1st Year MPhil Microeconomics Exam

I.O. questions and answers 2000/01 –2009[1]

These are all the 1st Year MPhil Microeconomics Exam questions (and answers) on Industrial Organization, including Internal Organization,[2] set since 2001, inclusive, and also the 2000 essay question. (Some of these questions may be included in the revision problem set which will be covered in the revision class.)

Most, but not all, of these questions were set by me (the most recent ones weren't); the answers are also generally by me except where stated.

Paul Klemperer.

[1] No exam question was set on I.O. in 2009. So the last question in this set is from 2008.

[2] Internal Organization may sometimes be taught in the "Bargaining and Contracts" course.

2008 1st year micro exam Industrial Organisation problem

3. Consider a Cournot oligopoly in which each of n firms faces a constant marginal cost of c. Given the industry's total output Q, price satisfies $P(Q) = \alpha + \beta Q^{-\gamma}$ where $\beta, \gamma > 0$.

(i) Calculate the curvature (that is, the elasticity of the slope) of the inverse-demand curve.

(ii) How is the curvature related to (a) the slope of marginal revenue and hence the concavity of a firm's profits in its output; and (b) the slope of a firm's reaction function?

Now assume that $\gamma < 1$ and $(c - \alpha)\gamma > 0$.

(iii) Show that the social surplus (profits plus consumer surplus) from the supply of Q units is

$$S(Q) = \frac{\beta Q^{1-\gamma}}{1-\gamma} - (c - \alpha)Q.$$

Hence calculate the socially efficient output $Q^{\dagger}$ and the maximum feasible social surplus $S^{\dagger}$.

(iv) Calculate the Nash equilibrium output Q^{*} and per-firm profits π^{*}. Show that

$$Q^{*} = \left[1 - \frac{\gamma}{n}\right]^{1/\gamma} \times Q^{\dagger} \quad \text{and} \quad n \times \pi^{*} = \frac{1-\gamma}{n-\gamma}\left[1 - \frac{\gamma}{n}\right]^{1/\gamma} \times S^{\dagger}.$$

(v) How does the curvature of demand affect the pass through of cost changes to consumers?

(vi) How does your answer to (iv) lead to a natural measure of monopoly power? How does the curvature of demand influence this power? (In answering the second question, you may find it useful to note that $\log[1 - x] + [x / (1 - x)]$ is positive for all $x < 1$.)

(vii) "Market power is enhanced by inelastic demand." Discuss this claim.

HECN 5372
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2008 1st year micro exam Internal Organization Essay

PART B

7. There is a firm that produces gadgets. It needs widgets as inputs and it buys them from another firm. Currently, the gadget producer is unhappy with the behaviour of the widget producer. Should they merge?

2007 1st year mica exam
Industrial Organisation problem

3. Consider a simultaneous-move Cournot duopoly in which firm $i \in \{1,2\}$ chooses output $q_i \geq 0$ and incurs a constant marginal cost of c_i. The inverse-demand curve is linear, so that the price satisfies $p = 1 - Q$ where $Q = q_1 + q_2$.

(i) Show that firm i's best-reply $\mathrm{BR}_i(q_i)$ to output q_j from its competitor satisfies

$$q_i = \mathrm{BR}_i(q_j) = \max\left[\frac{1-c_i-q_j}{2}, 0\right]$$

(ii) Find the Nash equilibrium, ensuring that you distinguish between cases where either (a) the inequality $2\max[c_1,c_2] < 1 + \min[c_1,c_2]$ holds, and (b) the inequality fails.

Now suppose that each firm can choose its production technology. Firm i can choose any marginal cost c_i satisfying $0 \leq c_i \leq 1$. However, by choosing a lower marginal cost it increases its fixed costs. Its fixed cost F_i satisfies:

$$F_i = \frac{a(1-c_i)^2}{2}$$

(iii) Show that a monopolist is strictly profitable if and only if $a < \frac{1}{2}$.

Assume that $a < \frac{1}{2}$. Consider a game in which the Cournot duopolists simultaneously choose their outputs and production technologies.

(iv) Show that each firm $i \in \{1,2\}$ will choose either $c_i = 0$ or $c_i = 1$.

(v) Show that there is an equilibrium in which one firm monopolizes the industry (so that $q_i > 0$ and $q_j = 0$ for $i \neq j$) if and only if $a \geq \frac{1}{8}$.

(vi) Show that there is an equilibrium in which both firms supply positive output (so that $q_1 = q_2 > 0$) if and only if $a \leq \frac{2}{9}$.

(vii) How might you choose between the multiple equilibria when $\frac{1}{8} \leq a \leq \frac{2}{9}$?

2006 1st year micro exam Industrial Organisation problem

[I told the examiners I thought this was a little long, and that they should probably shorten it. They didn't but, after the fact, I think they agreed I was right.]

Consider a linear city inhabited by two types of consumers, male (m) and female (f), each distributed uniformly in the interval $[0, 1]$. There are two nightclubs located at the extremes of the interval, that compete to attract customers. They both have the same constant marginal cost, c, of supplying each type of customer. Let p_i^l be the price charged by the club in location l to each type $i = m, f$. Then, a consumer of type i located in $x \in [0, 1]$ has a total cost of $p_i^0 + tx$ for going to club 0, and $p_i^1 + t(1 - x)$ for going to club 1, where t is the marginal transport cost (same for everyone). The value of clubbing is so high that all consumers visit one or the other venue.

a. Calculate the demand function for each of the clubs.

b. Compute the equilibrium prices for each club.

Now, suppose that male consumers enjoy an additional benefit αn^l, when going to club l, where n^l is the number of women going to club l, and α is a constant. Women's preferences remain the same as before (they are indifferent to men). Assume that $2t > \alpha$.

c. Calculate the demand function for each club.

d. Calculate the equilibrium prices. (Hint: assume a symmetric equilibrium, i.e. $p_m^0 = p_m^1 = p_m$ and $p_f^0 = p_f^1 = p_f$.)

e. Interpret your results in parts (c) and (d). How do prices depend on α and t?

   Explain also what is the role of the condition $2t > \alpha$. Without doing any calculation, could you argue what would happen if this condition was not satisfied?

f. Without doing any more calculations what do you think would happen if one club decided to break up into two, one catering only to men, and the other only to women (both in the same location)? Could lowering the quality of the offering to men (without affecting the quality of the offering to women) possibly increase a firm's profits? Why or why not? (Hint: think about what could happen when α approaches $2t$.)

2006 1st year micro exam Internal Organisation essay

Consider a car company that is considering outsourcing the manufacturing of some of its cars' parts. Discuss the costs and benefits that this strategy could bring to the company. How could they manage the relationship with suppliers to make the outsourcing decision more profitable?

2005 First-Year Microeconomic Theory Exam Problem Question

PART A

1. A cow produces milk, which can be used as an input by a cream (c) or a yogurt (y) producer. Neither firm can produce the other's output, and milk has no alternative use. A final consumer wants to buy exactly one pound of cream and one pound of yogurt, and values each of these at $v_c(I_c, s_c) = I_c + s_c$ and $v_y(I_y, s_y) = I_y + s_y$, respectively, where $I_c < 1$ and $I_y < 1$ are the investments of each producer, and s_c, s_y are shocks. With probability $\frac{1}{2}$, c receives a favourable shock $s_c = 1$, while $s_y = 0$ (notice that in this case, $v_c > v_y$ for all values of I_c and I_y). Otherwise (also with probability $\frac{1}{2}$), the shocks are $s_c = 0$ and $s_y = 1$ (in which case $v_c < v_y$).

Each of the firms can extract the entire consumer surplus. The cost of the investments for c and y are I_c^2 and $2I_y^2$, respectively. The production of one pound of c or y can be made at zero cost, but requires the input of one gallon of milk (a firm without milk cannot produce so earns zero). However, the cow has a capacity constraint. With probability μ, the cow produces only one gallon of milk, and with probability $1-\mu$, it produces two. Therefore, with probability μ there is a shortage of milk. The cow's capacity (one gallon or two) is independent of the realization of the shocks (so there are four different states of the world).

Consider two types of integration: we have c-integration if the cream producer owns the cow; if the yogurt producer owns the cow we have y-integration. Whoever owns the cow can decide what to do with the milk being produced. Assume that any bargaining yields the Nash bargaining solution with equal bargaining powers, and bargaining occurs after the shock and the cow's capacity constraint are realized.

(i) Describe who should be producing in each state of the world. Hence set up the social planner's problem and solve for the first-best levels of investment.

(ii) Suppose that c owns the cow. What are the payoffs to c and y that arise from the bargaining stage in each state of the world?

(iii) Compute the incentives to invest under c-integration, and under y-integration.

(iv) Compare your results in part (iii) with the first best in part (i). Is there over- or under-investment? Why?

(v) Denote by W^c and W^y the welfare from each type of integration. Calculate $W^c - W^y$ and determine the optimal ownership structure. Is the difference $W^c - W^y$ increasing or decreasing in μ? Interpret your results.

PART B

5. What interpretations are given to the following two measures: $L_i = \frac{p_i - C_i'}{p_i}$ and $H = \sum_i \left(\frac{q_i}{Q}\right)^2$, in which p_i is the price, q_i the quantity, and C_i' the marginal cost of firm i, and $Q = \sum_i q_i$ is aggregate output in the industry? How are these measures related under Cournot competition? Discuss the extent to which this relationship holds more generally.

2004 First-Year Microeconomic Theory Exam Essay Question

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5. "Since economists cannot tell whether oligopolistic competition is better modelled by price competition or quantity competition, they cannot tell whether oligopolists have incentive to over- or under-invest in cost reduction." Do you agree?

2003 Micro Questions for Industrial Economics

Part A (Problem)

4. Consider a Hotelling model in which consumers who wish to purchase one unit each are uniformly distributed on a line from 0 to 1 and buy from the firm which gives them the best deal, including travel costs and price. Travel costs are 1 per unit. Firms A and B are located at 0 and 1 respectively. A has a constant marginal cost of production, $c < 1$. B's costs are zero.

   (i) Assume each firm simultaneously sets a price p_A, and p_B respectively.

      (a) Compute firms' equilibrium profits and market shares.
      (b) How much will a small increase in A's marginal cost reduce A's profits? Is this more than, less than, or equal to the change in A's cost times A's market share? Why?

   (ii) Now assume that instead of the firms offering the same prices to all consumers, firms price separately to each individual consumer (i.e., firms can price discriminate perfectly). Compute firms' equilibrium profits and market shares for this case.

   (iii) Are industry profits higher in case (i) or case (ii)?

   Without making any more calculations,

   (I) Is case (i) or case (ii) more efficient?
   (II) Are consumers better off on average in case (i) or case (ii)?

