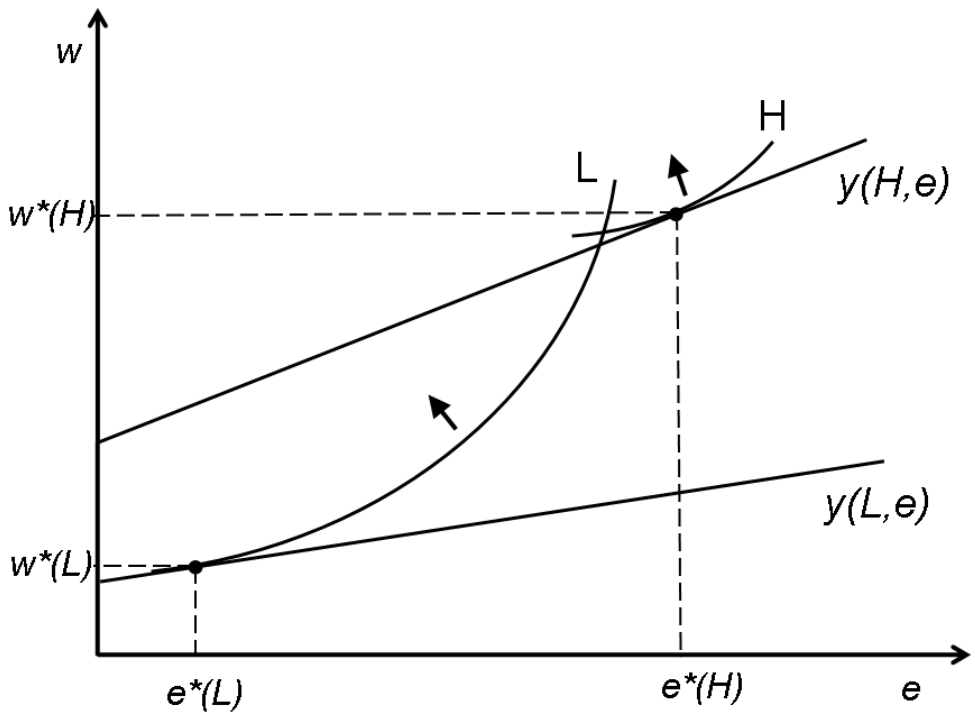
1. $\mu(H|e) \geq 0$; $\mu(L|e) \geq 0$; $\mu(H|e) + \mu(L|e) = 1$, $\forall e$
2. For each $e \geq 0$, $w_1^*(e)$ is optimal for firm 1, given its belief $\mu(n|e)$ and given $w_2^*(e)$, and $w_2^*(e)$ is optimal for firm 2, given its belief $\mu(n|e)$ and given $w_1^*(e)$.
3. For each $n \in \{H, L\}$, $e^*(n)$ maximizes worker utility, given firms' strategies $w_1^*(e)$ and $w_2^*(e)$.
4. For each $e \geq 0$, if there exists $n \in \{H, L\}$ such that with positive probability, n chooses e, then firms' belief if they observe e follows from Bayes' rule and the worker's strategy $e^*(n)$.

Since firms are identical, Reqt. 2. implies that competition btw. firms drives expected profit to 0:

$$w_1(e) = w_2(e) = \mu(H|e)y(H, e) + \mu(L|e)y(L, e) \equiv w(e)$$

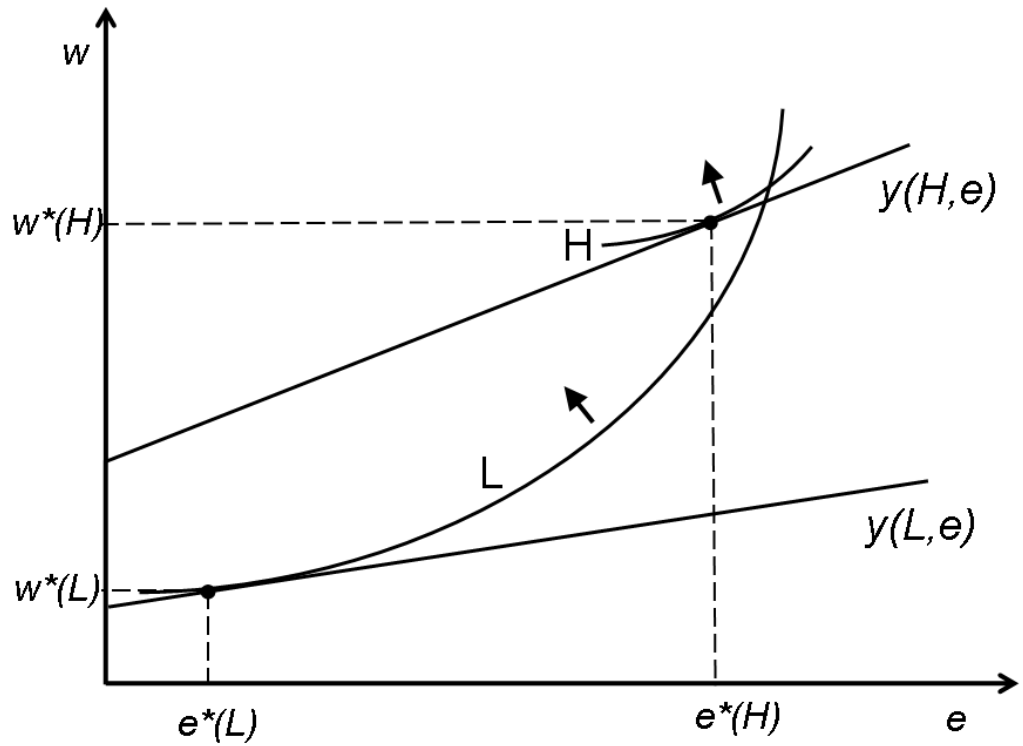
Full-information benchmark

- If firms both knew n, competition between firms would lead to $w(e) = y(n, e)$, $\forall n, e$
- Consequently, worker of ability n would choose e to maximize $y(n, e) - C(n, e)$
- **"No-envy" case:** Neither type of worker envies the other type's full-information (w, e) pair



Full-information benchmark cont.

"Envy" case: L-type worker envies the H-type's full-information (w, e) pair. (NB: It cannot be that H-type envies L-type.)



What features of workers' preferences and firms' technology make the "envy" case more likely to arise?

