Relational Contracts with Subjective Peer Evaluations

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Introduction

• Objective performance measures are hard to obtain in most jobs.
• Peer evaluations have become an important source of information about the performance of co-workers.
• However, peer evaluations are difficult for the firm to elicit and use.
• A hierarchical structure vs team environment.
• Key question: How should a firm optimally use public subjective measures along with dispersed private subjective measures (such as peer evaluations) to offer sharper work incentives?
Main Contributions

• Allow public and private performance measures in multi-agent setting.

• Introduce the idea of incentives for truthful reporting.

• Interaction between agent’s incentives for effort and incentives for truthful reporting.

• Illustrates how this interaction affects contracting.
Overview

• Study two different settings.
  • Baseline Model:
    - Setup: Separation of productive and informative roles.
    - First-best outcome can be achieved even in the static setting.
  • General Model:
    - Setup: Agents play both the informative and productive role.
    - Optimal static contract is necessarily inefficient due to the threat of double deviation and the possibility of the firm manipulating peer evaluations.
    - Optimal contract tries to minimize surplus destruction.
    - Dynamic setting: Firm’s patience plays a key role.
• Key takeaway: Peer evaluations sparingly used in relational contracts due to surplus destruction.
• Extensions and Criticisms.
Baseline Model: Setting

• Consider a setting with organizational hierarchy
  • e.g. firm (F) - manager (M) - worker (A)
• Key: separation of informative and productive roles
• Static setting
• Desired outcome for the principal:
  • Supervisor reports truthfully
  • Agent exerts effort
• Result: first best is achievable with subjective evaluation
Baseline Model: set-up (1)

- Project outcome $Y \in \{y_1, y_2, \ldots, y_N\}$ with $y_1 < y_2 < \ldots < y_N$
- Publicly observable but non-verifiable
  - Depends only on effort $e \in \{0,1\}$
  - $\Pr(Y = y_j \mid e) = \begin{cases} \alpha_j & \text{if } e = 1 \\ \beta_j & \text{otherwise} \end{cases}$
- Effort is costly and privately observed
  - $c(e) = \begin{cases} c & \text{if } e = 1 \\ 0 & \text{otherwise} \end{cases}$
- M observes a noisy signal after the agent has chosen effort
  - $\Pr(s_M = s \mid e) = \begin{cases} p_s & \text{if } e = 1 \\ q_s & \text{if } e = 0 \end{cases}$
Baseline Model: set-up (2)

• Assumptions:
  • Y and $s_M$ satisfy the monotone likelihood ratio property
    • $\frac{\alpha_i}{\beta_j} > \frac{\alpha_k}{\beta_k}$ for any $j > k$ and $\frac{\nu_s}{q_s} > \frac{\nu_r}{q_r}$ for any $s > r$
  • $\{Y, s_M\}$ mutually independent conditional on effort
  • $\sum_{j=1}^{N} \alpha_j y_j - c > \sum_{j=1}^{N} \beta_j y_j$ so that it is efficient to implement $e = 1$

• Contracts: $w_s^M(Y)$ and $w_s^A(Y)$

• Total payroll expense: $w_s^M(Y) + w_s^A(Y) \leq w$
  • w cannot vary with Y since output non–verifiable
  • w cannot vary with s since the firm would have an incentive to lie

• All players are risk neutral with payoffs:
  • $\Pi = E[ Y|e] - w$  
  • $u_M = E[w_s^M(Y)|e, m]$  
  • $u_A = E[w_s^A(Y)|e, m] - ce$
Baseline Model: Timing

- **Beginning of stage 1.** Firm offers a contract to both agents
  - Stage 1.1. Manager and the agent accept or reject. If both players accept, the game continues to the next stage.
  - Stage 1.2. The agent exerts effort $e$
  - Stage 1.3. The manager privately obtains signal $s_M$ and privately sends her evaluation $s$ to the firm.
  - Stage 1.4. Output $Y$ is realized.