Extra Questions (NOT on Exam)

(A) Answer question (i)(b) for the situation of part (ii).

(B) Is firm A's market share higher in case (i) or (ii)? Why?

Part B (Essay)

8. **Should policy makers seek to promote the entry of more firms into industries than would arise naturally?**

2002 Micro Questions for Industrial Economics

Part A (Problem)

5. The City of Oxford is visited by many tourists, who buy Q sweatshirts with "University of Oxford" emblazoned across the front. Sweatshirts may be produced at a constant marginal cost of c. The inverse demand curve for sweatshirts satisfies $P = 1 - Q$. There are N official sweatshirt suppliers in Oxford, and M unofficial suppliers. All suppliers simultaneously choose quantities.

(i) Characterize the Cournot Nash equilibrium among the $N + M$ sweatshirt suppliers.

The University now has the opportunity to regulate the N official suppliers. In fact, it can specify in advance that they are to produce a total supply of Q_D sweatshirts. The University cares about the profits made by the official suppliers, but does not care about the welfare of tourists or the unofficial suppliers.

(ii) Given the regulated supply of Q_D sweatshirts from the official suppliers, what will be the equilibrium quantities offered by the unofficial suppliers?

(iii) Show that the University will set Q_D equal to the monopoly supply of sweatshirts.

Suppose now that the University is unable to regulate the official supply of sweatshirts. Instead, it is able to charge a royalty fee t on each sweatshirt sold by an official supplier. It cares equally about profits made by official local retailers and the income it raises from royalties.

(iv) Using your answer to part (iii), or otherwise, calculate the optimal choice of royalty.

(v) Show that the University will wish to subsidise the official retailers (i.e. set $t < 0$) whenever $N < M + 1$.

Part B (Essay)

9. What is ownership, and how does it matter?

2001 Micro Problem Question for Industrial Economics

2. Assume that a market has an infinite number of potential entrants, who are lined up sequentially. When it is entrant j's turn to enter it must pay a sunk cost of F if it wishes to enter. If it enters, it (and only it) learns its constant marginal cost c_j, which is drawn independently from a uniform distribution on [0,1]. If firm j declines to enter no subsequent firm is allowed to enter the market; after this, these firms which have entered the market, i.e., firms $1, 2, \ldots j-1$, learn each others' marginal costs. There is one customer who has a reservation value of 1 for a single unit.

   (i) Assume that no firm can make a credible commitment about price until entry has ceased. After entry ceases, firms engage in Bertrand competition, in which the firm with the lowest actual cost sells to the customer at a price equal to the cost of the second lowest firm. How may firms will enter?
   Hint: If N independent draws are taken of a random variable uniformly distributed between 0 and 1, then the expected value of the Kth lowest draw will be $\frac{K}{N+1}$ for all $K \leq N$.

   (ii) Now assume that after each entrant enters, it announces a price which it can neither raise nor lower. Describe an equilibrium in which entry continues until someone announces a price that is low enough to deter future entry.

   (iii) Now assume that after each entrant enters it announces a price that cannot be raised, but can be lowered in Bertrand competition if anyone else enters. Now what price will deter entry?

   (iv) Which equilibrium is more efficient, that in part (ii) or part (iii)?

2001 Micro Essay Questions for Industrial Economics

7. The Herfindahl Index of an industry Is $H = \sum_i \left(\frac{q_i}{Q}\right)^2$ in which q_i is firm i's market share and Q is total industry output. Explain why anti-trust regulators might think industries with higher Herfindahls are 'worse' industries. Should they?

2000 1st Year Microeconomic Theory Exam Essay Question

(i) Consider a differentiated products duopoly. Does the price-setting game typically yield the same prices or higher prices or lower prices than the quantity-setting game?

(ii) Why?

(iii) Which is the better model?

Question set – and answer supplied by David Myatt

Notes on the Suggested Problem for MPhil Micro Paper 2008

Differentiating the inverse-demand function we obtain

$$P'(Q) = -\beta\gamma Q^{-(1+\gamma)} \quad \text{and} \quad P''(Q) = \beta\gamma(1+\gamma)Q^{-(2+\gamma)}.$$

The curvature (elasticity of $|P'(Q)|$) is then

$$-\frac{QP''(Q)}{P'(Q)} = 1+\gamma.$$

This answers part (a). Marginal revenue for firm i is $P(Q) + q_i P'(Q)$. This is decreasing in the firm's own output q_i if and only if $2P'(Q) + q_i P''(Q) < 0$. Substituting in the expressions above and re-arranging this is true if and only if

$$2 > \frac{(1+\gamma)q_i}{Q}.$$

This holds so long as the curvature of demand is not too large. This is hardest to satisfy for a monopolist, when $q_i = Q$, in which case we need $\gamma < 1$. A firm's reaction function is downward sloping if and only if its marginal revenue is decreasing in the output of others, which is true if and only if $P'(Q) + q_i P''(Q) < 0$. Using this expressions derived above again, we need

$$1 > \frac{(1+\gamma)q_i}{Q}.$$

A critically sufficient condition for this to hold is $\gamma < 0$. More generally, reaction functions are downward sloping local to a symmetric Cournot-Nash equilibrium if $\gamma < n-1$. Similarly, marginal revenue is decreasing local to an equilibrium if $\gamma < 2n-1$. The basic conclusion to part (b) is that the standard Cournot properties holds so long as demand is not too convex.

Turning to part (c), we can verify by claim by checking that $S'(Q) = P(Q) - c$:

$$S'(Q) = \beta Q^{-\gamma} - (c-\alpha) = [\alpha + \beta Q^{-\gamma}] - c = P(Q) - c.$$

We obtain the socially efficient output by setting $S'(Q) = 0$, and hence equating price to marginal cost. Solving this and plugging back in to $S(Q)$ leads to

$$Q^\dagger = \left[\frac{\beta}{c-\alpha}\right]^{1/\gamma} \quad \text{and} \quad S^\dagger = \frac{\gamma}{1-\gamma}\left[\frac{\beta}{(c-\alpha)^{1-\gamma}}\right]^{1/\gamma}.$$

For part (d) we equate marginal revenue to marginal cost in the usual way. Hence

$$P(Q) + \frac{QP'(Q)}{n} = \alpha + \beta Q^{-\gamma} - \frac{\gamma\beta Q^{-\gamma}}{n} = c \quad \Leftrightarrow \quad \left[1 - \frac{\gamma}{n}\right] \beta Q^{-\gamma} = c - \alpha.$$

This solves straightforwardly to given the solution for Q^* stated in the question, and substituting yields industry profits as claimed.

For part (e) we plug the solution for Q^* back in to $P(Q)$ to obtain

$$P(Q^*) = \alpha + \frac{n(c-\alpha)}{n-\gamma}.$$

The derivative with respect to c is $n/(n-\gamma)$, which in turn is increasing in γ. Hence the extent of cost pass-through is increasing in the curvature of demand.

For part (f), the solution for part (d) suggests that for $n = 1$ (we are asked about monopoly power) a monopolist a fraction m of maximum feasible social surplus $S^\dagger$ where $m = (1-\gamma)^{1/\gamma}$. Differentiating this we obtain

$$\frac{\partial \log m}{\partial \gamma} = -\frac{\log(1-\gamma)}{\gamma^2} - \frac{1}{\gamma(1-\gamma)} \leq 0 \quad \Leftrightarrow \quad \log(1-\gamma)\gamma^2 + \frac{\gamma}{(1-\gamma)} \geq 0.$$

This last inequality holds since we are told it does so in the question. Hence the fraction of surplus captured by the monopolist is decreasing in the curvature of demand. (This is also true when we consider the fraction of surplus extracted by a Cournot oligopoly.)

For the final part, note that when $\alpha = 0$ we obtain a constant-elasticity specification with elasticity of demand $1/\gamma$. Hence a reduction in elasticity corresponds to an increase in γ, and hence an increase in curvature. But this reduces the fraction of surplus extracted by a monopolist (this is also true for oligopolists) and so contradicts the statement.

Solution to Essay Question 2008 (Set - and answer supplied by Misha Drugov)

The essay could start by saying that if there are other producers of widgets and any of them can be used in the production of gadgets, the best option would probably be simply to switch to another widget supplier.

If this is not the case, i.e., it is impossible or very costly to use other widgets, it is efficient to continue the relationship. The merger is then one of several options. Whether the merger is the best option depends on the particular situation. The essay could consider the following:

- They cannot write a detailed contract that would specify the price of the widgets, the quantity, the terms of delivery, etc. and the widget producer often engages into bargaining with the gadget producer trying to extract a better price. This leads to delays in delivery, uncertainty over future supplies and other inefficiencies. Williamson argues that they should merge in this case because inside the firm one side has the authority to impose its will on the other. Integration avoids ex-post haggling. This is the usual interpretation of the Fisher Body case (the essay might cite Klein, Crawford & Alchian (1978)).

- The widget producer does not invest enough into developing better modifications of widgets. If they can contract on this investment, they should do it and the gadget producer should finance the investment (if these better widgets cannot be used by other gadget producers). If not, they can consider contracting on the prices of different models of widgets (and quantities supplied). This will insure the widget producer against the hold-up by the gadget producer. If this is not possible (for example, different modifications are not distinguishable by an outside party), the merger is optimal. This is the view of the property right theory of the firm (Grossman, Hart &Moore). Moreover, if there is a widget manager that has some specific knowledge about the widget production, it is the widget producer that should own the merged firm.

- The widget producer does not put too much effort into making quality widgets but it also has to maintain the widget production line. We are in the situation of multitasking. Probably, the maintenance is not easily observable. Then, integration could provide better incentives for making quality widgets but it could also destroy the incentives to maintain the widget production line since the widget producer is not its (sole) owner anymore. The merger is then not optimal. This is the view of incentive system theory of the firm (Holmstrom and Milgrom).

The essay could conclude that two (necessary) conditions should be satisfied for the integration to be optimal. First, the two parties are locked-in in the relationship; second, ("reasonably") complete contracts are not possible.