Perfect Bayesian Equilibrium: multiplicity

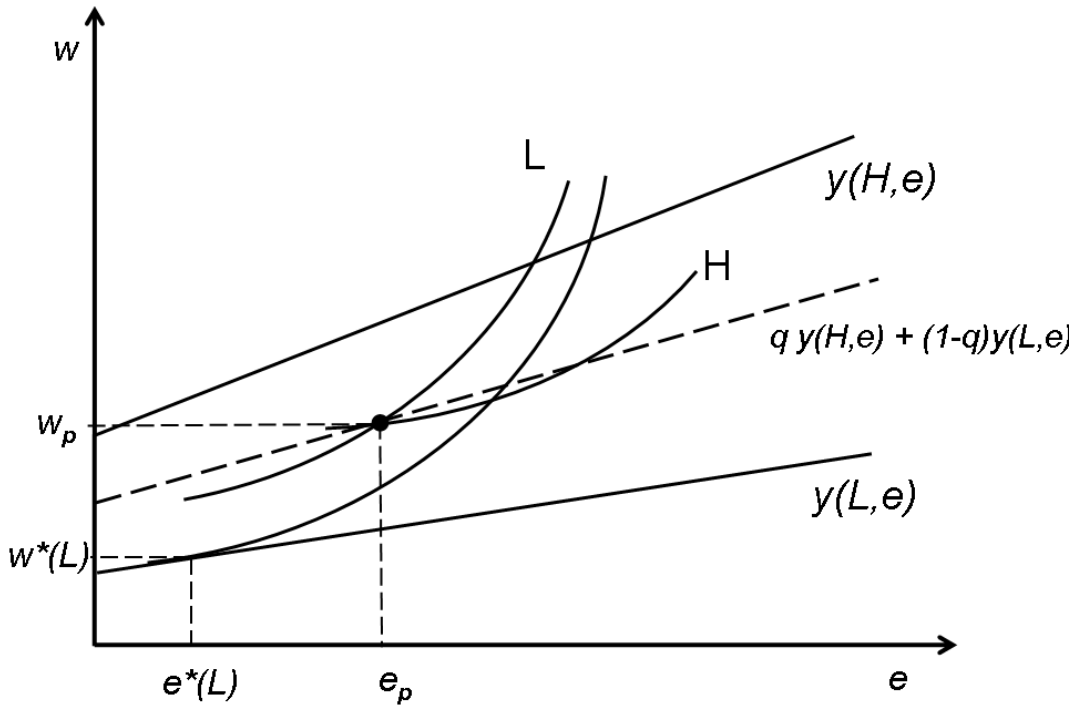
There are three different *types* of PBE in the signalling model:

- **Separating**: L- and H-types choose different levels of e
- **Pooling**: L- and H-types choose the same level of e
- **Hybrid**: one type chooses a level of e with certainty; the other type randomizes between pooling with the first type and separating from the first type

There are a multiplicity of PBE of each type.

Pooling PBE: example

Worker's strategy is $e(H) = e(L) = e_p$, so by Reqt. 4, $\mu(H|e_p) = q$, and by Reqt. 2, $w(e_p) = w_p \equiv qy(H, e_p) + (1 - q)y(L, e_p)$.



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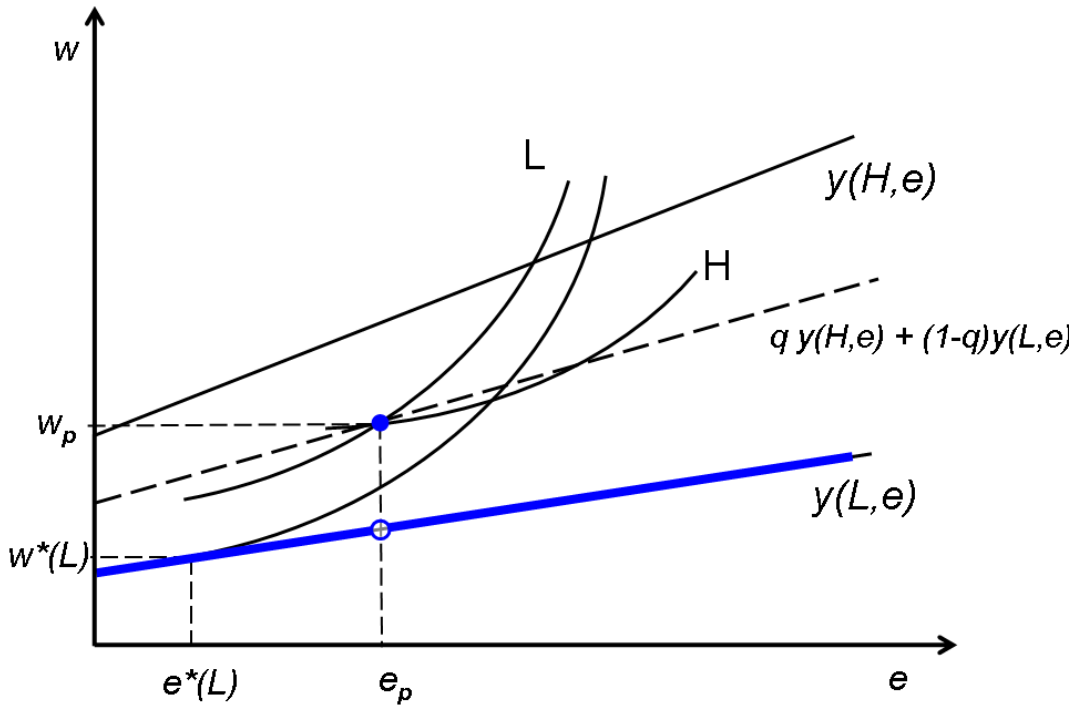

Blue function (discontinuous at e_p) describes wage worker would obtain as a function of e in this pooling PBE: it corresponds to off-eqm-path beliefs $\mu(H|e) = 0$ for $e \neq e_p$, implying $w(e) = y(L, e)$ for $e \neq e_p$.

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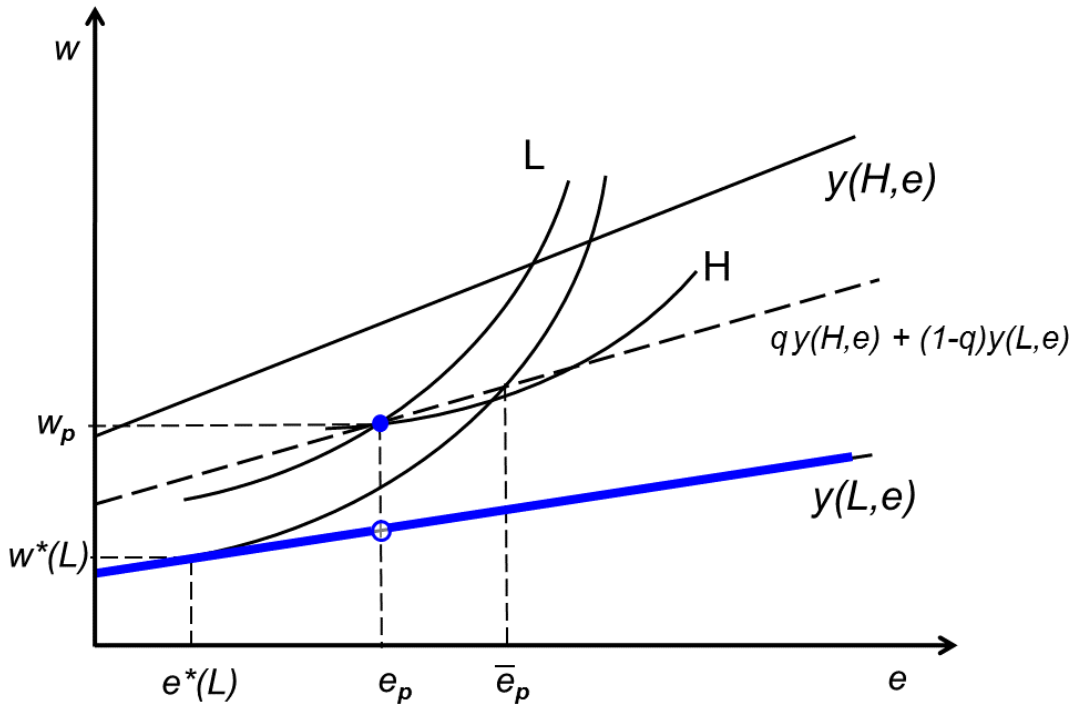

