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A1. (i) Explain what it means to say that bidders' private signals are *affiliated*. Are ascending or first-price sealed-bid auctions more profitable in this circumstance? What is the intuition for this result?

(ii) Each of n risk-neutral bidders has a privately-known value v_i for a single object. For each i, $v_i = \theta + t_i$ in which the t_i are independently drawn from a uniform distribution on [0,1], but bidders have a "diffuse prior" for θ (that is, all values of θ are equally likely,[1] and bidder i knows only v_i, not t_i or θ). The seller's value of the object is θ.

(a) What is the seller's profit from an ascending auction?

(b) What is the seller's profit from a first-price sealed-bid auction? [Note that each bidder bids $v_i - x$ for some x, since the 'diffuse prior' assumption means she has no knowledge of how high or low her value is relative to others'.]

(iii) Repeat (ii)(a) and (b) under the assumption that θ is common knowledge to all bidders (but v_i remains private information to bidder i). Explain why the answers to (ii)(b) and (iii)(b) are different.

(iv) Repeat (ii)(a) and (b) under the assumption that all the v_i are common knowledge to all bidders.

(v) Describe how the results of (ii), (iii), and (iv) would change qualitatively if the t_i were affiliated.

[Oxford, Economic Theory, 2004]

A2. (i) Sketch a proof of the Revenue Equivalence Theorem.

(ii) For each of the following cases, briefly discuss why the theorem can break down and the extent (if any) to which the theorem applies.

(a) Bidders' signals are not independent.

(b) Bidders are risk-averse.

(c) Bidders can collude.

(d) The seller has multiple units for sale.

(e) The distribution from which bidders' valuations are drawn is not both strictly-increasing and atomless.

[Oxford, Economic Theory, 2004]

[1] Formally θ can be thought of as uniformly distributed on $[-m, +m]$ in the limit as $m \to \infty$.

A3. Risk-neutral bidders $i = 1, 2$ each receive a private signal z_i independently drawn from a uniform distribution on $[0, 1]$. Both bidders have the same value for an object, $v = z_1 + z_2$.

(i) Assume the object is sold by English (i.e. Japanese, or ascending) auction. Compute symmetric equilibrium bidding strategies.

(ii) Assume the object is sold by Dutch (i.e. descending) auction. Use your solution in (i) together with the Revenue Equivalence Theorem to deduce equilibrium bidding strategies.

(iii) Assume the object is sold by an All-pay auction (in which each bidder pays her bid whether or not she wins). Use your solution in (i) together with the Revenue Equivalence Theorem to deduce equilibrium bidding strategies.

(iv) Solve for the (symmetric) All-pay equilibrium strategies directly, by writing out i's surplus when her signal is z and she bids as if her type were $\widetilde{z}$, and then obtaining a differential equation from the first-order condition.

(v) If now bidder 1's value is $v = z_1 + z_2 + 0.01$ (but bidder 2's value is unchanged), what happens in the ascending auction? Can we use the Revenue Equivalence Theorem to compute the Dutch auction's outcome from the ascending auction's outcome? Why or why not?

[Oxford, 1st Year Micro, 2004]

A4. Risk-neutral bidders $i = 1, ..., 4$ each receive a private signal t_i independently drawn from a common distribution $F(\cdot)$ and compete for two objects, each of value $v = \sum_{i=1}^{4} t_i$. Each bidder can win at most one object. Write $t_{(i)}$ for the ith highest actual signal.

(i)(a) Describe symmetric equilibrium bidding strategies when both objects are sold in a single ascending auction (i.e., the price rises continuously until just two bidders remain; the remaining bidders each win one object at the final price).
(b) Describe symmetric equilibrium bidding strategies when the bidders are randomly divided into two pairs, and each pair competes in a single ascending auction for a single object.
(c) Hence compute expected revenue for cases (a) and (b) when $F(t) = t$.