MPhil Microeconomics 2007: IO Sketch Solution

Question set — and answer supplied — by David Myatt

(i) If j produces q_j units then $p \leq 1 - q_j$. Hence, if $c_i > 1 - q_j$ then firm i cannot profitably supply positive output, so its best reply is $q_i = 0$. However, if $c_i < 1 - q_j$,

$$\pi_i = q_i(1 - q_i - q_j - c_i).$$

This is strictly concave in q_i, and the FOC is necessary and sufficient:

$$\frac{\partial \pi_i}{\partial q_i} = 1 - 2q_i - q_j - c_i = 0 \quad \Leftrightarrow \quad q_i = \frac{1 - c_i - q_j}{2}.$$

(ii) Label the firms so that $c_1 \leq c_2$. One possible equilibrium is for firm 1 to act as a monopolist, so that $q_2 = 0$. Applying the reaction function derived in part (i),

$$q_1 = \frac{1 - c_1}{2}.$$

It is a best reply for firm 2 to set $q_2 = 0$ so long as $c_2 \geq 1 - q_1$, or equivalently

$$c_2 \geq 1 - \frac{1 - c_1}{2} = \frac{1 + c_2}{2} \quad \Leftrightarrow \quad 2c_2 \geq 1 + c_1.$$

This corresponds to case where $2\max[c_1, c_2] \geq 1 + \min[c_1, c_2]$. For case (a), both firms can produce in equilibrium. Hence we need to solve simultaneously:

$$q_1 = \frac{1 - c_1 - q_2}{2} \quad \text{and} \quad q_2 = \frac{1 - c_2 - q_1}{2}.$$

Doing so yields the familiar solution:

$$q_i = \frac{1 - 2c_i + c_j}{3} \quad \text{for } i \in 1, 2 \text{ and } j \neq i.$$

Substituting to obtain a firm's profits leads to $\pi_i = q_i^2$ where q_i satisfies the equality given above. Checking that $q_2 > 0$ leads to $2c_2 < 1 + c_1$, or equivalently $2\max[c_1, c_2] < 1 + \min[c_1; c_2]$, confirming that we are considering case (a).

(iii) As a monopolist, firm i would choose output satisfying

$$q_i = \frac{1 - c_i}{2},$$

and its variable profits will be q_i^2. Taking fixed costs into account,

$$\pi_i = \frac{(1 - c_i)^2}{4} - \frac{\alpha(1 - c_i)^2}{2} = \frac{(1 - c_i)^2}{2}\left[\frac{1}{2} - \alpha\right].$$

By inspection, net profits are strictly positive only when $\alpha < \frac{1}{2}$.

(iv) Consider the problem faced by firm i, and fix the output q_j of its opponent. Given its choice of c_i, it will choose its output q_i to satisfy $q_i = \text{BR}_i(q_j)$, where $\text{BR}_i(\cdot)$ is the best-reply derived in part (i). Its profits, as a function of c_i, satisfy

$$\pi_i = q_i(1 - q_i - q_j - c_i) - \frac{\alpha(1-c_i)^2}{2} \quad \text{where} \quad q_i = \max\left[\frac{1 - c_i - q_j}{2}, 0\right]$$

Differentiating with respect to c_i, we obtain

$$\frac{\partial \pi_i}{\partial c_i} = \alpha(1 - c_i) - q_i,$$

where we have applied the envelope theorem. By inspection, π_i is strictly increasing in c_i whenever $q_i = 0$, or equivalently when $c_i \geq 1 - q_j$. Hence, an interior choice of c_i must satisfy $c_i < 1 - q_j$. For $c_i < 1 - q_j$, we can evaluate the second derivative:

$$\frac{\partial^2 \pi_i}{\partial c_i^2} = -\alpha - \frac{\partial q_i}{\partial c_i} = \frac{1}{2} - \alpha > 0.$$

Hence π_i is convex in c_i within the range $c_i < 1 - q_j$. Any c_i satisfying a FOC for firm i is a local minimum. So, firm i chooses either $c_i = 0$ or $c_i = 1$. Note that:

$$c_i = 1 \quad \Rightarrow \quad F_i = \frac{\alpha}{2}$$

(v) Consider an equilibrium in which $q_1 > 0$ and $q_2 = 0$. Firm 1 is producting positive output, and hence must choose $c_1 < 1$, which implies that $c_1 = 0$, and so $q_1 = \frac{1}{2}$. Firm 2 chooses $c_2 = 1$ and earns zero profits. If firm 2 deviated, it would choose $c_2 = 0$ and produce a best-reply output of $q_2 = \frac{1}{4}$, yielding variable profits of $1/16$. This deviation is non-profitable so long as

$$\frac{1}{16} \leq \frac{\alpha}{2} \quad \Leftrightarrow \quad \alpha \geq \frac{1}{8}$$

(vi) Consider an equilibrium in which both firms operate. Given part (iv), we must have $c_1 = c_2 = 0$. Applying the Cournot solution derived in part (ii), $q_1 = q_2 = \frac{1}{3}$, and firms each earn variable profits of $\frac{1}{9}$. This is profitable so long as

$$\frac{1}{9} \geq \frac{\alpha}{2} \quad \Leftrightarrow \quad \alpha \leq \frac{2}{9}.$$

(vii) One possibility is to consider a two-stage game in which firms first play a small cost to enter, and then consider a forward-inductive argument.

2006 1st year micro exam Industrial Organisation problem solution

Part (a).

Since each of the two types of consumers are independent, we can compute the demands and equilibrium separately. Let x_i^* be the marginal consumer of type i that is indifferent between going to club 0 and 1. Then x_i^* satisfies $p_i^0 + tx_i^* = p_i^1 + t\left(1 - x_i^*\right)$. In an interior solution, we have $x_i^* = \frac{1}{2} + \frac{p_i^1 - p_i^0}{2t}$. However, when the price differences are large enough, one of the firms may end up with the entire market. We have:

$$x_i^* = \begin{cases} \frac{1}{2} + \frac{p_i^1 - p_i^0}{2t} & \text{if } \frac{1}{2} + \frac{p_i^1 - p_i^0}{2t} \in [0,1] \\ 0 & \text{if } \frac{1}{2} + \frac{p_i^1 - p_i^0}{2t} < 0 \\ 1 & \text{if } \frac{1}{2} + \frac{p_i^1 - p_i^0}{2t} > 1 \end{cases} \tag{1}$$

Then, total demand for each firm when they set prices $\left(p_m^0, p_f^0\right)$ and $\left(p_m^1, p_f^1\right)$ are $D_0 = x_m^* + x_f^*$ and $D_1 = 2 - x_m^* - x_f^*$, respectively.

Part (b).

Firms maximize total profits: $\pi^0 = \pi_m^0 + \pi_f^0 = \left(p_m^0 - c\right) x_m^* + \left(p_f^0 - c\right) x_f^*$, and analogously for firm 1. Since there is no interaction across markets, this is just the standard Hotelling model for each type of consumer, male and female. We get:

$$p_m = p_f = t + c$$

Part (c).

The marginal male consumer satisfies: $\alpha n^0 - p_m^0 - tx_m^* = \alpha n^1 - p_m^1 - t\left(1 - x_m^*\right)$, which yields

$$x_m^* = \frac{1}{2} + \frac{p_m^1 - p_m^0 + \alpha\left(n^0 - n^1\right)}{2t} \tag{2}$$

For the women, we get the same answer as in part (a):

$$x_f^* = \frac{1}{2} + \frac{p_f^1 - p_f^0}{2t} \tag{3}$$

Now, $n^0 = x_f^*$, and $n^1 = 1 - x_f^*$. Substituting x_f^* into the expression for x_m^* and solving the equation yields:

$$x_m^* = \frac{1}{2} + \frac{t\left(p_m^1 - p_m^0\right) + \alpha\left(p_f^1 - p_f^0\right)}{2t^2}, x_f^* = \frac{1}{2} + \frac{p_f^1 - p_f^0}{2t} \tag{4}$$

Notice, also, that the demands for firm 1 are symmetric: $1 - x_m^* = \frac{1}{2} + \frac{t\left(p_m^0 - p_m^1\right) + \alpha\left(p_f^0 - p_f^1\right)}{2t^2}$ and $n^1 = 1 - x_f^* = \frac{1}{2} + \frac{p_f^0 - p_f^1}{2t}$.

Part (d).

For firm 0, the first-order conditions for the new profit-maximization problem are:

$$\begin{aligned} x_m^* + \left(p_m^0 - c\right) \frac{\partial x_m^*}{\partial p_m^0} + \left(p_f^0 - c\right) \frac{\partial x_f^*}{\partial p_m^0} &= 0 \\ x_f^* + \left(p_m^0 - c\right) \frac{\partial x_m^*}{\partial p_f^0} + \left(p_f^0 - c\right) \frac{\partial x_f^*}{\partial p_f^0} &= 0 \end{aligned} \tag{5}$$

(Second-order conditions are dealt with in part e.) If we assume a symmetric equilibrium, then $p_m^0 = p_m^1 = p_m$ and $p_f^0 = p_f^1 = p_f$, and the first-order conditions become:

$$\begin{aligned} \frac{1}{2} + (p_m - c)\left(-\frac{1}{2t}\right) &= 0 \\ \frac{1}{2} + (p_m - c)\left(-\frac{\alpha}{2t^2}\right) + (p_f - c)\left(-\frac{1}{2t}\right) &= 0 \end{aligned}$$

From the first line we get that $p_m = t + c$. Substituting in the second line, we get that $p_f = t + c - \alpha$.

Part (e).

Prices of female consumers are decreasing in α. Competition for women becomes more aggressive, because this helps each firm attract men (and hence, charge them higher prices). In equilibrium, however, both firms charge the same prices to women, and hence, get the same demand as before. Their profits consequently suffer.

The role of the condition $2t > \alpha$ is twofold. First, it guarantees that both firms make non-negative profits ($\pi^i = \frac{1}{2}(2t - \alpha)$) in equilibrium. But, more than that, it also guarantees that the maximization problem of the firms is concave, and hence, the solution is characterized by the previous first-order conditions. This can be obtained from the condition that the Hessian matrix is negative definite. Consider, for instance, π^0. Then:

$$H = \begin{pmatrix} -\frac{1}{t} & -\frac{\alpha}{2t^2} \\ -\frac{\alpha}{2t^2} & -\frac{1}{t} \end{pmatrix}$$

H is negative definite if and only if $\det(H) = \frac{1}{t^2}\left(1 - \frac{\alpha^2}{4t^2}\right) > 0$, which gives the condition $2t > \alpha$. The conditions says that the "network" effects (the preference of men for having women around) cannot be too strong, compared with the differentiation effect (measured by the transport cost, or preference for going to one club or another).

If this condition is not satisfied, then the preferences of men for having women around are stronger than the effects of the transport cost. As a result, the equilibrium would involve a corner solution: all men and women go to the same club, e.g., $x_m^* = x_f^* = 1$.[1]

[1] In this case, club 0 captures both markets, and must set prices such that club 1 does not want to attract any of its customers. This means $p_f^0 \leq c - t$ and $p_m^0 \leq c + \alpha - t$ (as club 1 could lower both

Part (f).

[Students are not expected to do any of this maths. But the very best students should see the two effects on profits, and that the net effect could increase profits even for the firm that breaks up.]