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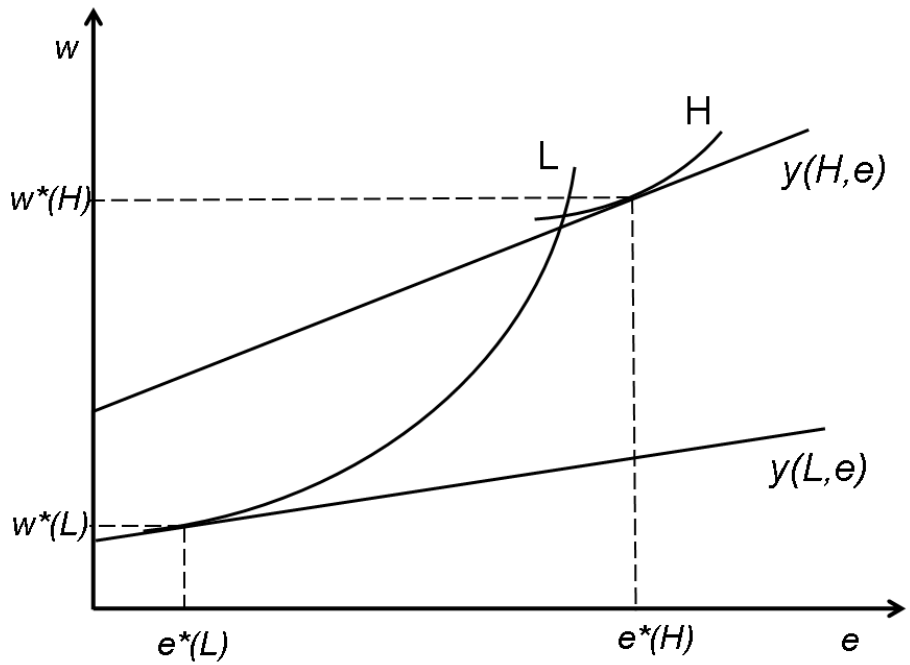
Education $\bar{e}_p$ is largest education level supportable in a pooling PBE.

Separating PBE: no-envy case

Strategies, beliefs and wages:

- $e(L) = e^*(L)$, $e(H) = e^*(H)$
- $\mu(H|e) = 0$ for $e < e^*(H)$ and $\mu(H|e) = 1$ for $e \geq e^*(H)$
- $w(e) = y(L, e)$ for $e < e^*(H)$ and $w(e) = y(H, e)$ for $e \geq e^*(H)$



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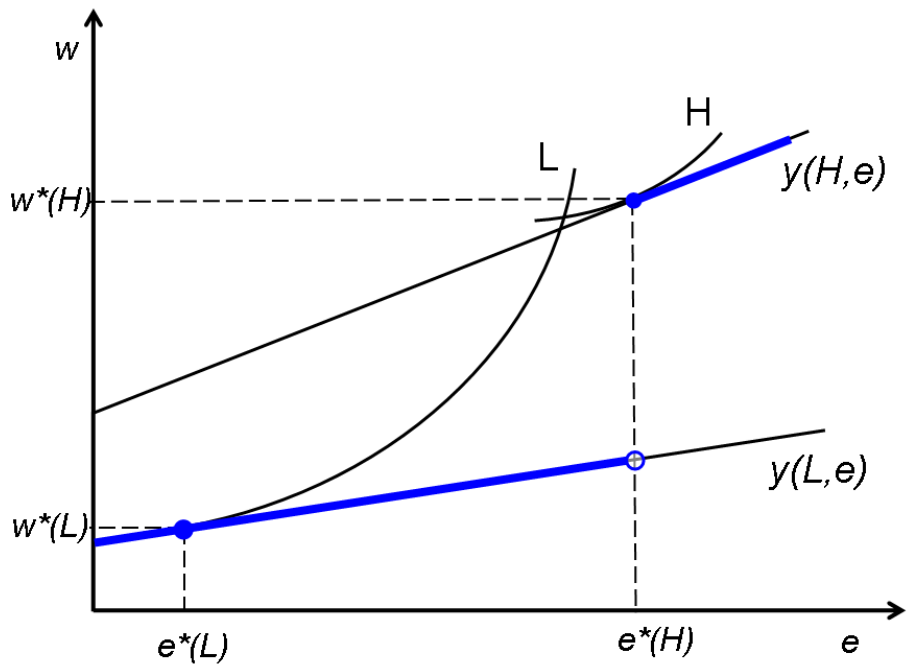
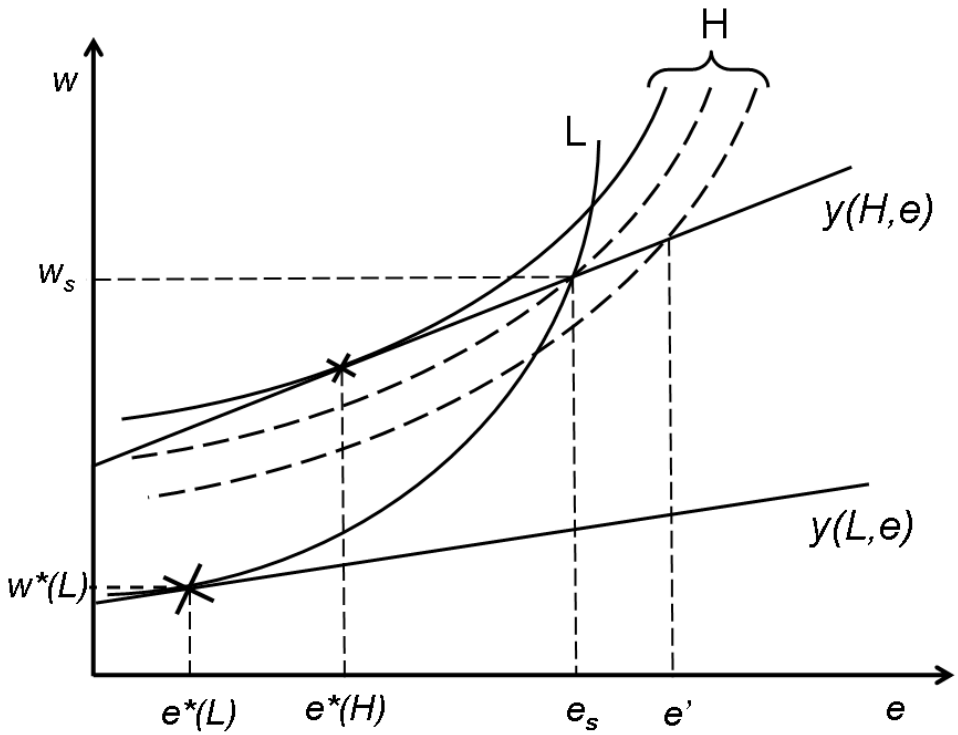