- **End of Stage 1.** Transfers paid to the agents and the game ends.
Baseline Model: Principal’s Problem

\[
\text{max } \pi = E[Y|e = 1] - w \\
\text{s.t. (B), (T), (IC), (IR}_A\text{), (IR}_M\text{)}
\]

\[
w_S^M(Y) + w_S^A(Y) \leq w \quad \forall s \in S \text{ and } Y \in \{y_1, y_2, \ldots, y_N\} \quad (B)
\]

\[
E_Y[w_S^M(Y)|e = 1, s_M = s] \geq E_Y[w_S^M(Y)|e = 1, s_M = s'] \quad \forall s \text{ and } s' \in S \quad (T)
\]

\[
E_{\{Y,s\}}[w_S^A(Y)|e = 1] - c \geq E_{\{Y,s\}}[w_S^A(Y)|e = 0] \quad (IC)
\]

\[
u_M = E[w_S^M(Y)|e = 1] \geq 0 \quad (IR_M)
\]

\[
u_A = E[w_S^A(Y)|e = 1] - c \geq 0 \quad (IR_A)
\]

• We seek a Perfect Bayesian Equilibrium (PBE)
• Using the Revelation Principle focus on equilibria with \(m(s) = s \quad \forall s \in S\)
Proposition 1: The optimal contract induces first best. Under the optimal contract, (i) the firm commits to a payroll expense of $c$, (ii) the manager reports truthfully and receives the transfer

$$w^M_S(Y) = \begin{cases} -\Delta_M & \text{if } Y = y_N \text{ and } s = 1 \\ \Delta_A & \text{if } Y = y_1 \text{ and } s = 1 \\ 0 & \text{otherwise} \end{cases}$$

and (iii) the agent exerts effort and receives the transfer:

$$w^A_S(Y) = \begin{cases} c + \Delta_M & \text{if } Y = y_N \text{ and } s = 1 \\ c - \Delta_A & \text{if } Y = y_1 \text{ and } s = 1 \\ c & \text{otherwise} \end{cases}$$

where $\Delta_M = \frac{\alpha_1 c}{q_1(\alpha_0\beta_1 - \alpha_1\beta_N)}$ and $\Delta_A = \frac{\alpha_N c}{q_1(\alpha_0\beta_1 - \alpha_1\beta_N)}$}

<table>
<thead>
<tr>
<th>$(u_M,u_A)$</th>
<th>$Y = y_1$</th>
<th>...</th>
<th>$Y = y_N$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S = s_1$</td>
<td>$(\Delta_A, c - \Delta_A)$</td>
<td>$(0, c)$</td>
<td>$(\Delta_M, c + \Delta_M)$</td>
</tr>
<tr>
<td></td>
<td>$[\alpha_1 p_1]$</td>
<td></td>
<td>$[\alpha_N p_1]$</td>
</tr>
<tr>
<td>$S = s_M$</td>
<td>$(0, c)$</td>
<td>$(0, c)$</td>
<td>$(0, c)$</td>
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</table>

Key Lessons:
1. Does not require money burning
2. Implication – in a repeated setting even if the firm has the possibility of offering a relational contract, this would be redundant
Overview

• Assume now that both workers play both a productive and informative role (by evaluating the other’s performance).
• Manager and worker are modelled as symmetrical agents (ignoring hierarchy), denoted $A_1$ and $A_2$
• Agents are long-lived, and play the stage game in all periods $t \in \{1, 2, \cdots\}$

Technology

• Each agent privately exerts effort: $e_i \in \{0, 1\}$, at an identical cost $c$
• The project outcome is discretely chosen from $Y \in \{y_1, \cdots, y_N\}$, and is jointly determined by the effort profile of both agents, $e = (e_1, e_2)$:

$$
\Pr(Y = y_j | e_1, e_2) = \begin{cases} 
\alpha_j & \text{if } e = (1,1) \\
\beta_j & \text{if } e = (1,0) \text{ or } (0,1) \\
\gamma_j & \text{if } e = (0,0)
\end{cases}
$$
General Model: set-up (2)

Subjective performance signals

• Both agents privately observe a noisy signal $s_i$, where $s \in \{1, \cdots, n\}$, about their counterpart:

$$\Pr(s_i = s | e_{-i}) = \begin{cases} p_s & \text{if } e_{-i} = 1 \\ q_s & \text{if } e_{-i} = 0 \end{cases}$$

• Assume that it is optimal for $F$ to implement the maximum effort case

• Assume that $\{Y, s_1, s_2\}$ are mutually independent conditional on effort

Contract

• Maintains key elements of the baseline model allowing for two workers:

  • Effort levels $e$, reporting strategies for each agent $m$
  • Transfer payment $w_{rs}^i(Y)$ depends on both $r$, the peer evaluation received from co-worker, and $s$, the peer evaluation sent by the worker.
  • By assumption, consider only symmetric contracts: $w_{rs}^1(Y) = w_{rs}^2(Y) = w_{rs}(Y)$
General Model: set-up (3)