(ii)(a) Give a condition in terms of "marginal revenues" $MR_{(i)} \equiv \left[v - \frac{1-F(t_{(i)})}{f(t_{(i)})} \right]$ (since $\frac{\partial v}{\partial t_{(i)}} = 1$ here) such that the sales mechanism of (i)(a) is more profitable than that of (i)(b).
(b) Hence (I) confirm your result in (i)(c) about which sales mechanism is more profitable if $F(t) = t$, and (II) determine which sales mechanism is more profitable if

$F(t) = 1 - t^{-2}$, for $t \geq 1$. (III) For each of these two distributions, $F(\cdot)$, which sales mechanisms do bidders prefer? (IV) For each of these two distributions, which sales mechanisms would be more profitable if bidders i's value was simply $v_i = t_i$, $i = 1, ...4$. (c) Comment.

[Oxford, Economic Theory 2006]

A5. Why is a Vickrey auction often thought a good sales mechanism in theory? Why are they often less desirable in practice? Your answer should consider both single-unit and multi-unit contexts.

[Oxford, Economic Theory 2006]

A6. Two risk-neutral partners $i = 1, 2$ each own 50% of a company. Each partner i has a private value, v_i, for the whole company that is drawn independently and identically from the uniform distribution on [0,1]. The partners decide they will run an auction between them, with one partner buying the other out. Assume that the auction will be efficient.

(i) Define the expected gross surplus, $S_i(v_i)$, of partner i as his probability, $P_i(v_i)$, of winning the whole firm times his value, v_i, plus any expected payments he receives from his former partner minus any expected payments he makes to the former partner. On average, what is the combined expected gross surplus of the two bidders? How does the *per bidder* expected surplus compare with the expected per bidder surplus the bidders would receive if they were competing to buy the same asset, but it was owned by a third party?

(ii) Assuming bidders do not know their partners' values, explain carefully why $S_i(v_i) = S_i(0) + \int_0^{v_i} P_i(x)dx$. Hence obtain a simple expression for $S_i(v_i)$, assuming $S_i(0)$ is the same for both partners.

(iii) Assuming that the partners make sealed bids, and the high bidder pays his bid to buy out his partner's share of the firm, obtain an expression for $S_i(v_i)$ in terms of each partner's bid $b_i(v_i)$. (Note that efficiency implies that the partners use the same bidding function, so that the bidder with the higher value always wins.) Combine this with your result for part (ii) to show that each partner's bid $b_i(v_i)$ is linear in his value v_i.

(iv) If instead the auction is an ascending auction, at what price does a bidder with value v_i quit? Why does each partner always bid up to a price that exceeds his value for the half of the partnership that he is buying (i.e., bid further than $\frac{v_i}{2}$)?

(v) On average which "type", v_i, of partner gains least from the dissolution (assume a partner with a 50% share would have a value of $\frac{v_i}{2}$ if the partnership were not dissolved)?

(vi) If partners' shares were unequal how, if at all, would the analysis be affected?

Explain how unequal partners can adjust either of the mechanisms in parts (iii) and (iv) above so that $S_i(0)$ is different for different partners, and hence so that a (somewhat) larger partner is always willing to participate. How unequal can the shares be before efficient dissolution becomes impossible by any mechanism that both partners would agree to, if the partners know their own values before agreeing to a mechanism?

[Oxford, Economic Theory, 2007]

A7. A magazine recently (slightly mis)quoted an (Oxford) academic as saying (about the outcome of a simultaneous ascending auction) "Companies that bid for a few large regions to obtain nationwide spectrum paid almost twice as much as firms with the equivalent amount of spectrum assembled from hundreds of licences in smaller areas. This suggests an inefficiency in the auction design, since the same coverage ought to cost the same amount."

(i) What do you think the economist meant to convey by his comments?

(ii) What might have caused this outcome?

(iii) Is it always true that an auction design that sells identical objects at different prices is an inefficient one?

(iv) What determines whether a commodity like spectrum is best sold as a small number of licences each of which covers a large region, or as a large number of licences each of which covers a small part of a region?