Suppose the club in location 1 breaks up into two. This eliminates the value αn_f^1 that men enjoyed when going to club 1 before. Consequently, the club's demand from men is altered, and (2) becomes:

$$x_m^* = \frac{1}{2} + \frac{p_m^1 - p_m^0 + \alpha n^0}{2t} \tag{6}$$

Notice that, compared to the case of a single club 1, x_m^* increases (for given prices p_m^0 and p_m^1): demand for club 1 decreases. On the other hand, demand from women stays the same as (3), since they are perfectly happy on their own. Therefore, it is costly for club 1 to break up since, ceteris paribus, its total demand decreases. This direct effect tends to decrease 1's profits.

Solving for (6) and (3) as before, we get

$$x_m^* = \frac{1}{2} + \frac{\alpha}{4t} + \frac{2t\left(p_m^1 - p_m^0\right) + \alpha\left(p_f^1 - p_f^0\right)}{4t^2}, x_f^* = \frac{1}{2} + \frac{p_f^1 - p_f^0}{2t} \tag{7}$$

It follows that $\frac{\partial x_m^*}{\partial p_m^0} = -\frac{1}{2t}$ remains unchanged. But $\frac{\partial x_m^*}{\partial p_f^0} = -\frac{\alpha}{4t^2} > -\frac{\alpha}{2t^2}$ increases (decreases in absolute terms). The absolute value of $\frac{\partial x_m^*}{\partial p_f^0}$ measures the incentives of club 0 to cut female prices in order to increase the appeal to men, and increase their demand. When club 1 breaks up, it decreases the aggressiveness of club 0 (with respect to the female market). This is so because 0 has now less to lose from letting one woman go to the alternative club: increasing the female price p_f^0 increases both the demand for women AND men for club 1 (for women, club 1 is now relatively cheaper; for men, club 1 becomes relatively more attractive since there is fewer women in 0, and more in 1). This makes club 0 aggressive in the pricing of women. But when men and women are not together in location 1, club 0 has less to lose by increasing p_f^0 (for club 1, the extra demand from the women does not translate in stronger demand from men). As a result, club 0 is less aggressive in the pricing of females. This is the strategic effect, and works in the opposite direction: it increases profits for club 1.

In general, either of the two effects could dominate, so the decision to break up into two clubs has an ambiguous effect on profits. Yet the strategic effect can dominate the direct effect, and cause profits to increase. To see this, consider what happens when $\alpha = 2t - \varepsilon$, for $\varepsilon > 0$ small. When both clubs are mixed, competetition yields profits

prices down to c). Furthermore, club 0 must make zero profits (otherwise, club 1 could charge slightly lower prices and capture both markets). Hence, $p_f^0 + p_m^0 = 2c$. Any pair of prices $\left(p_f^0, p_m^0\right)$ satisfying these three conditions will sustain an equilibrium.

of $4t - \alpha = \varepsilon$ for each of them; in the limit, when $\varepsilon = 0$, the initial equilibrium yields zero profits. On the other hand, when breaking up, club 1 would make strictly positive profits. To see this, notice that now club 1 must set $p_f^1, p_m^1 \geq c$. Then club 0 must make strictly positive profits (for instance, setting prices $p_f^0 = p_m^0 = c + \varepsilon$ achieves this) so must charge either $p_f^0 > c - t$ or $p_m^0 > c + \alpha - t$. Suppose, then, that $p_f^0 > c - t$ (the other case is identical). Then club 1 can also make strictly positive profits by setting a price p_f^1 slightly above c but low enough to capture some demand (i.e., $p_f^1 \in \left(c, p_f^0 + t\right)$). Therefore, club 1 would like to break up.

In fact, the strategic effect may dominate even when α is not close to $2t$: suppose that $\alpha = t = c = 1$. When both men and women go to the same club, the equilibrium prices are $p_m = 2$ and $p_f = 1$. Profits are then equal to 0.5 for both clubs. But if club 1 breaks up into two, the equilibrium profits for club 1 are 0.6312.[2]

[For other parameter values, I guess clubs might be in a prisoners' dilemma in which both individually gain by not breaking up, but they would be best off collectively if both broke up.]

[Note that this is a curious model in which, because of the perfectly inelastic demand, oligopoly profits are independent of cost. Whilst the effect of costs on profits is ambiguous, the more usual result is that lower costs raise industry profits. With downward-sloping demand, or of course with more monopoly power, it would be less likely that night clubs would lose from men's preferences to be around women.]

[2] Pablo Casas-Arce did this for me; I haven't checked it. He writes:

Club 0 uses the same first order conditions in (5), replacing for the new demands. (It is easy to see that the second order conditions are satisfied as long as $\alpha < 4t$.) This yields:

$$\begin{aligned} 2p_m^0 &= t + \frac{\alpha}{2} + c + p_m^1 + \frac{\alpha}{2t}\left(p_f^1 - p_f^0\right) \\ 2p_f^0 &= t + c\left(1 + \frac{\alpha}{2t}\right) + p_f^1 - \frac{\alpha}{2t}p_m^0 \end{aligned}$$

Club 1 now maximizes male and female profits ($\left(p_m^1 - c\right)\left(1 - x_m^*\right)$ and $\left(p_f^1 - c\right)\left(1 - x_f^*\right)$) independently. This yields the first order conditions:

$$\begin{aligned} 2p_f^1 &= t + c + p_f^0 \\ 2p_m^1 &= t - \frac{\alpha}{2} + c + p_m^0 + \frac{\alpha}{t}\left(p_f^0 - p_f^1\right) \end{aligned}$$

Substituting for the values of the parameters, and solving the system of equations gives us the equilibrium prices:

$$p_m^0 = \frac{13}{6}, p_f^0 = \frac{29}{18}, p_m^1 = \frac{125}{72}, p_f^1 = \frac{65}{36}$$

and the equilibrium demands:

$$x_f^* = \frac{43}{72}, x_m^* = \frac{7}{12}$$

2006 1st year micro exam Internal Organisation essay solution

[by Pablo Casas-Arce]

The essay might start with a discussion of the role of incomplete contracts in the theory of the firm. When the parties cannot specify what to do under every possible contingency, it becomes difficult to manage the relationship through contractual arrangements, and ownership matters. Outsourcing is simply a transfer of ownership of the productive assets from the company to the new suppliers.

Among the costs and benefits that could be discussed:.

the ownership structure will affect relationship specific investments (Grossman and Hart, 1986)– you might discuss this theory in detail.

It may also affect the type of incentives that can be provided where there are multitasking problems (Holmstrom and Milgrom, 1994): for instance, if the machines need to be maintained properly and utilized to optimal capacity (better to outsource), or when the producer can vary the quality of the parts (better to integrate).

The car company could also suffer problems with supply when outsourcing (Bolton and Whinston, 1993). When producing in-house, instead, it would be assured to get the parts, while an outside producer might decide to sell to another buyer.

This can also lead to bargaining costs (Williamson), as the suppliers could delay deliveries to bargain for higher prices. Integration could mitigate this by using fiat, but instead, it may generate other costs due to bureaucracy,...

One way in which some of these costs associated with outsourcing could be mitigated is through the use of long-term relationships with the suppliers. By doing so, we introduce a repeated game element, and could improve upon spot transactions by creating a system of punishment and reward (this could sustain high specific investments, for instance). Also, the company could invest in a monitoring system, to make sure the parts arrive on a timely manner, and the quality is high, thereby reducing multitasking problems, and haggling costs.

[Grossman and Hart, Holmstrom and Milgrom, and (to a lesser extent) Williamson were covered in the lectures. The Bolton and Whinston paper inspired the problem question on the 2005 exam (but was – at least in 2006 – only in the "Additional References" section of the reading list).]

2005 First-Year Microeconomic Theory Exam Problem Solution

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Solution to Question 1.

Part (a).

With probability $(1-\mu)$, there is no shortage of milk, and both c and y can produce, resulting in an expected surplus of $I_c + I_y + 1$ (notice that the sum of both shocks is 1). With probability μ there is a shortage of milk. In that case, it is optimal to produce cream with probability $\frac{1}{2}$ (when $s_c = 1$), and yogurt with probability $\frac{1}{2}$ (when $s_y = 1$). The expected surplus is then $\frac{1}{2}[I_c + 1 + I_y + 1]$.

As a result, the first best levels of investment solve:

$$\max_{I_c, I_y} \left\{ \begin{array}{c} (1-\mu)[I_c + I_y + 1] \\ +\mu\frac{1}{2}[I_c + 1 + I_y + 1] \\ -\frac{1}{8}I_c^2 - \frac{1}{4}I_y^2 \end{array} \right\}$$

This yields the following optimal investments:

$$\begin{aligned} I_c^{FB} &= \frac{1}{2}\left[(1-\mu) + \mu\frac{1}{2}\right] = \frac{2-\mu}{4} \\ I_y^{FB} &= \frac{1}{4}\left[(1-\mu) + \mu\frac{1}{2}\right] = \frac{2-\mu}{8} \end{aligned}$$

Part (b).

Consider now c-integration. In this case, the c-producer can decide whether he sells the milk to the y-producer, and in the bargaining process will be able to extract some rents. If the cow produces 2 gallons of milk, the c-producer uses 1 gallon, and the outside option for the extra milk would be to throw it away and get zero. In that case, the c-producer could extract $\frac{1}{2}(I_y + s_y)$ of the surplus from the milk used by the y-producer (the same that the y-producer would get). If there is only one gallon being produced, the outside option for the c-producer is to use the milk himself and get $I_c + s_c$. He would keep the milk whenever $s_c = 1$ (in that case, $I_c + s_c > I_y + s_y$). Otherwise, if $s_c = 0$, he would sell the milk to the y-producer for a price $I_c + \frac{1}{2}(I_y + 1 - I_c)$.

Part (c).

The c-producer solves:

$$\max_{I_c} \left\{ \begin{array}{c} (1-\mu)\left[I_c + \frac{1}{2}I_y + \frac{3}{4}\right] \\ +\mu\left[\frac{1}{2}(I_c + 1) + \frac{1}{2}\left(I_c + \frac{1}{2}(I_y + 1 - I_c)\right)\right] \\ -I_c^2 \end{array} \right\}$$

which yields:

$$I_c = \frac{1}{2}\left[(1-\mu) + \mu\frac{3}{4}\right] = \frac{4-\mu}{8}$$

The y-producer solves:

$$\max_{I_y}\left\{\begin{array}{c}(1-\mu)\frac{1}{2}\left[I_y+\frac{1}{2}\right]\\ +\mu\left[\frac{1}{2}0+\frac{1}{2}\left(\frac{1}{2}\left(I_y+1-I_c\right)\right)\right]\\ -2I_y^2\end{array}\right\}$$

which yields:

$$I_y = \frac{1}{4}\left[(1-\mu)\frac{1}{2} + \mu\frac{1}{4}\right] = \frac{2-\mu}{16}$$

Notice that the c-producer over-invests, while the y-producer under-invests, relative to the first best.