Separating PBE: envy case

Define e_s as the education level at the point on $y(H, e)$ which would give L-type the same utility as from $(e^*(L), w^*(L))$:

$$y(H, e_s) - C(L, e_s) = w^*(L) - C(L, e^*(L)).$$

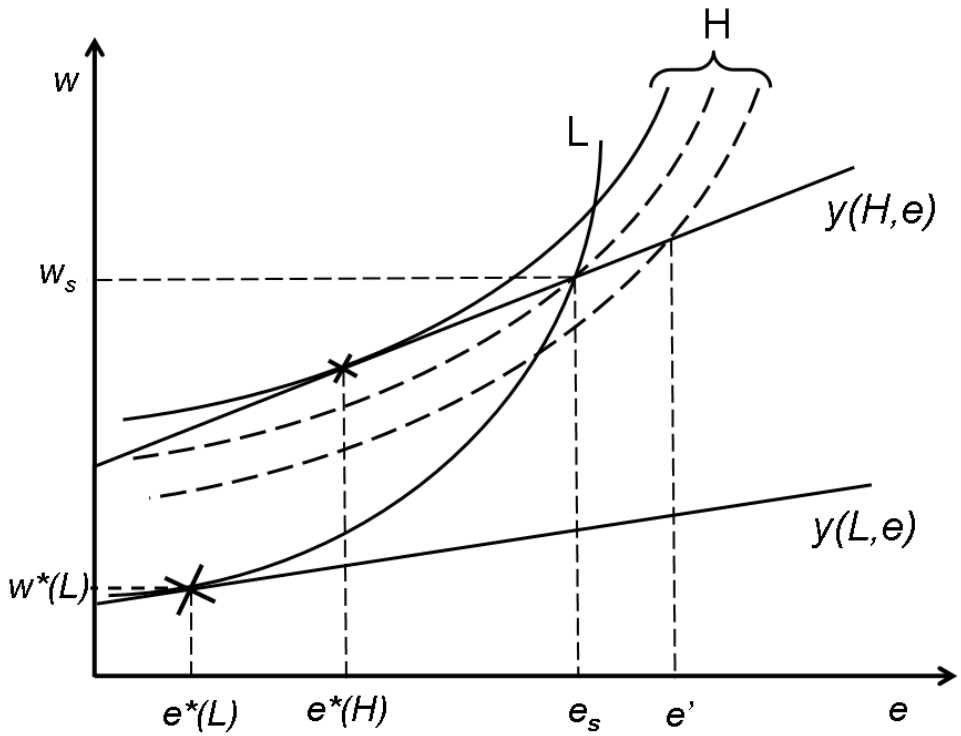
Compared with $(e^*(L), w^*(L))$, L-type would never strictly benefit from choosing $e \geq e_s$, even if doing so convinced the firms that he was H-type.



Separating PBE: envy case

Strategies, beliefs and wages:

- $e(L) = e^*(L)$, $e(H) = e_s$ $(> e^*(H))$
- $\mu(H|e) = 0$ for $e < e_s$ and $\mu(H|e) = 1$ for $e \geq e_s$
- $w(e) = y(L, e)$ for $e < e_s$ and $w(e) = y(H, e)$ for $e \geq e_s$

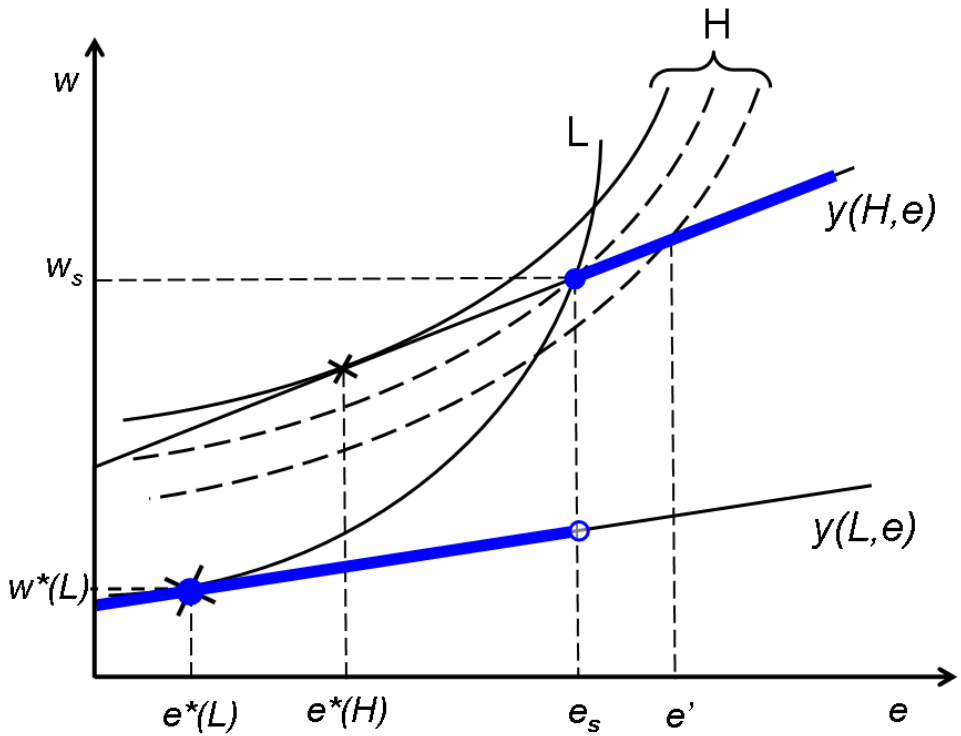


Separating PBE: envy case

Strategies, beliefs and wages:

- $e(L) = e^*(L)$, $e(H) = e_s$ $(> e^*(H))$
- $\mu(H|e) = 0$ for $e < e_s$ and $\mu(H|e) = 1$ for $e \geq e_s$
- $w(e) = y(L, e)$ for $e < e_s$ and $w(e) = y(H, e)$ for $e \geq e_s$