Contract
• Adds a public performance bonus element $B(Y) \geq 0$
• Offered as a relational contract because $Y$ is not verifiable and cannot be contracted on
• Let $b^1(Y)$ be the public performance bonus offered to agent $i$, where $b^1(Y) + b^2(Y) = B(Y)$
• By assumption, limit attention to symmetric contracts where $b^1(Y) = b^2(Y) = b(Y)$
• A contract is characterised by the tuple:
  $$\phi = \{e, m, w, b(Y), w_{rs}(Y)\} \in \Phi^*$$

Payoff
$$\pi = E[Y|e] - w - 2E[b(Y)|e,m]$$
$$u_i = E[b(Y) + w_{rs}(Y)|e,m] - ce_i$$
General Model: set-up (4)

Repeated game

• Assume constant discount factor between all players $\delta \in (0,1)$
• History of play of period $t$ is given by $h_t = \{\phi, Y, \hat{b}\}_t$, where $\hat{b}$ is the bonus paid out at $t$
• History of play in period $t$ is the concatenation of the history of play in each period:

$$h^t = \left\{ \{\phi, Y, \hat{b}\}_\tau \bigg| \tau = 1, \ldots, t - 1 \right\} \in H^t$$

Strategies and equilibrium

• Restrict the class to strategies to only strategies that depend on the entire public history, but only the most recent private history (comes with loss of generality)
• By the Revelation Principle, focus on truthful equilibria such that $m_i(s) = s$
• Assume that following a publicly observable deviation (such that $\hat{b} \neq b(Y)$), all players take their outside option forever (without loss of generality) (Abreu, 1988)
General Model: set-up (5)

Strategies and equilibrium
To summarise, the strategies of the firm and the agents in the stage game has the following components:

• Firm:
  • Chooses a contract to offer to each agent: $H_t \rightarrow \Phi^*$
  • After output is realised, choose a level of public performance bonus to offer: $H_t \times Y \rightarrow R_+$
  • After output is realised and signals are received, choose a subjective performance bonus to give: $H_t \times Y \times S \times S \rightarrow R_+$

• Agent:
  • Chooses whether or not to accept the contract offered by F: $H_t \times \Phi^* \rightarrow \{\text{accept, reject}\}$
  • Chooses a level of effort to put in: $H_t \times \Phi^* \rightarrow \{0, 1\}$
  • Chooses a reporting strategy: $H_t \times \Phi^* \times \{0, 1\} \rightarrow M_i$
General Model: Optimal Contract

Overview:
1. The firm’s contracting problem; differences from the baseline model
2. The stage game equilibrium when $\forall Y: b(Y) = 0$
3. The optimal contract in the repeated game
General Model: Optimal Contract (1)

Firm’s contracting problem:

• Focus on stationary relational contracts where the contract is invariant over time, with no loss of generality in the class of semi-public strategies.

• We carry over three constraints essentially unchanged from the baseline model:
  • Budget constraint (B’): \( w \geq w_{rs}(Y) + w_{sr}(Y), \forall Y \in \{y_1, \ldots, y_N\}, \forall r, s \in S \)
  • Truth-telling for both agents (T’): \( E_{\{Y,r\}}[w_{rs}(Y)|e = 1, s_i = s] \geq E_{\{Y,r\}}[w_{rs}(Y)|e = 1, s_i = s], \forall s, s' \in S \)
  • Participation constraints for both agents (IR’): \( u_i = E_{\{Y,r,s\}}[b(Y) + w_{rs}(Y)|e = 1] - c \geq 0, \forall i \)

• We need to alter the incentive constraint to reflect the fact that the agent can now also deviate by changing his reporting strategy on his co-worker (i.e. double deviate): (IC)

\[
E_{\{Y,r,s\}}[b(Y) + w_{rs}(Y)|e_i = 1, e_{-i} = 1] - c \geq \max_{s'} E_{\{Y,r\}}[b(Y) + w_{rs'}(Y)|e_i = 0, e_{-i} = 1]
\]
General Model: Optimal Contract (2)

Firm’s contracting problem:

• Dynamic enforcement constraint for F to not renege on payment of public performance incentives (DE): $\frac{\delta}{1-\delta} \pi \geq \max_j 2b(y_j)$

• The firm’s contracting problem is thus to choose the profit-maximising contract:

$$\max_{\phi \in \Phi^*} \pi \quad \text{subject to} \quad (B'), (T'), (IR'), (IC), (DE)$$

• Note that since the participation constraints must bind, the objective function reduces to maximisation of total surplus generated by the contract:

$$\pi = [E[Y|e = 1] - 2c] - [w - 2E_{Y,r,s}[w_{rs}(Y)|e = 1]]$$

• Denote by $z = w - 2E_{Y,r,s}[w_{rs}(Y)|e = 1]$ the surplus that is destroyed when the firm relies on subject performance evaluations.