Now consider y-integration. The returns on the investments of each of the two producers are now reversed. The solution is then:

$$\begin{aligned} I_c &= \frac{1}{2}\left[(1-\mu)\frac{1}{2} + \mu\frac{1}{4}\right] = \frac{2-\mu}{8} \\ I_y &= \frac{1}{4}\left[(1-\mu) + \mu\frac{3}{4}\right] = \frac{4-\mu}{16} \end{aligned}$$

Now its the y-producer that over-invests, and the c-producer under-invests.

Part (d).

From part (b), the investments for c-integration are:

$$\begin{aligned} I_c &= \frac{1}{2}\left[(1-\mu) + \mu\frac{3}{4}\right] = I_c^{FB} + \frac{\mu}{8} > I_c^{FB} \\ I_y &= \frac{1}{4}\left[(1-\mu)\frac{1}{2} + \mu\frac{1}{4}\right] = \frac{1}{2}I_y^{FB} < I_y^{FB} \end{aligned}$$

From part (c), the investments for y-integration are:

$$\begin{aligned} I_c &= \frac{1}{2}\left[(1-\mu)\frac{1}{2} + \mu\frac{1}{4}\right] = \frac{1}{2}I_c^{FB} < I_c^{FB} \\ I_y &= \frac{1}{4}\left[(1-\mu) + \mu\frac{3}{4}\right] = I_y^{FB} + \frac{\mu}{16} > I_y^{FB} \end{aligned}$$

Notice that the owner of the cow over-invests, while the other producer under-invests with respect to the first best levels of investment. The owner of the cow receives the entire return of his investment when 2 gallows of milk are being produced. When there is only one gallon, he only produces (and receives the full return on his investment) with probability $\frac{1}{2}$. With probability $\frac{1}{2}$ he should not be producing, but his prior investment

increases his outside option for the milk he owns, and hence he is able to hold-up the other producer. It is this increase in the bargaining power that creates too much incentives to invest.

On the other hand, the producer that does not own the cow is being held-up by the owner under every circumstance. And as a result under-invests.

In this model, ownership gives supply assurance. The owner of the asset is assured of having the input for production, without suffering a hold-up problem.

Part (e).

We can obtain the welfare under each type of integration by plugging the results in part (c) into the objective function in (a). For c-integration we obtain:

$$\begin{aligned} W^c &= (1-\mu)\left[\frac{26-3\mu}{16}\right] + \mu\frac{1}{2}\left[\frac{42-3\mu}{16}\right] - \left(\frac{4-\mu}{8}\right)^2 - 2\left(\frac{2-\mu}{16}\right)^2 \\ &= \frac{43}{32} - \frac{5}{32}\mu + \frac{9}{128}\mu^2 \end{aligned}$$

For y-integration we get:

$$\begin{aligned} W^y &= (1-\mu)\left[\frac{24-3\mu}{16}\right] + \mu\frac{1}{2}\left[\frac{40-3\mu}{16}\right] - \left(\frac{2-\mu}{8}\right)^2 - 2\left(\frac{4-\mu}{16}\right)^2 \\ &= \frac{21}{16} - \frac{1}{8}\mu + \frac{9}{128}\mu^2 \end{aligned}$$

And therefore,

$$W^c - W^y = \frac{1}{32} - \frac{1}{32}\mu = \frac{1}{32}(1-\mu) > 0$$

This difference can also be computed faster as follows (superscript denotes the type of integration):

$$\begin{aligned} W^c - W^y &= \left(\frac{2-\mu}{2}\right)\left(I_c^c + I_y^c - I_c^y - I_y^y\right) - (I_c^c)^2 - 2\left(I_y^c\right)^2 + (I_c^y)^2 + \left(I_y^y\right)^2 \\ &= \left(\frac{2-\mu}{2}\right)\frac{1}{8} - \left(\frac{4-\mu}{8}\right)^2 - 2\left(\frac{2-\mu}{16}\right)^2 + \left(\frac{2-\mu}{8}\right)^2 + 2\left(\frac{4-\mu}{16}\right)^2 \\ &= \left(\frac{2-\mu}{16}\right) - \frac{1}{2}\left(\frac{4-\mu}{8}\right)^2 + \frac{1}{2}\left(\frac{2-\mu}{8}\right)^2 \\ &= \left(\frac{2-\mu}{16}\right) - \frac{1}{128}\left[(4-\mu)^2 - (2-\mu)^2\right] = \left(\frac{2-\mu}{16}\right) - \frac{1}{128}\left[2(6-2\mu)\right] \\ &= \left(\frac{2-\mu}{16}\right) - \left(\frac{3-\mu}{32}\right) = \frac{1-\mu}{32} \end{aligned}$$

And therefore, c-integration is always preferred to y-integration (except for $\mu = 1$, when indifference arises).

The derivative $\frac{d(W^c - W^y)}{d\mu} = -\frac{1}{32} < 0$.

When $\mu = 0$, there is never a shortage of milk. In that case, we do not have an over-investment problem, but only an under-investment. In that case, the welfare loss is minimized by transferring ownership of the cow to the producer that is most efficient (has the lowest cost of investment), and hence benefits most from avoiding the under-investment problem. In this case, it is the c-producer.

When $\mu > 0$, there is sometimes a shortage of milk. In that case, there is both an over-investment problem and an under-investment problem. To minimize the under-investment problem, you want to transfer ownership to the most efficient producer. However, to minimize the over-investment cost, ownership needs to go to the least efficient producer (this is the one that finds investing to gain bargaining power most costly, and hence will do less of it). The under-investment problem turns out to be more severe, and hence, the cow should still go to the most productive producer. However, the relative benefit decreases, as we can see in the negative derivative of $W^c - W^y$ with respect to μ.

2005 First-Year Microeconomic Theory Exam Essay Solution

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Solution to Question 5.

The Lerner index, L_i, measures the marginal profitability of the firm and is a proxy for the inefficiency caused by the markup, while the Herfindahl index, H, measures the level of concentration in the industry and is often used to measure "market power".

In equilibrium, in a Cournot model, the Lerner index is given by $L_i = \frac{s_i}{\eta}$, where s_i is firm i's market share, and η is the elasticity of demand. Therefore, if we aggregate the price-cost margins for each firm, weighted by the firm's market share, we obtain $\sum_i s_i L_i = \frac{1}{\eta} \sum_i s_i^2 = \frac{H}{\eta}$, i.e., in a Cournot model, industry concentration is proportional to the average markup in the industry (see handout attached). So we expect to see low profitabilities in markets with many firms, each having a small share, and vice versa (indeed, as the number of firms in the market increase, both indices go to zero).

This result can be generalized to a constant "conjectural variation" of λ (where $\lambda = 0$ represents Cournot), and the average price-cost margin would then be $\frac{(1+\lambda)H}{\eta}$. (We would not expect students to necessarily know this result, but they should know the next one.) But, with Bertrand competition, only when going from monopoly to duopoly would we observe a positive relationship, and profits are always zero in perfectly contestable markets. (This is the $\lambda = -1$ case.) More generally, the threat of new entry is obviously important (if, e.g., there is any limit-pricing behaviour).

There are other effects that push in the same direction as the (one-shot game Nash equilibrium) Cournot result. Having a small number of firms in the industry facilitates collusion for several reasons, hence higher price-cost margins. However, if the firms become more asymmetric, this would increase H, but probably make it more difficult to sustain cooperation (firms may find it more difficult to, e.g., agree on what price to collude).

With product differentiation, and monopolistic competition, we have positive price-cost margins, despite having a large number of firms. Also, η will vary across industries. For example, electricity suppliers may have a lot of market power (at least in the short run) even with low H, because demand is so inelastic.

Note that a positive price-cost margin need not imply economic rents, e.g., if there are fixed costs of entry into a market. Also, if firms vary in their productivities, the more productive (efficient) firms will tend to both be more profitable and increase faster, so a positive relationship between profitability and concentration need not be caused by market power.

There is more that could be said, but a good essay need not say as much as this.

(See also the answer to the similar 2001 essay question, but note that this question was a little different - and so also should the answer be!)

2004 Industry Solution

It is true that we neither have good theory nor good empirical evidence to distinguish between price and quantity competition. A variety of theoretical models addressing this question were discussed in class (see also the question set in 2000), and these could be discussed.

However, a more significant point is that distinction is not strictly prices vs. quantities, but rather strategic substitutes vs. strategic complements. And in fact it may be slightly easier (although perhaps not necessarily) to distinguish between strategic substitutes and complements – although it would be entirely reasonable to argue that this distinction too is very hard to make in practice.

A critical point is that, of course, the theory or strategic entry deterrence shows that a firm trying to prevent the entry of a rival has a clear incentive to over-invest, independent of strategic substitutes vs. strategic complements; the ambiguity in whether the firm wishes to over- or under-invest arises only when it is accommodating, rather than preventing, entry. A good answer would probably quickly give the main line of this argument (along the lines discussed in class).

There are additional issues that could be raised, for example how strategic entry deterrence works when there are several firms, or uncertainty, etc., etc., but students were not expected to discuss these points (and none of them did this in the exam).

Long Solution to IO Problem, Microeconomics 2003 (Question 4) by Max Tse

(i) Hotelling Analysis

(a) Profits and Market Shares

The indifferent consumer is located at x where the total cost of each good is the same:

$p_A + x = p_B + (1 - x) \qquad \Rightarrow \qquad x = \frac{1+p_B-p_A}{2}$

The demand and profit (in terms of prices) of each firm is thus:

$q_A = x = \frac{1+p_B-p_A}{2} \qquad \pi_A = (p_A - c)q_A = (p_A - c)\frac{1+p_B-p_A}{2}$

$q_B = 1 - x = \frac{1-p_B+p_A}{2} \qquad \pi_B = p_B q_B = p_B \frac{1-p_B+p_A}{2}$

Each firm maximizes its profit with respect to its own price, yielding two equations which can be solved simultaneously for prices:

$$\left\{ \begin{array}{l} \frac{\partial \pi_A}{\partial p_A} = \frac{1}{2}(1 + p_B - 2p_A + c) = 0 \\ \frac{\partial \pi_B}{\partial p_B} = \frac{1}{2}(1 - 2p_B + p_A) = 0 \end{array} \right\} \Rightarrow \left\{ \begin{array}{l} p_A = 1 + \frac{2c}{3} \\ p_B = 1 + \frac{c}{3} \end{array} \right\}$$

Finally substitute back in for demands (= market shares) and profits:

$q_A = \frac{1}{2}(1 - \frac{c}{3}) \qquad \pi_A = \frac{1}{2}(1 - \frac{c}{3})^2$

$q_B = \frac{1}{2}(1 + \frac{c}{3}) \qquad \pi_B = \frac{1}{2}(1 + \frac{c}{3})^2$

(b) Small Increase in MC

The derivative of A's profit with respect to its MC is:

$\frac{\partial \pi_A}{\partial c} = -\frac{1}{3}(1 - \frac{c}{3}) = -\frac{2}{3}q_A$

which is less than the change in cost times market share ($\frac{\partial(-cq_A)}{\partial c} = -q_A$). This is the consequence of firms competing in strategic complements (i.e. prices). A rise in A's MC causes an increase in A's price which softens competition in prices and results in an increase in B's price ($c \uparrow \Rightarrow p_A \uparrow \Rightarrow p_B \uparrow$). The direct effect on A's profit of increasing c is $-q_A$, but the adjustment of p_A has a second-order effect in raising p_B by $\frac{1}{3}$ (NB $\frac{\partial p_B}{\partial c} = \frac{1}{3}$). This raises A's demand by $\frac{1}{6}$ and profits by $\frac{1}{3}q_A$, giving a total effect of $-\frac{2}{3}q_A$on A's profits.