(v) Does a "package auction" resolve the problems of both these approaches?

[Oxford, Economic Theory, 2007]

A8. "The revenue equivalence theorem proves that auction design is irrelevant." Does it?

[Oxford, 1st Year Micro, 2007]

A9. (i) How, in theory, can an auction extract all the surplus from risk-neutral bidders when their information is affiliated?

(ii) Why might this not work well in practice?

(iii) Discuss what recommendations are usually made about how to run an auction in practice when bidders' information is affiliated.

[Oxford, Economic Theory, 2008]

A10. (i) A woman has a value drawn from a uniform distribution on [40,80] for an object. A man has a value (independently) drawn from a uniform distribution on [0,40] for the same object. The seller's valuation for the object is 10.

A. Describe an expected-profit maximising auction for the object.

B. Describe how the auction could be run as a standard ascending auction, if we give the man a (fixed) discount conditional on him being the winner.

C. Explain why discriminating in favour of the man is necessary to maximise expected revenue.

(ii) Now assume both bidders' values are independently drawn from the same distribution, which with probability 20% is uniform on [40,80], and with probability 80% is uniform on [0,40]. As before, the seller's valuation for the object is 10.

A. Explain why expected profit cannot be maximised by an auction that always sells to the higher-value bidder whenever it sells to either bidder, but can be maximised if it does this with the exception that it randomises equally between the bidders when both have values between 35 and 50. What prices should be charged in the latter auction (as a function of the bidders' values) to maximise expected profit?

B. Explain why it would not harm revenue if, instead of randomising equally, the object were always given to the man in the event both bidders' values were between 35 and 50. What prices should be chosen in this case?

[Oxford, Economic Theory, 2008]

A11. (i) State the Revenue Equivalence Theorem.

(ii)Three risk-neutral firms compete for a single indivisible object which is worth v_i to firm i; $i = 1, 2, 3$, where each v_i is a (privately-known) independent draw from the uniform distribution on $[0, V]$.

(a) What is the expected payment of the winning firm, if the object is sold using an ascending auction? What is the winner's expected profit?
(b) Assume, instead, that the object is sold in an auction in which firms simultaneously choose bids, B_i. The firm that makes the highest bid wins the object and pays θB_i, where $\theta > 0$. Other firms each pay φB_i, where $\theta > \varphi > 0$ and win nothing. What is the total of the expected payments made by the three firms? What is the net industry expected profit?
(iii) Each of three risk-neutral firms has φ "locked in" consumers who will buy from it at any price up to £1. There are also $\theta - \varphi$ "uncommitted" consumers who will all buy from the firm that sets the lowest price, if that price is not more than £1. Each firm's per unit production cost c_i is a (privately-known) independent draw from the uniform distribution on $[0, C], C < 1$. All firms set all prices simultaneously.
(a) What is expected industry profit when firms can price discriminate

between their customers? (That is, each firm i will set price £1 to its own "locked in" consumers, who will buy from it anyway, and will set a different price P_i to the "uncommitted" consumers.)
(b) Now assume, instead, firms cannot price discriminate between consumers. (That is, each firm i must set a price Q_i that both applies to its own "locked in" consumers, who will buy from it anyway, and also applies to the "uncommitted" consumers.) Explain how this problem can be described as an auction. What is the prize? What are the winner's and the losers' payments? What is expected industry profit?
(iv) Very briefly, comment on whether the ability to price discriminate against consumers with switching costs necessarily raises a firm's profits, and give intuition for your answer.