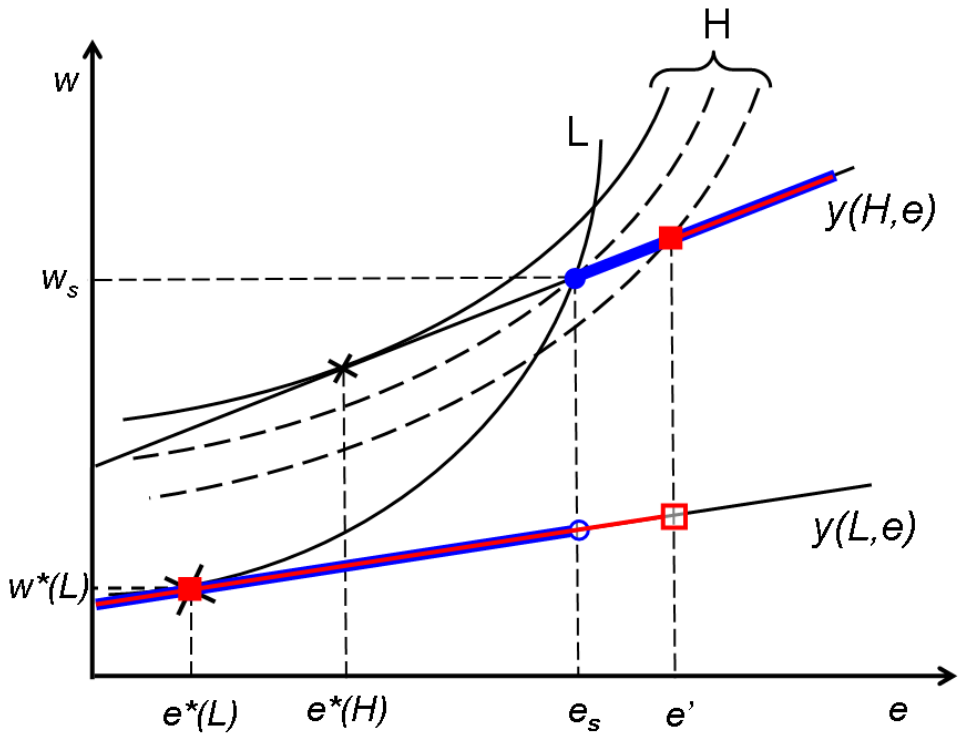
Blue function (discontinuous at e_s) describes wage worker would obtain as a function of e in this separating PBE.



Multiplicity of separating PBE: envy case

Red function (discontinuous at e') describes wage worker would obtain as a function of e in the alternative separating PBE.

- The PBE with $e(H) = e_s$ Pareto-dominates the PBE with $e(H) = e'$. In fact, the PBE with $e(H) = e_s$ Pareto-dominates all other separating PBE.



General results for PBEs

Proposition

a) *In any PBE, L-type gets utility of at least* $w^*(L) - C(L, e^*(L))$.

b) *In any separating PBE, L-type chooses* $e^*(L)$ *and receives* $w^*(L)$.

Proof.

a) By Reqt 1, $w(e) \geq y(L, e)$. So L-type, by choosing $e^*(L)$, can ensure a wage of at least $w^*(L)$, hence utility of at least $w^*(L) - C(L, e^*(L))$.

b) Since $y_n > 0$, H-type does not want to be thought to be L. If PBE is separating, then if L-type chooses $e(L)$, $w(e(L)) = y(L, e(L))$. Optimal choice for L-type is therefore $e = e^*(L)$, yielding $w = w^*(L)$.

□

Part a) above implies that $\bar{e}_p$ on pooling PBE diagram is the largest education level supportable in a pooling PBE.

We are still left with a large multiplicity of PBE.

Refinements of Perfect Bayesian Equilibrium

Definition: Education level (signal) e' is **dominated** for type n if $\exists e$ such that type n's lowest possible payoff from e is greater than type n's highest possible payoff from e', i.e. $y(L,e) - C(n,e) > y(H,e') - C(n,e')$.

Requirement 5: If e' is off equilibrium path and is dominated for type n (but not for all types), then the firms' belief should place 0 probability on type n after e' is observed, i.e. $\mu(n|e') = 0$.

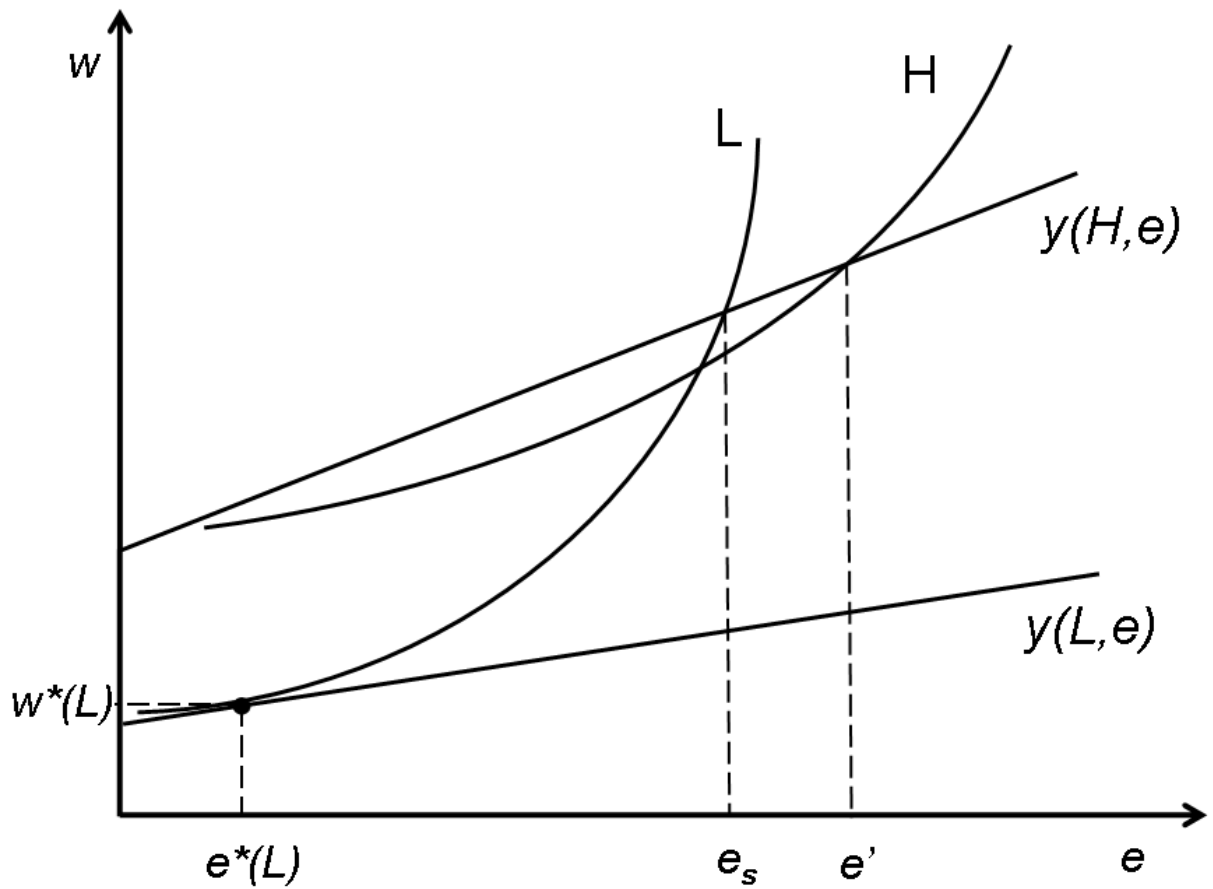
Definition: Given a PBE in a signalling game, education level e' is **equilibrium-dominated** for type n if type n's equilibrium payoff, denoted $U^*(n)$, is greater than type n's highest possible payoff from e', i.e. $U^*(n) > y(H,e') - C(n,e')$.

Requirement 6: If e' is off equilibrium path and is equilibrium-dominated for type n (but not for all types), then the firms' belief should place 0 probability on type n after e' is observed, i.e. $\mu(n|e') = 0$.

- Reqt. 6 is a stronger refinement (stronger restriction on off-eqm-path beliefs) than Reqt. 5. Hence any PBE satisfying Reqt. 6 satisfies Reqt. 5, but not vice versa.