• We can also think of the problem as minimising $z$ subject to $(B'), (T'), (IR'), (IC), (DE)$. 
**General Model: Optimal Contract (3)**

*Proposition 2* (Optimal contract in the stage game)

In the optimal contract that induces effort in the stage game, the firm commits to a payroll expense:

\[ \hat{w} = \frac{2\beta_1 q_1}{\beta_1 q_1 - \alpha_1 p_1} c \]

The agents’ compensation is given by:

\[ \hat{w}_{rs}(Y) = \begin{cases} 
  \frac{1}{2} \hat{w} - \frac{c}{\beta_1 q_1 - \alpha_1 p_1} & \text{if } Y = y_1 \text{ and } r = 1 \\
  \frac{1}{2} \hat{w} & \text{otherwise}
\end{cases} \]
General Model: Optimal Contract (4)

Comments:
• The optimal contract’s reward for agent $i$ only depends on his co-worker’s report and not his report on his co-worker.

• **Double-deviation**: Compared to the baseline model, an agent can deviate by shirking and by deviating from the contracted reporting strategy.
  - Given the contract in the baseline case, an agent can shirk and avoid/mitigate punishment by sending a bad report on his co-worker.

• **Surplus destruction**: Punishing agents involves “burning money”, where a part of $w$ is wasted, i.e. $w > w_{rs}(Y) + w_{sr}(Y)$.

• **Wage compression**: The firm punishes the agents only in the case that is most informative of shirking. This minimises surplus destruction while still giving the agent sufficient incentive to exert effort.
General Model: Optimal Contract (5)

• Expected Surplus Destruction under the contract is given by-

\[
\hat{z} = \hat{w} - 2E_{\{y, r, s\}}[w_{rs}(Y); e = 1] = \frac{2\alpha p_1 c}{\beta_1 q_1 - \alpha_1 p_1}
\]

Notice that expected surplus destruction decreases as \(\beta_1 q_1\) becomes larger compared to \(\alpha_1 p_1\) i.e. There is a greater chance of the worst output and signal being realised, given the agent shirks.

• The firm’s payoff \(= v - 2c - \hat{z} \geq 0\) iff \(\frac{\beta_1 q_1}{\alpha_1 p_1} \geq \frac{v}{v-2c}\)
Proposition 3 (Stage Game)

Given *surplus destruction* $z$, the firms payoff $\pi = v - 2c - \hat{\mathcal{Z}} \geq 0$ iff

$$\frac{\beta_1 q_1}{\alpha_1 p_1} \geq \frac{v}{v-2c}.$$  Two cases arise as a result.

<table>
<thead>
<tr>
<th>$\frac{\beta_1 q_1}{\alpha_1 p_1} &lt; \frac{v}{v-2c}$</th>
<th>$\exists$ Nash equilibrium where the two workers do not put in any effort</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{\beta_1 q_1}{\alpha_1 p_1} \geq \frac{v}{v-2c}$</td>
<td>$\exists$ multiple equilibria- the worst where agents put in no effort and the best where there is no surplus destruction and firm earns $v - \hat{\mathcal{W}}$</td>
</tr>
</tbody>
</table>
• Recall that $\beta_1 = \Pr(y = y_1; e = (0,1) \text{ or } (1,0))$, $q_1 = \Pr(s_i = s; e_{-i} = 0)$
  $\alpha_1 = \Pr(y = y_1; e = (1,1))$ and $p_1 = \Pr(s_i = s; e_{-i} = 1)$

• Precision of the performance-signals matters. As $\frac{\beta_1 q_1}{\alpha_1 p_1}$ increases, there is a greater relative chance of getting the worst output + worst signal if the agent doesn’t put in effort

• Reward and punishment would have a “relatively small spread” in that case and this would involve less surplus destruction.