[Oxford, 1st Year Micro, 2009]

A12. i. Two bidders A and B compete in an ascending auction to buy a company. i's actual value is $\alpha z_i + (1-\alpha)z_j,\ i,j = A,B,\ i \neq j,\ \alpha \in (1/2, 1]$ in which z_i and z_j are private information to i and j respectively.

a. If B follows the strategy of quitting the auction when the price reaches $k_B z_B$, for some constant k_B, what is A's optimal strategy?

b. Provide intuition for how your result depends upon α.

c. Find a perfect Bayesian Nash equilibrium of the auction.

ii. Now assume that synergies between A and its existing assets mean that A's actual value is $\phi(\alpha z_A + (1-\alpha)z_B)$ for some constant $\phi > 1$. (B's actual value is $\alpha z_B + (1-\alpha)z_A$, as before.)

a. What is A's optimal strategy, if B still follows the strategy of quitting the auction at $k_B z_B$?

b. Show that if $\phi < \alpha/(1-\alpha)$ then i quitting at $k_i z_i,\ i = A,B$, with $k_A = (2\alpha - 1)\phi/(\alpha + \phi\alpha - \phi),\ k_B = (2\alpha - 1)\phi/(\alpha + \phi\alpha - 1)$, is a perfect BayesianNash equilibrium.

c. Explain what happens as $\phi \to \alpha/(1-\alpha)$, and why.

iii. Assume z_i and z_j are independent draws from the same uniform distribution with lower bound zero.

a. By considering the two cases $\phi = 1$, and ϕ large, show that the seller's expected profits need not be monotonic increasing in ϕ.

b Without making any more calculations, suggest what possible strategies might raise the seller's revenue when $\alpha/(1-\alpha) \approx 1$

[Oxford, 2nd Year Advanced Micro, 2011]

A13. Does good auction design depend on whether bidders' valuations are private or common? Explain why or why not.

[Oxford, 1st Year Micro, 2011]

A14. (i) Describe how a simultaneous ascending auction (SAA) works.

(ii) Give conditions under which an SAA is efficient, and explain why each of your conditions is necessary.

(iii) Is an SAA likely to work well under more general conditions?

[Oxford, 2nd Year Advanced Micro, 2012]

A15. (i) What are the advantages and disadvantages of the Vickrey auction design?

(ii) How can it be modified to alleviate its problems?

[Oxford, 2nd Year Advanced Micro, 2013]

A16. Each of three students $i = 1, 2, 3$ has a value, v_i, that is independently drawn from a uniform distribution on [£5, £15], for a bottle of wine, and each student bids independently and non-co-operatively in an auction.

(i) as a function of v_1, what is student 1's optimal bid in a second-price sealed-bid auction?

(ii) as a function of v_1, what is the expected gain to student 1 from participating in the auction?

(iii) what additional assumption(s) do you need to make for the expected value to student 1 of participating in the auction to be the same as if the auction were a first-price sealed-bid auction?

(iv) making the additional assumption(s) made in part (iii), use the Revenue Equivalence theorem to find student 1's optimal bid as a function of v_1, in a first-price sealed-bid auction?

(v) qualitatively, if the additional assumption(s) made in part (iii) did not hold, how would the optimal bid in part (iv) and the associated expected value change?

[Oxford, 1st Year Micro, 2013]

A17. Consider the following first-price sealed-bid auction. Two players $i = 1, 2$ simultaneously make bids for a single good which player i values at v_i. Each player's valuation v_i is drawn independently from the uniform distribution on $[0, 1]$, so that $\Pr(v_i \leq v) = v$ for $v \in [0, 1]$.

The players have preferences represented by the following vN-M utility functions: $u_1(x) = x^{\alpha_1}$ and $u_2(x) = x^{\alpha_2}$ where $x \geq 0$ and $0 < \alpha_1, \alpha_2 \leq 1$. The player that makes the higher bid receives the good and pays the price bid, whereas the losing player receives nothing: so if b_i was bid and was higher than b_j, then $x = v_i - b_i$ for player i and $x = 0$ for player j.

(a) Define what is meant by a bidding strategy for each player.

(b) Write down the expected utility for player i with valuation v_i as a function of the strategies of both players.

(c) Suppose that player i bids an amount that is proportional to his valuation v_i. What is the other player's best response strategy?