Refining equilibrium using Requirement 5 in the envy case

- The "envy" case is defined by the condition $e_s > e^*(H)$.
- Any $e' > e_s$ is **dominated** for L-type by $e^*(L)$:
  $y(L, e^*(L)) - C(L, e^*(L)) > y(H, e') - C(L, e')$ for $e' > e_s$.



Refining equilibrium using Requirement 5 in the envy case

Proposition

a) *In the only separating PBE satisfying Requirement 5, the H-type worker chooses e_s. This is the Pareto-dominant separating PBE, since in any other separating PBE, the H-type would be worse off, given $e(H) > e_s$.*

b) *In any PBE satisfying Requirement 5, the H-type worker must receive utility of at least $y(H, e_s) - C(H, e_s)$.*

Proof.

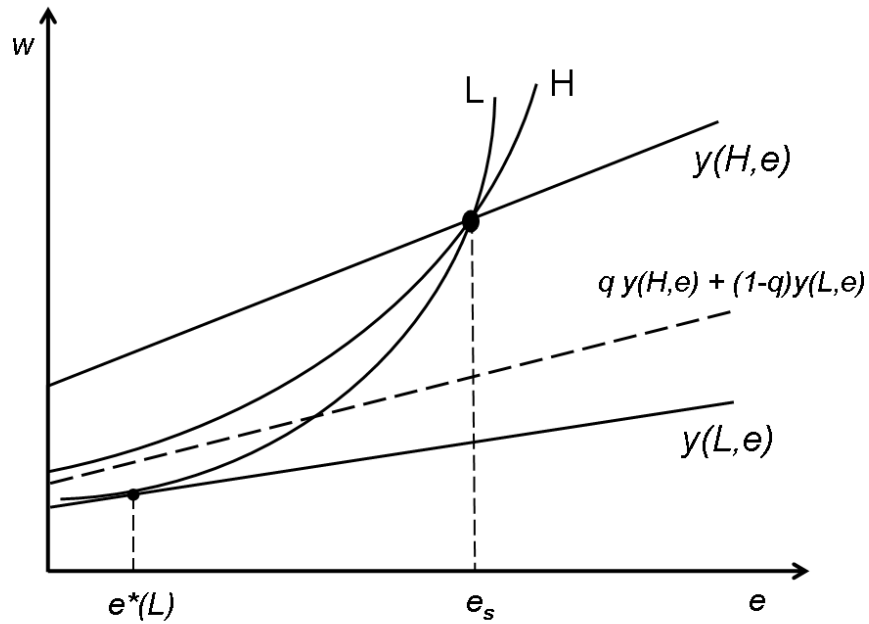
a) Suppose there were a separating PBE, satisfying Reqt 5, in which $e(H) = e' > e_s$. Because any $e' > e_s$ is **dominated** for L-type, Reqt. 5 implies $\mu(H, e') = 1$ for $e' > e_s$. Since in the envy case, $e_s > e^*(H)$, e_s maximizes $y(H, e') - C(H, e')$ s.t. $e' \geq e_s$. Hence for any $e' > e_s$, H-type would benefit by reducing $e(H)$ very slightly below e'. Contradiction.

b) If H-type were receiving utility strictly below $y(H, e_s) - C(H, e_s)$, would benefit by deviating to $e' > e_s$ but arbitrarily close to e_s, since $\mu(H|e') = 1$ for $e' > e_s$.

□

Refining equilibrium using Requirement 5 in the envy case

Case A (low prior prob q of H-type): H-type's indifference curve through $(e_s, y(H, e_s))$ lies everywhere *above* $qy(H, e) + (1 - q)y(L, e)$.



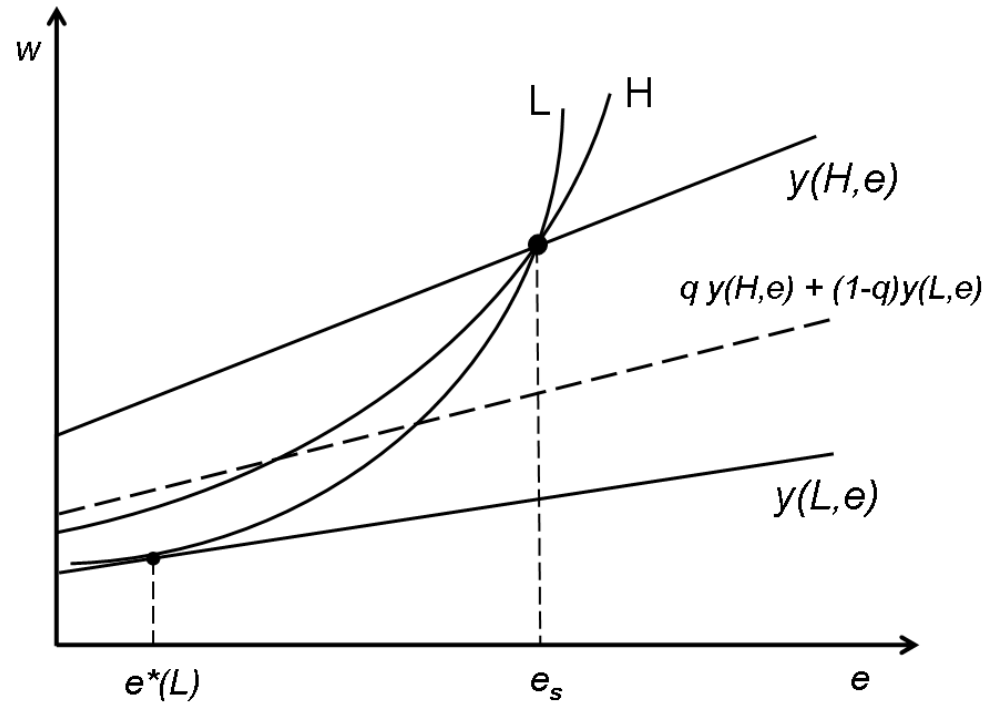
Proposition In Case A, no pooling PBE satisfies Requirement 5.

Proof Such a pooling PBE would give H-type less than $y(H, e_s) - C(H, e_s)$. Contradiction. □

Proposition In Case A, the only PBE (of any type) satisfying Requirement 5 is the Pareto-dominant separating PBE: $e(L) = e^*(L)$ and $e(H) = e_s$.

Refining equilibrium using Requirement 5 in the envy case

Case B (high prior prob q of H-type): H-type's indifference curve through $(e_s, y(H, e_s))$ dips *below* $qy(H, e) + (1 - q)y(L, e)$.