(ii) Perfect Price Discrimination

Under price discrimination each consumer is treated as a separate market. The indifferent consumer in this case faces both firms pricing at MC (else one or other firm would lower its price and gain a customer). Thus the indifferent consumer is located at x^* such that both firms are pricing at MC and $c + x^* = 0 + (1 - x^*)$ which implies $x^* = \frac{1}{2}(1 - c)$. Below this value consumers purchase from firm A and firm B prices at MC. Above this value consumers purchase from firm B and firm A prices at MC. The market share of each firm is: $q_A = x^* = \frac{1}{2}(1 - c)$ and $q_B = 1 - x^* = \frac{1}{2}(1 + c)$.

Now consider the price charged by firm A to one of its customers (i.e. $x < \frac{1-c}{2}$). Even if B is pricing at MC firm A can charge $1 - 2x$ and still attract the customer (because of transport costs). The closer a consumer is to A (i.e. the lower x

becomes) the higher the price A is able to charge without losing the customer.

A's average price-cost margin to the consumers he serves is $\frac{1}{2}(1-c)$ and thus profit for firm A is $\pi_A = \frac{1}{2}(1-c)q_A = \frac{1}{4}(1-c)^2$. Similarly profit for firm B is $\pi_B = \frac{1}{4}(1+c)^2$.

[It may be helpful to note that in calculating π_A when there are linear transport costs and uniformly distributed consumers we are in effect taking the area under a triangle of height $1-c$ (the highest price-cost margin A can charge to its closest customer) and base $\frac{1}{2}(1-c)$.]

(iii) Comparison

In part (i): $\pi_{(i)} = \frac{1}{2}(1-\frac{c}{3})^2 + \frac{1}{2}(1+\frac{c}{3})^2 = 1 + \frac{c^2}{9} > 1$

In part (ii): $\pi_{(ii)} = \frac{1}{4}(1-c)^2 + \frac{1}{4}(1+c)^2 = \frac{1}{2}(1+c^2) < 1$

Price discrimination lowers industry profits.

(I) Efficiency

The second case is efficient because each consumer purchases from the firm with the lowest social cost of provision to that particular consumer. Without price discrimination firm A supplies some consumers that could be supplied by firm B at a lower social cost.

(II) Consumer Surplus

Consumers must be better off on average in the second case. This is because (ii) is more efficient and yields lower industry profits, leaving more surplus for consumers.

Extra Questions NOT on Exam

(A) Small Increase in MC (with Price Discrimination)

$\frac{\partial \pi_A}{\partial c} = -\frac{(1-c)}{2} = -q_A$

This is exactly the change in (Market Share x Costs) because there is no strategic effect in prices. B continues to charge 0 to all the customers that buy from A.

(B) Market Share

A's share is higher in the first case ($\frac{1}{2}(1-\frac{c}{3}) > \frac{1}{2}(1-c)$). This is explained by considering the treatment of marginal consumers. In (i) B takes advantage of lower costs to charge a higher price-cost margin than A to the marginal consumer, and this means that firms' shares are less sensitive to MC differences than in (ii) where the price-cost margin is always 0.

In (i) firms set MR=MC and the firm with a larger market share will have a lower MR at the same price (because of the effect on inframarginal units), and consequently a greater price-cost margin. A lower MC may translate into some increase in market share but this is mitigated by the rise in price-cost margins. By contrast in (ii) firms price at MC (p=MR=MC). Lower MC translates entirely into more competitive pricing and greater market share for firm B.

Solution to 2003 Industry Problem

(i)(a) $q_A = \frac{1 + p_B - p_A}{2}$ & $\pi_A = (p_A - c) q_A$

$\Rightarrow \frac{\partial \pi_A}{\partial p_A} = \frac{1}{2}(1 + p_B - 2p_A + c) = 0$ } $p_A = 1 + \frac{2c}{3}$

for B: $\frac{1}{2}(1 + p_A - 2p_B) = 0$ } $p_B = 1 + \frac{c}{3}$

$\Rightarrow \pi_A = \frac{1}{2}\left(1 - \frac{c}{3}\right)^2$; $\pi_B = \frac{1}{2}\left(1 + \frac{c}{3}\right)^2$; $q_A = \frac{1}{2}\left(1 - \frac{c}{3}\right)$; $q_B = \frac{1}{2}\left(1 + \frac{c}{3}\right)$

(b) $\frac{\partial \pi_A}{\partial c} = \frac{-2}{3} q_A$ $\Rightarrow$ A's profits reduced by less than change in costs × market share / Firms are competing in strategic complements.

$c \uparrow \Rightarrow p_A \uparrow \Rightarrow p_B \uparrow$.

$c \uparrow$ | costs A q_A in direct costs

increase p_B by $\frac{1}{3}$ which earns A $\frac{1}{6}$ customers, worth $\frac{1}{3} q_A$

(adjustment of p_A has second-order effect)

(ii) competing à la Bertrand, A charges c for consumers $\in \left[\frac{1-c}{2}, 1\right]$

B charges 0 — — — — $\in \left[0, \frac{1-c}{2}\right]$

Consumer at $\left[\frac{1-c}{2}\right]$ is indifferent between firms when both charge marginal cost

for consumer at $x < \frac{1-c}{2}$, A can charge $2\left(\frac{1}{2} - x\right)$ and still make sale

$\Rightarrow$ A's average price − cost margin where he wins $= \frac{1-c}{2}$

$\Rightarrow \pi_A = \frac{1}{4}(1-c)^2$; $\pi_B = \frac{1}{4}(1+c)^2$; $q_A = \frac{1}{2}(1-c)$; $q_B = \frac{1}{2}(1+c)$

(iii) $\pi_{(ii)} = \frac{1}{4}(1-c)^2 + \frac{1}{4}(1+c)^2 = \frac{1}{2}(1+c^2) < 1$

$\pi_{(i)} = \frac{1}{2}\left(1 - \frac{c}{3}\right)^2 + \frac{1}{2}\left(1 + \frac{c}{3}\right)^2 = 1 + \frac{c^2}{9} > 1$

} $\Rightarrow \pi_{(i)}$ larger

(I) (ii) is more efficient since the lower (social-) cost firm wins every consumer.

(II) since (ii) is more efficient, and firms are worse off, consumers must be better off on average.

Solution to Extra Bits NOT on Exam

(A) $\frac{\partial \pi_A}{\partial c} = -q_A$ $\Rightarrow$ A's profits reduced by exactly change in costs × market share / There's no strategic effect. B continues to charge 0 to all customers A sells to.

(B) A's share is higher in case (i). B takes advantage of his lower costs to take a higher price-cost margin than A on the marginal consumer; so firms' shares do not respond as much to costs as in (ii). In (i), firms set MR = MC and the firm with a larger share has a lower MR (So the firm with the larger share has a larger price-cost margin.) at the same price. / By contrast, in (ii) each consumer is a separate market so MR = p and the price is set by p = MC by the marginal firm (∴ p = MC (i.e. the price-cost margin is zero) / for both firms for the marginal consumer).

IO Essay, Microeconomics 2003 (Question 8)

Should policy makers seek to promote the entry of more firms into industries than would arise naturally?

The lecture/class handout (attached) suggests that there may be excessive entry in the absence of regulation (see also Armstrong, Cowan and Vickers, 1994, pp.106-111). This is due to the marginal firm having a business-stealing incentive to enter, even if entry is socially undesirable because it lowers the efficiency of production (e.g. by duplicating fixed costs). This result does not depend on the nature of competition (provided $p > c'$ and $q' < 0$).

However as discussed in lectures/classes the argument ignores other effects that may make the promotion of entry desirable. First, the argument ignores the possibility of an integer constraint. In particular, there may be no entry at all ($n = 0$) when a monopoly would be preferable. (There is always too little incentive for the very first firm to enter, since this firm has no business-stealing incentive and it does not fully appropriate the consumer surplus it creates.) Second, it assumes that social welfare weights consumer surplus and profits equally. Third, there may be scope for product variety that benefits consumers. Fourth, costs may be affected by entry. Increased dynamic incentives for cost reduction may outweigh the static inefficiencies of more firms. Higher entry may also help select of more efficient firms.

Perhaps the most important consideration for policy makers is the possibility of strategic entry deterrence which may reduce excessive entry or lead to too little entry occurring. A nice illustration of the opposing effects of business-stealing and entry deterrence is provided by a circular city model of product differentiation (see Salop, 1979). In the basic model with free entry there is excessive entry (see Tirole, 1988, pp.183-4) but if firms are able to commit to particular locations they can cut the number of entrants, sometimes achieving the socially optimal level of entry (see Schmalensee, 1978). There are many other strategic entry deterrence models (eg Dixit, 1980) that might also be mentioned.

Finally contestability (see Baumol et al., 1982) suggests that regulators should concentrate on the conditions for entry rather than the number of firms in an industry. The possibility of entry may discipline incumbents without entry actually needing to occur. In such circumstances the promotion of entry itself would be unnecessary.

Symmetric one-period model with free entry

$W(n) = [\text{consumer surplus}] + [\text{profits}]$

$= [\text{consumers' gross utility} - \text{consumers' payments}]$
$+ [\text{firms' revenues - costs}]$

$W(n) = U(nq(n)) - nC(q(n))$

$\Longrightarrow \frac{dW}{dn} = U'(nq(n)).\left[q(n) + nq'(n)\right]$

$-C(q(n)) - nC'(q(n))q'(n)$

$= [\text{price}.q(n) - C(.)] + [\text{price} - C'(.)][nq'(n)]$

$\Longrightarrow$ too many firms

Problem Question, Microeconomics 2002 (Question 5).
Solution by David Gill.