(d) Show that there is a Bayesian-Nash equilibrium in which each player's bid is given by $b_i(v_i) = \dfrac{v_i}{1 + \alpha_i}$ if $\alpha_1 = \alpha_2$.

(e) Is $b_i(v_i) = \dfrac{v_i}{1 + \alpha_i}$ an equilibrium if $\alpha_1 \neq \alpha_2$?

(f) What do players need to know in order to justify the solution of part (d)?

[*corrected version of* Oxford, 1st Year Micro, 2014; *I believe this is the only Oxford MPhil exam question on auctions not set by me (prior to 2025).*])

A18. Explain qualitatively how optimal bidding differs between uniform-price auctions and discriminatory auctions for multiple units of a homogeneous product, when each bidder has private values and desires multiple units. Which auction form is likely to (i) be more robust against collusion, (ii) result in more bidders participating, (iii) be more efficient. Explain your answers.

[Oxford, 2nd Year Advanced Micro, 2014]

A19. Two spectrum licences "East" and West" are to be auctioned. There are three bidders, namely (a) a bidder only interested in the "East" licence (and who has no value for the other licence), (b) a bidder only interested in the "West" licence (and who has no value for the other licence), and (c) a bidder who would value winning both licences, but has no use for just one license. Each bidder has a "private value" (i.e., knows the value to itself of each license, and of the pair of licences) and doesn't care about the identity of other winner(s), or how much other winner(s) pay.

(a) Under what circumstances might a Vickrey auction be a good idea?

(b) What problems might there be with a Vickrey auction?

(c) Discuss the merits of modifications that have been suggested for dealing with these problems.

[Oxford, 2nd Year Advanced Micro, 2015]

A20. (i) Under what circumstances and in what sense(s) are first-price sealed-bid auctions equivalent to descending auctions?

(ii) Under what circumstances and in what sense(s) are second-price sealed-bid auctions equivalent to ascending auctions?

(iii) Under what circumstances and in what sense(s) are all four auctions equivalent?

Give a sketch proof for your answer to (iii).

[Oxford, 1st Year Micro, 2016]

A21. Consider 3 bidders, A,B,C bidding in an auction of three identical objects.

Each bidder knows its own valuations, but not others' valuations. Bidder A has value 5 for any single item, but no incremental value for winning a further item. Bidder B has value 7 for winning any two items, but no incremental value for winning a third item, and places no value on winning any single item on its own. Bidder C places no value on winning just one or two items, but has value 10 for winning all three items.

(i) What would the outcome of the Vickrey auction be? (i.e., who wins how many objects, and what payments do the bidders make?)

(ii) Assuming bidders would bid in the same way as in (i), give the outcome of a core-selecting package auction, under each of TWO alternative pricing rules that have been suggested.

(iii) How do your answers change if bidder B is replaced by two separate bidders, B' and B", where B' and B" are each interested in only at most a single object, for which they have values of 3 and 4 respectively.

(iv) Discuss the RELATIVE merits of the THREE pricing rules you have considered in this setting, and more generally.

[Oxford, 2nd Year Advanced Micro, 2016]

A22. What are the advantages and disadvantages of running a Simultaneous Multiple Round Auction relative to running a Product-Mix Auction?

[Oxford, 2nd Year Advanced Micro, 2017]

A23. To what extent can we understand the distinctions between the behaviour in, and outcomes of, uniform and discriminatory auctions (for multiple homogeneous objects) by analogy with the distinctions between the behaviour in, and outcomes of, first-price and second-price auctions (for a single object)?

[Oxford, 2nd Year Advanced Micro, 2018]

A24. You have a ticket to a boxing match that you are not very interested in seeing. So it is worth onlv £10 to you. You conjecture that any man interested in bidding for the ticket has a value that is drawn from a uniform distribution on [£20, £40], whilst any woman interested in bidding has a value that is (independently) drawn from a uniform distribution on [£40,£80]. All bidders are risk neutral.

(a) You are facing two bidders and you know one is a man, and one is a woman, and you know which is which. Explain how to maximise your expected profits (net of the value to you of the ticket if you sell it).