Refining equilibrium using Requirement 5 in the envy case

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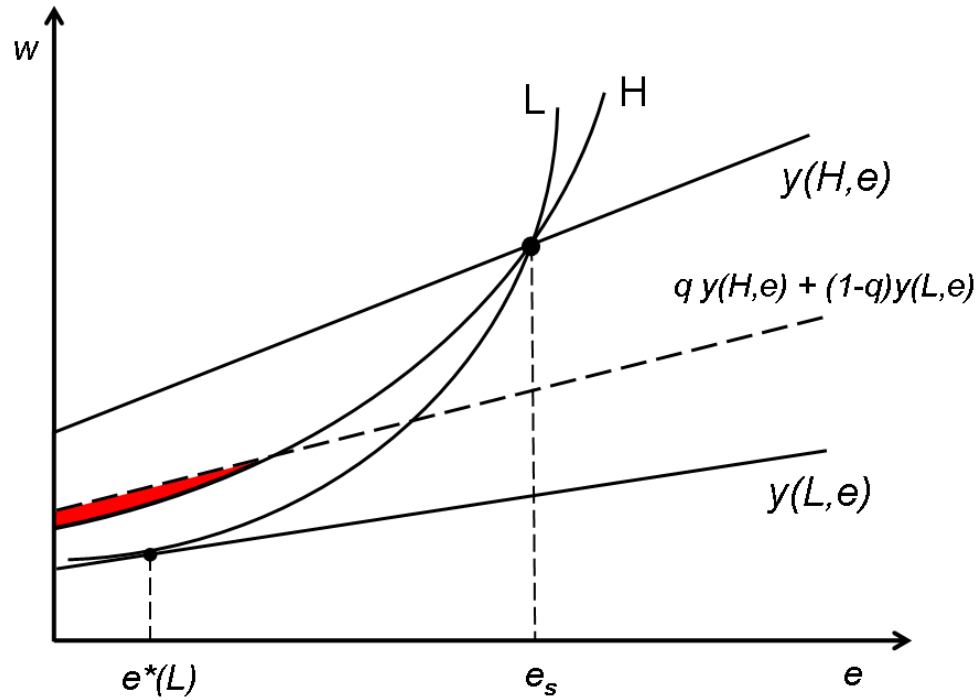


In Case B, there are pooling PBE satisfying Requirement 5—they lie along the upper boundary of the red wedge.

- Note that these pooling PBE Pareto-dominate the Pareto-dominant separating PBE.

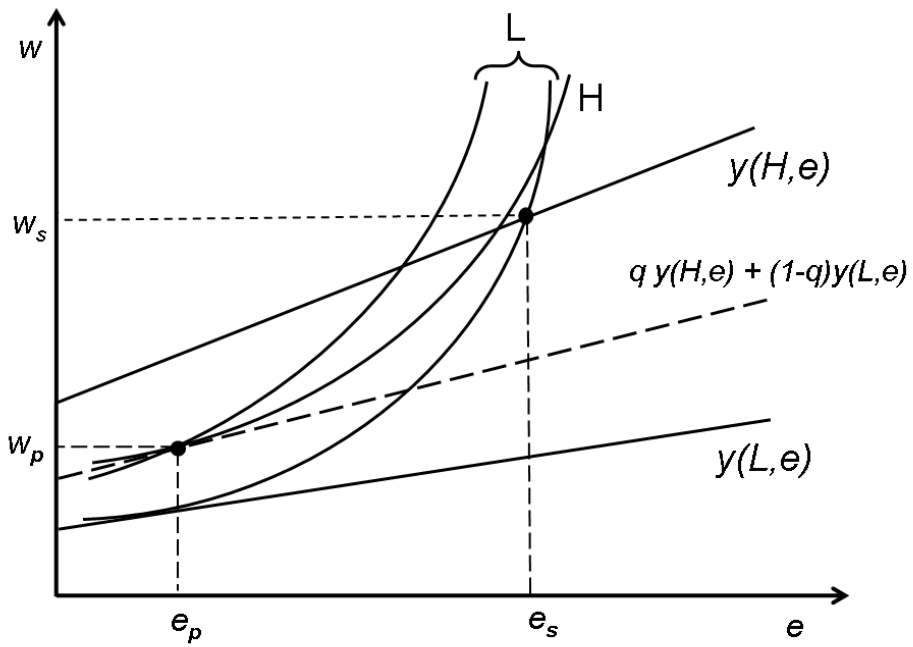
Refining equilibrium using Requirement 6 in the envy case

Proposition

No pooling PBE satisfies Requirement 6.

Proof.

Consider a candidate pooling eqm (w_p, e_p). Any $e' > g_1$ is eqm-dominated for L-type, but $e' \in (g_1, g_2)$ is not eqm-dominated for H-type. Hence, Requirement 6 implies $\mu(H|e') = 1$ for $e' \in (g_1, g_2)$, so H-type will deviate from e_p. □



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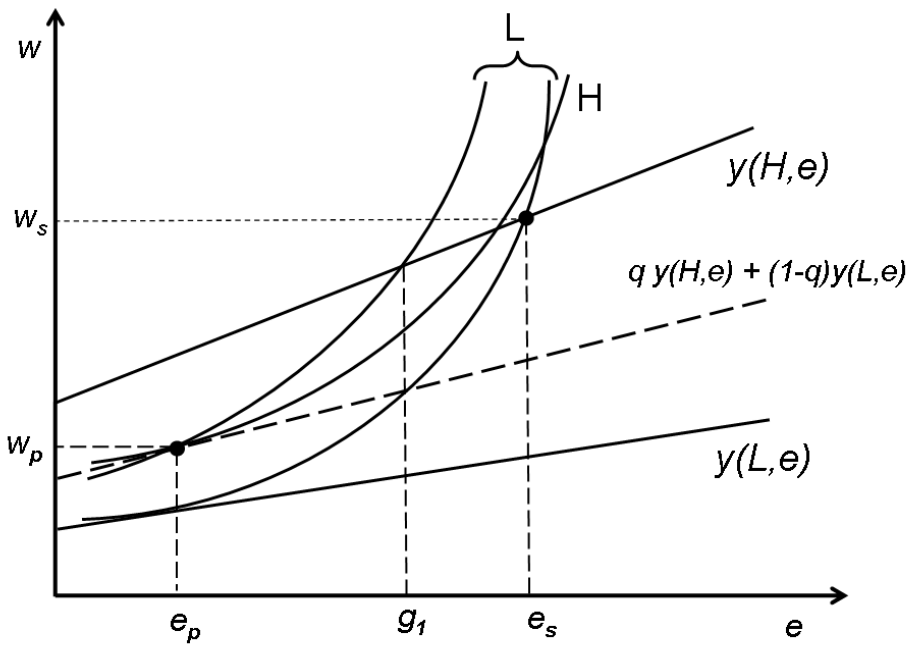

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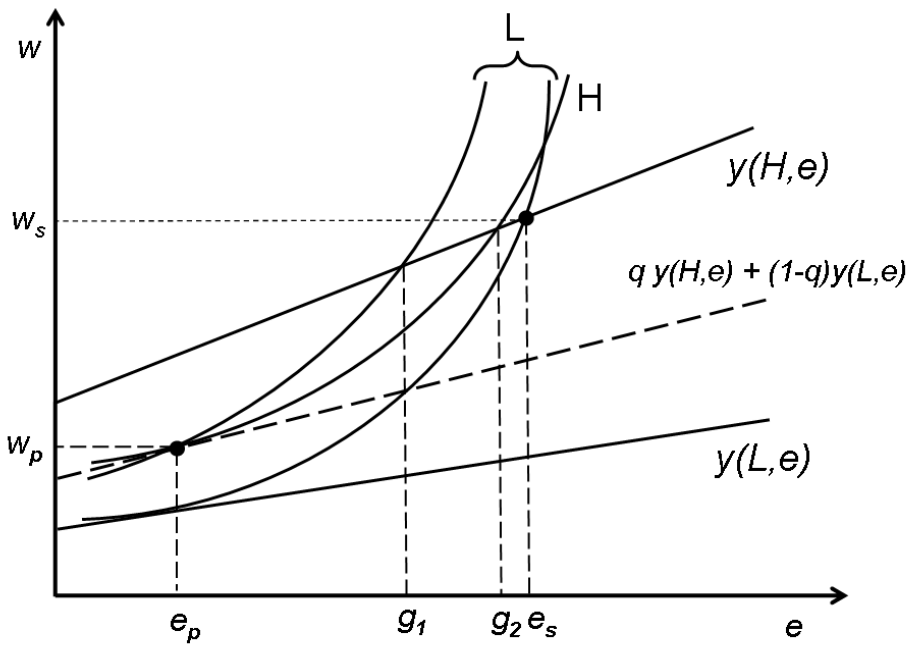