(i)

More generally, let $P = \alpha - Q$.

Here, official and unofficial suppliers are strategically equivalent, so let the total number of suppliers be $Z = N + M$.

Each individual supplier will:

$$\max_{q_i} \pi_i = (P - c)\, q_i = (\alpha - Q - c)q_i$$

Using the product rule and the fact that $Q = q_i + \sum_{j \neq i} q_j$, the first order condition is:

$$\frac{\partial \pi_i}{\partial q_i} = (\alpha - Q - c) - q_i = 0$$

which gives:

$$q_i^* = \alpha - Q - c$$

The second order condition for a maximum is satisfied as the second derivate of π_i with respect to q_i is $-2 < 0$.

Note that equilibrium must by symmetric, as q_i^* for all suppliers takes the above form with respect to a common Q. Thus:

$$q_i^* = \alpha - Zq_i^* - c$$

So, re-arranging

$$q_i^* = \frac{\alpha - c}{Z + 1} \tag{1}$$

and

$$Q^* = \frac{Z}{Z+1}(\alpha - c)$$

Also,

$$P^* = \alpha - \frac{Z}{Z+1}(\alpha - c) = \frac{\alpha + Zc}{Z+1}$$

Here $\alpha = 1$, so $q_i^* = \frac{1-c}{Z+1}$, $Q^* = \frac{Z}{Z+1}(1-c)$ and $P^* = \frac{1+Zc}{Z+1}$.

(ii)

Let Q_U be the total supply of the M unregulated suppliers.

Given Q_D, the unregulated suppliers effectively face $P = 1 - Q_D - Q_U$, i.e. from their perspective, $\alpha = 1 - Q_D$. Thus, using (1), for each unofficial supplier:

$$q_m^* = \frac{1 - Q_D - c}{M + 1}$$

and

$$Q_U^* = \frac{M}{M+1}(1 - Q_D - c) \tag{2}$$

(iii)

The University will:

$$\max_{Q_D} \sum_{n=1}^{N} \pi_n = (P - c)Q_D = (1 - Q_D - Q_U - c)Q_D$$

Using (2), the University thus maximises:

$$\begin{aligned} & \left[1 - Q_D - \frac{M}{M+1}(1 - Q_D - c) - c\right] Q_D \\ = & \left[\frac{M + 1 - Q_D M - Q_D - M + Q_D M + cM - cM - c}{M+1}\right] Q_D \end{aligned}$$

Simplifying, and getting rid of the constant multiplicative term $\frac{1}{M+1}$, the University's problem is:

$$\max_{Q_D}(1 - Q_D - c)Q_D$$

which is the same as the monopolist's maximisation problem over Q.

(iv)

Let Q_R be the total supply of the N regulated suppliers.

In equilibrium, from each group's perspective, the output of the other group is fixed. Thus, we can use the same reasoning as behind (2) to find that:

$$Q_R^* = \frac{N}{N+1}(1 - Q_U^* - c - t)$$

$$Q_U^* = \frac{M}{M+1}(1 - Q_R^* - c)$$

where we also use the fact that the regulated suppliers now face an effective marginal cost of $c + t$. Therefore:

$$Q_R^* = \frac{N}{N+1}\left[1 - \frac{M}{M+1}(1 - Q_R^* - c) - c - t\right]$$

So,

$$Q_R^*\left[\frac{(N+1)(M+1) - MN}{(N+1)(M+1)}\right] = \frac{N}{(N+1)(M+1)}\left[M + 1 - M + cM - cM - c - tM - t\right]$$

And hence:

$$Q_R^* = \frac{N}{1 + N + M}[1 - c - t(1 + M)] \tag{3}$$

Now, the University wants to:

$$\max_t (P - c - t)Q_R + tQ_R = (P - c)Q_R$$

But note that from (3), it can control Q_R through the use of t. Thus, its problem boils down to choosing Q_R to maximise $(P - c)Q_R$, which is exactly the same problem it faced in (iii) with respect to the directly regulated Q_D. Thus, by the same reasoning as in (iii), it will set Q_R at the monopoly level of output, which from (1) is $\frac{1-c}{2}$, as the monopolist faces $Z = 1$ and $\alpha = 1$.

Thus, the University will set:

$$Q_R^* = \frac{N}{1 + N + M}[1 - c - t(1 + M)] = \frac{1 - c}{2}$$

So,

$$1 - c - t(1 + M) = (1 - c)\left[\frac{1 + N + M}{2N}\right]$$

$$(1 - c)\left[\frac{2N - (1 + N + M)}{2N}\right] = t(1 + M)$$

Finally, we find:

$$t^* = (1 - c)\left[\frac{N - M - 1}{2N(1 + M)}\right]$$

(v)

Trivially, $t^* < 0$ iff $N < M + 1$.

Micro Essay Answer 2002

Q) What is ownership, and how does it matter?

A) The lectures discussed Grossman-Hart-Moore's theory that identifies ownership with residual control rights, hence ownership affects incentives for non-contractible investments.

A good answer would also discuss other theories that were mentioned more briefly. For example, ownership confers the rights to receive an asset's returns (Holmstrom-Milgrom) hence directly affects incentives to invest in and maintain the asset and so also affects incentives for other uses of an agent's time (e.g. producing output). Alternatively, ownership confers discretionary authority, hence affects subordinates' incentives for influence activities (Milgrom-Roberts).

Problem Question, Microeconomics 2001 (Question 2).
Solution by David Gill. (Long Version)

(i)

Suppose $n-1$ firms have already entered. Let the nth firm assume that it will be the last entrant. Then, on entry it has a $\frac{1}{n}$ chance of being the lowest cost firm. Conditional on this, expected price is the expected cost of the 2nd lowest cost firm. Thus, expected profits conditional on being lowest cost are $\frac{2}{n+1} - \frac{1}{n+1}$, so overall expected profits are $\frac{1}{n}\left(\frac{1}{n+1}\right)$. This means that the nth firm will definitely not enter if $\frac{1}{n.(n+1)} < F$ (if further entry were to occur, expected profits would be even lower). Let j be the lowest integer such that $\frac{1}{j}\left(\frac{1}{j+1}\right) < F$. Then, firms j and above will definitely not enter. By definition, for firm $j-1$, $\frac{1}{(j-1)}\left(\frac{1}{(j-1)+1}\right) \geq F$, so $j-1$ will enter as it understands that it will be the last entrant. Firm $j-2$ understands that if it enters, so will $j-1$, but not j and subsequent firms, so it gets the same expected profits from entry as $j-1$ and so enters. We can then work backwards by a similar argument to show that firms 1 to $j-1$ all enter.

(ii)

To simplify, we suppose that if two firms announce the same lowest price, then the firm to have announced first makes the sale.

Let the nth firm assume it will be the last entrant, and let the current lowest price be p. Then, if it enters, $c_n < p$ with probability p, in which case expected costs are $\frac{p}{2}$ (given the uniform distribution on costs), and it will just undercut the current price by ε. Thus, pre-entry expected profits are $\pi = p.(p - \varepsilon - \frac{p}{2}) - F = \frac{p^2}{2} - F - p\varepsilon$, and so $\pi < 0$ iff $p \leq \sqrt{2F}$ (for any $p > \sqrt{2F}$, the entrant can set ε small enough to give $\pi > 0$). If further entry were to occur, this could only lower $n's$ profits, so $p \leq \sqrt{2F}$ always deters entry.

An equilibrium in which entry continues until someone announces a price that is low enough to deter future entry is then as follows. Each firm n enters iff the current lowest price $p > \sqrt{2F}$, in which case it sets its own price $p_n = \sqrt{2F}$ iff $c_n \leq \sqrt{2F}$. If $c_n > \sqrt{2F}$, it sets price at cost - it will be undercut by a lower cost later entrant in any case. Any potential entrant facing $p > \sqrt{2F}$ who follows this strategy deters any future entry if $c_n \leq \sqrt{2F}$ (so it sets the entry deterring price), and hence gets an expected payoff of $\sqrt{2F}(\sqrt{2F} - \frac{\sqrt{2F}}{2}) - F = 0$. It cannot set a higher price, as given the equilibrium strategies followed by the other firms, firms would then continue entering until one of them had costs lower than $\sqrt{2F}$, and thus set $p = \sqrt{2F}$, undercutting the price and leading to profits of $-F$.

(iii)

Suppose there is an equilibrium of the following form in which entry continues until someone announces a price p^* that is low enough to deter future entry. Each firm n enters iff the current lowest price $p_L > p^*$. If firm n enters, then if $c_n \leq p^*$, it sets $p = p^*$, otherwise it sets price at cost (a lower cost entrant will come in, in any case).

In such an equilibrium, suppose the lowest price is p^*. An entrant n could come in and would

face the lowest cost firm (who set p^*) with $c_L \in [0, p^*]$ - all other firms will have higher costs. With probability p^*, the entrant would also have cost in this range, in which case, using the hint in (i), expected lowest cost is $\frac{1}{3}$ of the range $[0, p^*]$, and expected second lowest cost (which will be the winning price plus ε), will be $\frac{2}{3}$ of this range. Given $c_n \in [0, p^*]$, the entrant will win $\frac{1}{2}$ the time. Putting all this together, expected profits for the entrant are

$$\frac{1}{2}.p^*.p^*.\left(\frac{2}{3} - \varepsilon - \frac{1}{3}\right) - F = \frac{(p^*)^2}{6} - \frac{(p^*)^2}{2}.\varepsilon - F$$

Thus,

$$\frac{(p^*)^2}{6} - \frac{(p^*)^2}{2}.\varepsilon - F \geq 0 \Leftrightarrow p^* > \sqrt{6F}$$

Thus, entry is deterred iff $p^* \leq \sqrt{6F}$.

Setting $p^* = \sqrt{6F}$, let's check our equilibrium works. Clearly, once $p_L = p^*$, further entry is deterred. So, we just need to check that where $p_L > p^*$, firms want to enter. If they do, then of course if $c_n > p^*$, a lower cost entrant will eventually come in and set p^*, so the firm might as well set price at cost. If $c_n \leq p^*$, then the firm is lowest cost and p^* deters future entry, so the firm wins the Bertrand competition at p^*. There is no point setting a lower price, as this just reduces profits. There is no point setting a higher price, as this will simply induce further entry, so eventually a firm will come in and set p^*, and this is clearly strictly worse as price cannot be higher, but the firm might lose the competition! Thus, expected profits from entry are $p^*\left(p^* - \frac{p^*}{2}\right) - F = \frac{(p^*)^2}{2} - F > 0$, so entry is optimal.