• If the value of the fraction is low i.e lower relative chance of the worst output + worst signal being realised upon shirking (higher relative chance of worst output+ signal being realised upon putting in effort) then the principal would have to increase the threat of punishment to garner effort $\rightarrow$ more surplus destruction

• This surplus destruction can be allowed until the firm’s payoff is zero.
Proposition 4 (Repeated Game)

<table>
<thead>
<tr>
<th></th>
<th>( \delta &lt; \delta^* )</th>
<th>( \delta \geq \delta^* )</th>
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<tbody>
<tr>
<td>( \frac{\beta_1 q_1}{\alpha_1 p_1} ) &lt; ( \frac{v}{v-2c} )</td>
<td><em>Subjective bonus is not used</em> (P3 violated) i.e. ( w=w^* ) and each agent gets ( w^*/2 ).</td>
<td>Subjective bonus is not used.</td>
</tr>
<tr>
<td></td>
<td><em>Public performance bonus</em> not used either (Dynamic enforcement constraint will be violated)</td>
<td>Public performance bonus is used in the following manner-</td>
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<tr>
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<td></td>
<td>( b^<em>(y) = b^</em> ) if ( Y \geq y_{k^*} )</td>
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<td></td>
<td></td>
<td>= 0 otherwise</td>
</tr>
<tr>
<td>( \frac{\beta_1 q_1}{\alpha_1 p_1} \geq ( \frac{v}{v-2c} )</td>
<td>Subjective bonus is used in same manner as in Proposition 2. Public performance bonus is used in the following manner-</td>
<td></td>
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<tr>
<td></td>
<td></td>
<td>( b^{<strong>}(y) = b^{</strong>}(\delta) ) if ( Y \geq y_{k^*} )</td>
</tr>
<tr>
<td></td>
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<td>= 0 otherwise</td>
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</tbody>
</table>
• **What is** \( y_{k^*} \)? It is the output level such that \( \alpha_m \leq \beta_m \) for all output levels \( y_m \) below it. At \( y_{k^*} \) and above \( \alpha_m > \beta_m \)

i.e. at or above \( y_{k^*} \), there is a greater chance of observing that particular output given the agents worked as opposed to one of them shirking → the firm pays a bonus

• **What is** \( b^* \)? **Why is it optimal?** \( b^* = \frac{C}{\sum_{j=k^*} (\alpha_j - \beta_j)} \). This comes from the IC constraint and the nature of the contract as specified in Proposition 4.
Lessons from Proposition 4

- Patient firms never use the subjective performance bonus (bonus based on peer evaluations). Impatient firms use it only when the condition in Proposition 3 is satisfied i.e. profit is weakly greater than zero.

- Public Performance bonus is used alone only when the firm is patient. It is equal to zero if output is below its threshold level.

- Subjective performance bonus has two contradictory roles involving the DE constraint. Can surplus destruction be avoided? (What if surplus is just transferred from shirking agent A to hard-working agent B? Agent A would prefer to lie.... “double deviation” problem again..)
When signals are correlated

• Will the agent’s pay now be dependent on the report he writes?
  
  Not necessarily.

• Depends on which of the two mechanisms is more useful in finding whether worker shirked or not- a bad report v/s a report mismatch

• That in turn depends on whether the correlation is strong or not. If “relative performance” matters and the worker’s report affects his payoff, he can shirk and just write a bad report about his co-worker. Only if there is high correlation among signals can the agent ensure this doesn’t happen.
Discussion: Collusion? (1)

- Can both agents shirk and then send good peer evaluations?
- Authors show a collusion scheme may not be profitable
- It encourages both to shirk – free riding problem – and this negatively affects the expected public performance bonus
  - Under optimal contract, both agents indifferent between working and shirking
  - Suppose $A_1$ always sends a good report. When $A_2$ works, her payoff increases by $\alpha_1 p_1 \Delta$. When $A_2$ shirks, her payoff increases by $\beta_1 q_1 \Delta$. Note that payoffs change only when the worst scenario is realised – low output and bad report received
  - So if $A_1$ covers, the other agent will shirk and the former’s expected public performance bonus will fall – the participation constraint does not hold
Discussion (2)

• Restricting class of strategies - implications

• Why is compensation to an agent not based on both the report sent and the report received?

• Public signal reflects agents’ joint effort choice is important for previous results – if multiple signals, baseline first best

• What if peer reports were public, not private?

• Is money burning necessary? – firm and several teams

• Can money burning be mitigated? – reduce evaluation frequency

• Firm truth telling via alternate monitoring approach
Main Results

• What is optimal combination of public subjective measure and private subjective measure? Key is both firm and agents need incentives for truth telling

• When productive and informative roles are separate, first best can be achieved – incentives for inducing effort not linked to incentives for inducing truth telling

• When the roles merge, incentives are linked and first best cannot be achieved – we note that peer evaluation considered only when firm is impatient and surplus destruction is small
Thank you