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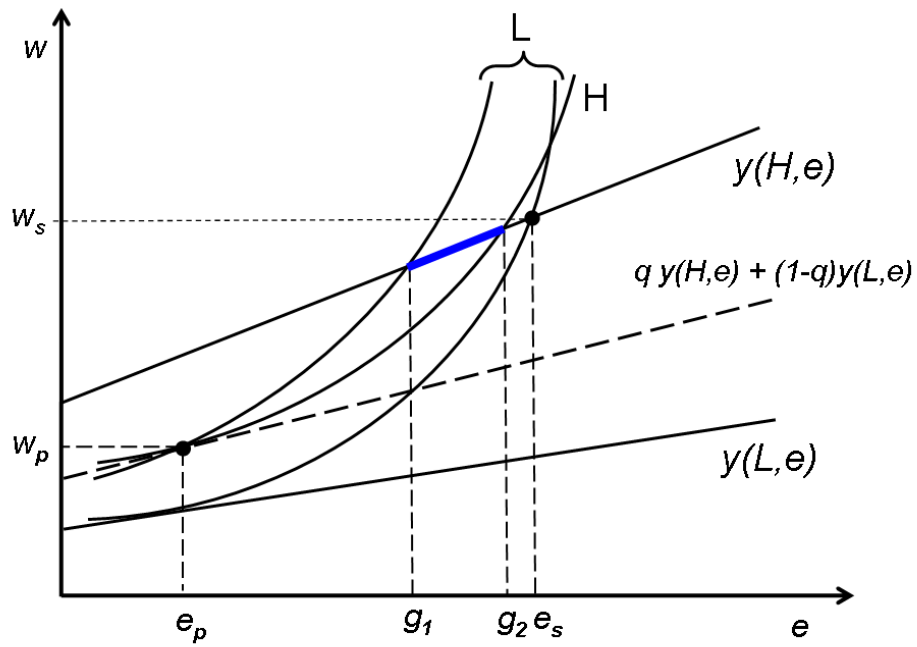

Uniqueness of PBE satisfying Requirement 6

Proposition

The only PBE (of any type) satisfying Requirement 6 is the Pareto-dominant separating PBE.

1. *In the no-envy case, this eqm has* $e(L) = e^*(L)$ *and* $e(H) = e^*(H)$.
2. *In the envy case, this eqm has* $e(L) = e^*(L)$ *and* $e(H) = e_s$.

- In the no-envy case, the Pareto-dominant separating PBE coincides with the full-information outcome.

- In the envy case, in the Pareto-dominant separating PBE,
  - i) the H-ability worker credibly signals his identity by **over-investing** in education relative to the optimal level under full information;
  - ii) the signal chosen by the H-type is the smallest level of education (e_s) which can deter imitation by the L-type.

Signalling model vs. optimal contracting model

Three important differences:

1. **Timing:** In signalling model, privately-informed party moves first; in optimal contracting model, uninformed party moves first.
   - *Implication of* **1.***:* multiple PBE in signalling model; unique eqm in optimal contracting model (except in knife-edge cases)
2. **Competition or not btw. uninformed parties:** In signalling, competition btw. firms drives profits to zero; in optimal contracting, P is a monopolist, makes a take-it-or-leave-it offer to A, and earns positive expected profit.
   - *Exercise: In P-A model of optimal contracting, if identical principals compete to contract with the single A, what happens?*
3. **Role of hidden information:** In signalling model, worker's type *directly* enters firms' profit, as well as worker's cost—it is a "common-value" environment; in optimal contracting model, A's type does not directly enter P's profit—it is a "private-value" environment.

Signalling model vs. optimal contracting model

Implication of differences **2.** *and* **3.***:*

- If there is envy under full information, then
  - in signalling model, bad worker envies good one: competing firms reward good worker, whose output is more valuable, with higher wage
  - in optimal contracting model, good worker envies bad one: firm "pampers" bad worker, for whom effort is more costly, with higher payment to induce him to accept contract

Key similarities btw. signalling and optimal contracting models:

- In both models, equilibrium involves *distortion in the allocation for the envied type*:
  - in signalling, inefficiently high education for good worker
  - in optimal contracting, inefficiently low output for bad worker
- In both models, the direction of distortion in the allocation for the envied type is the direction which allows self-selection (separation).

Criticisms of the signalling model

Two unappealing features of the signalling model:

1. **Assumptions:** The costly signal chosen by the informed party is the **only** information available to the uninformed party about the informed party's type.

2. **Results:** The unique PBE surviving the eqm refinements is **independent** of the prob. that the informed party is the bad type. Hence, in the envy case, no matter how small the (positive) prob. of the bad type, the good type's eqm behavior is very different from his behavior under full information.

A model of signalling with noisy tests

Daley and Green (*JET*, 2014) study "signalling with grades":

- Similar motivation in Feltovich et al (*Rand*, 2002), "Too Cool for School..." and Kurlat and Scheuer (*REStud*, 2020), "Signalling to Experts"

Assumptions:

- Firms observe both the costly signal (education) chosen by the worker and a noisy "grade" (evaluation or test result) correlated with the worker's ability but which he cannot influence.
- The worker chooses education before knowing his (noisy) grade.
- Two levels of worker ability, with education having lower marginal cost for the H-type than for the L-type.
- Education has no direct effect on productivity, so under full information, both types of worker would choose 0 education.

Now, high-ability worker faces a tradeoff between exploiting his cost advantage by signalling and relying on his expected test-result advantage.

"Signalling with grades": results

- If the noisy test is relatively uninformative about ability, then the earlier results continue to hold.
- If the noisy test is relatively informative, then separating PBE do not survive eqm refinements.
  - Choosing a separating level of education "looks bad", because it looks like you are trying to de-emphasize your grade.
- If, in addition, the prior probability of H is high enough, then the unique surviving PBE has full pooling. As this prob. goes to 1, the pooling level of education goes to 0 (the full-infor. outcome).
  - So a very small prob. that the worker is L results in only a very small amount of inefficiency in education choice.
- The more informative is the test,
  - the more H relies on the test and the less he relies on costly education to reveal his ability to firms;
  - the higher is H's expected payoff;
  - the lower is the amount of over-investment in education.

So the earlier model's results are robust if firms' tests are not very informative about ability but change substantively if tests are sufficiently informative.