(iv)

Socially, we want entry to continue so long as the expected reduction in costs is greater than F. So long as the lowest price p is greater than the entry deterring price in each case, then the lowest cost firm has $c = p$. Thus an entrant coming in will have lower cost with probability c, in which case expected cost is $\frac{c}{2}$. Thus, the expected reduction in cost is $\frac{c^2}{2} = \frac{p^2}{2}$. Thus, in the equilibrium in (ii) where $p^* = \sqrt{2F}$, until the point at which entry is deterred, entry continues so long as the expected cost reduction is greater than F. However, in the equilibrium in (iii), where the last entrant (who deters any further entry with $p^* = \sqrt{6F}$) has $c \in (\sqrt{2F}, \sqrt{6F}]$, the equilibrium in (ii) would have allowed further efficient entry, but the one in (iii) does not. Thus the equilibrium in (ii) is more efficient.

Another way of seeing this is to note that that the final entrant who comes in and sets the entry deterring price makes $\frac{(p^*)^2}{2} - F$ in each case. This gives zero profits in (ii), but strictly positive profit in (iii).

2001 Micro Problem Answer (Short Version)

(a) An entrant's expected profits are the expected cost advantage of the low cost firm times the probability of being the low cost firm. This must be set against entry costs of F.

Since expected profits for the winner are $\frac{1}{n+1}$ and the probability of winning is $\frac{1}{n}$, the number of entrants will be the largest integer such that

$$\frac{1}{n(n+1)} \geq F.$$

(b) Assume the critical price is p. Then a potential entrant would say "If I can have costs less than p, which will happen with probability p, I will be able to charge p and earn, on average, $\frac{p}{2}$ (since my costs will be $\frac{p}{2}$ on average if they are less than p). So my expected profits from entering if the price is p are $p^2/2 - F$. To deter entry, these expected profits must be zero, so $p^2/2 - F = 0$ and $p = \sqrt{2F}$.

(c) Now when a potential entrant observes a price of p she only knows that the incumbent's costs are not more than p and may be as low as zero. If the entrant comes in she will only win now with probability $p/2$. If she does enter and has costs below p the winner of the price competition will earn on average $p/3$. So expected profits are $p^2/6 - F$. Setting this equal to zero yields $p = \sqrt{6F}$, which is $\sqrt{3}$ times as great as the deterrence price in (b).

(d) Socially, we want more entry whenever the expected reduction in production costs exceeds the fixed cost F. If the lowest current cost is c then the expected cost reduction is $c^2/2$ and entry should occur until $c^2/2 - F \leq 0$. This is what happens in part (b).

2001 Microeconomics Essay Notes for an Answer

[Note: This is a non-exhaustive list of some points that might form part of an answer; they are neither necessary nor sufficient for a good answer.]

The simple theory is that the average price-cost margin in an industry is $\frac{H}{\eta}$ (see handout attached). This theory assumes Cournot behavior, but with constant "conjectural variation" of λ ($\lambda = 0$ in Cournot) the average $p - c$ margin is $\frac{(1+\lambda)H}{\eta}$ (since a firm's FOC is $0 = \frac{\partial \pi_i}{\partial q_i} = p + (1+\lambda)q_i p' - c_i'(q_i)$). (Furthermore, Encoua and Jacquemin (Intl E Rev 1980) point out that the Herfindahl (along with some other measures) satisfies some natural axioms that we might expect a concentration measure to satisfy.)

Of course, η will vary across industries. For example, electricity suppliers may have a lot of market power (at least in the short run) even with low H, because demand is so inelastic.

However, industry behavior may depend on the structure of the industry in other ways. One way of thinking about this is to say that λ may vary across industries. Equivalently, we can argue that some industries may be more "Cournot-like" than others, which may be more or less collusive. (I do not expect students to necessarily talk in terms of λ.)

The threat of new entry is important both in the short run (if, e.g., there is any limit-pricing behavior) and, more obviously, in the long run.

And, of course, price-cost margin is not a proper measure of welfare. For example, taking firms' costs as given, welfare is actually increasing in the Herfindahl for a *given* industry output. (Since consumer surplus is unaffected for a given output, and profit $= -Q^2 p' H$ is increasing in H for given Q—see second handout attached.) The point is just that a higher Herfindahl is reallocating even more output to the higher-market-share, lower-cost, firms.

Similarly, a change in which a low-cost firm changes from tacitly colluding with its high-cost rivals at a high price, to instead setting a lower price that drives its rivals out of the industry could raise all of H, average price-cost margin, consumer welfare, and social welfare. (If demand is inelastic, all these things are consistent with profits being lower in the low-price state. If demand is elastic, and if average price-cost margin (weighted by output) increases, then the low-cost firm prefers the "more competitive" outcome in the

simplest model[1] – but it then would be a shame if, e.g., the low-cost firm were reluctant to drive out its rivals for fear of being seen as "dominant" by the anti-trust authorities.)

Another related point is that too much entry, that is, too low a Herfindahl is generally a bad thing (and free entry generally implies too much entry—see Bulow, Geanakoplos and Klemperer (JPE 1985, esp. sec VI E) and Mankiw and Whinston (Rand 1986)—because of the business-stealing effect).

A student may wish to argue that regulators generally care about consumer surplus not profits (and that price-cost margin is a reasonable proxy for consumer welfare for a given cost structure, and given η).

But the broader point is that different industry structures may lead to different incentives to minimise costs (in a static model) or reduce costs (in a dynamic model).

There is a lot more that could be said. For example, a student could talk about switching costs and network externalities. Or discuss the Farrell & Shapiro (AER 1990, Rand 1990) papers.

A good essay would say less than this and **note that much of what I've said here will be beyond the knowledge of most, perhaps even all, students**. (For example, I do not teach the notion of "conjectural variation", λ, every year.)

However, a good essay would also be better organised!

2001mc1.tex

[1] Call the "tacit collusion" and "low price" states M and C, respectively, and write L^M and L^C for the weighted-average price-cost margins in these states, respectively. Then $L^C = \frac{p^C - c}{p^C}$ and $L^M > \frac{p^M - c}{p^M}\frac{q^M}{Q^M}$ where q and Q are the low-cost firm's output, and industry output, respectively, and c is the low cost. So (since $Q^C = q^C$), $L^C > L^M \Longrightarrow (p^C - c)q^C > (p^M - c)q^M \cdot \left(\frac{Q^C p^C}{Q^M p^M}\right) > (p^M - c)q^M$ if demand is elastic.

Cournot Equilibrium

Homogeneous goods. Inverse demand $p(Q)$

Firm i maximises $\pi_i = p(Q)q_i - C_i(q_i)$

FOC: $0 = \frac{\partial \pi_i}{\partial q_i} = p + q_i p' - C_i'(q_i)$

$\Longrightarrow$Lerner Index $\equiv \frac{p - C_i'}{p} = \frac{-q_i p'}{p} = \frac{s_i}{\eta}$

where $s_i \equiv \frac{q_i}{Q}$

and $\eta \equiv \frac{-p}{p'Q} = -$elasticity of demand

Productively inefficient since $s_i > s_j \Longrightarrow C_i' < C_j'$

Assume $C_i(q_i) = c_i q_i$,

then average price-cost margin

$= \sum_i s_i \left(\frac{p - c_i}{p}\right) = \sum_i \frac{s_i^2}{\eta} = \frac{H}{\eta}$

where H is the Herfindahl index

Industry profit $\Pi = \sum_i \pi_i = \sum_i (p - c_i) q_i$

$= \sum_i \left(\frac{p-c_i}{p}\right) p s_i Q = pQ\frac{H}{\eta} = -Q^2 p' H$

FOC $\Rightarrow q_i = \frac{p-c_i}{-p'} \Rightarrow Q = \frac{np - \sum_i c_i}{-p'}$

Assume symmetry, then $\Pi = \frac{-Q^2 p'}{n}$

Note $\frac{d}{dQ}\left(-Q^2 p'\right) = -2Qp' - Q^2 p''$

$= -Qp'\left(2 + \frac{Qp''}{p'}\right)$

So sign $\frac{d\Pi}{dc} = -\text{sign}\left(2 + \frac{Qp''}{p'}\right)$

For $p = kQ^{-\frac{1}{\eta}}$, $\frac{Qp''}{p'} = -\left(\frac{1}{\eta} + 1\right)$

$\Longrightarrow \frac{d\Pi}{dc} < 0 \Longleftrightarrow \eta > 1$

Essay Question, Microeconomics 2000 (IO).

Ideas that might form part of an answer. These are unofficial, and are certainly not necessary, and may not be sufficient for a good answer. In this case, I set but did not mark this question.

(i)

Price-setting yields lower prices under fairly general conditions. (See Cheng (Rand 1985), Klemperer (QJE 1987) etc.)

[Enthusiasts can also see Amir & Jin (*Int. J. Industrial Organization*, 2001), (which appeared after this question was set!)]

(ii)

This is difficult to do well.

A very good answer would note that each firm in an oligopoly acts as a monopolist against its residual demand. The firm is indifferent (absent uncertainty, etc.) between choosing a p or q but cares about its opponent's choice, which affects the shape of its residual demand. If the opponent has fixed q, then if you are more aggressive (i.e., increase q), his p falls (and q stays the same) which hurts you too. If the opponent has fixed p, then if you are more aggressive (i.e., decrease p), his p stays the same (and his q falls). This latter case is less bad for you (i.e., your residual demand is flatter) so you have more incentive, ceteris paribus, to be more aggressive in the latter case.

A less good answer would simply assume the result about residual demand based, perhaps, on the analogy with the undifferentiated products case.

(iii)

To discuss which is the better *representation* of reality, could refer to some of the following.

Kreps and Scheinkman (Bell 1983): Under a reasonable rationing assumption on residual demand, capacity setting followed by price setting leads to a Cournot outcome.

Singh & Vives (Rand 1984): If each firm is able to pre-commit to later competing in price or in quantity, then with substitute goods they will both choose quantity.

Klemperer and Meyer (Rand 1986): If each firm simultaneously chooses either any price or any quantity, then both the Bertrand and Cournot outcomes remain equilibria. But in the presence of demand uncertainty, firms have strict preferences between quantity setting and price setting, and the paper explains circumstances in which Cournot is more likely to arise than Bertrand, and vice versa.

Klemperer & Meyer (Ecta 1989): Firms choose supply functions, i.e., how much to produce as a function of the market price p, in the presence of demand uncertainty. The outcome is somewhere between Bertrand and Cournot competition, depending on the kinds of factors that are important in Klemperer and Meyer's previous paper.

As to which is the best *model*, I strongly believe in horses for courses, e.g., undifferentiated Cournot is fine if need to capture price decreasing smoothly in n, but not if you're modelling mergers, etc.