(b) You are facing just one bidder, and you think there is equal probability that the bidder is either a man or a woman (and you cannot find out which). Explain how to maximise your expected profits.

(c) You are facing two bidders, each of whom is equally likely to be a man or a woman (and their genders are independent of each other). Explain the *qualitative* features of how you would maximise your expected profits. (You are not required to compute the exact details.)

[*corrected version of* Oxford, 2nd Year Advanced Micro, 2019]

A25. Two risk-neutral potential bidders have valuations that are independently drawn from a uniform distribution on [0, 1] for an object.

After finding out their own valuation (but not the other bidder's valuation) they independently decide whether to enter and bid in an ascending auction.

a. Assume the reserve price is 0 (so, for example, if only one bidder enters the final price is 0).

What is the maximum entry fee the seller can charge so that all bidders with values greater than or equal to $\frac{1}{2}$ choose to enter?

b. What is the seller's expected revenue in this case (including the entry fees it collects)?

c. Now assume the seller charges no entry fee, but sets a reserve price of $\frac{1}{2}$. What is the seller's expected revenue?

d. Without doing any more calculations, does a bidder with a valuation strictly above $\frac{1}{2}$ prefer the entry fee that you calculated and no reserve price, or the reserve price of $\frac{1}{2}$ and no entry fee?

e. Now assume that the bidders are in fact risk-averse.

Rank the following three options in order of the seller's expected revenue.

(i) set a reserve price of 0, charge the entry fee you computed in part (a), and run an ascending auction

(ii) set a reserve price of $\frac{1}{2}$, and run an ascending auction if more than one bidder enters

(iii) set a reserve price of $\frac{1}{2}$, and run a sealed-bid auction if more than one bidder enters

Explain all your answers.

[Oxford, 1st Year Micro, 2019]

1.

A26. [**OPEN BOOK**] There are two bidders in a Combinatorial Clock Auction (CCA) with Vickrey Pricing for 3 units of a homogeneous good (there is no core-selection in the pricing rule). The clock prices only take integer values.

Bidder A values winning 1 unit at 6, values winning 2 units at 8.5, and values winning 3 units at 10. Bidder B values winning 1 unit at 5, values winning 2 units at 8.5, and values winning 3 units at 9. These values are also given in the following table.

Aggregate values		
	A	B
$V(1)$	6	5
$V(2)$	8.5	8.5
$V(3)$	10	9

(1) What do you expect the outcome (i.e., the allocation of the goods, and the bidders' payments) of a Vickrey auction (i.e., a CCA without a clock phase) to be?

(2) Which quantities would the bidders demand at the clock prices 0, 1, 2, 3, and 4 if they bid truthfully? At which of these prices does the clock phase end?

(3) Suppose the bidders bid truthfully in the clock phase, and do not make any additional bids in the sealed-bid phase. What will be the outcome of the CCA (with Vickrey Pricing)?

(4) Suppose, instead, bidder A bids for 3 units for $p \leq 3$ and 2 units for $p = 4$ in the clock phase. Bidder B bids as in part (2) in the clock phase. (Recall that prices are integers in the clock phase.) Neither bidder makes any additional bids in the sealed-bid phase. What is the outcome?

(5) Comment on your results.

(6) Note that, under standard activity rules, the additional bids that either bidder would be permitted to submit in the sealed-bid phase of the CCA in this example (in either part (3) or part (4)) would not increase that bidder's profits. Is it reasonable to assume that bidders will not submit additional bids in the sealed-bid phase of similar—but not identical—examples? Give the reason for your answer.

(7) Would you expect bidder A to behave similarly in an SMRA as in part (4)? (You are *not* asked to predict the outcome of an SMRA.)

[Oxford, 2nd Year Advanced Micro, 2021]

A27. [**OPEN BOOK**] Each of n risk-neutral bidders, $i = 1, \ldots, n$ independently receives a signal x_i drawn from distribution $F(.)$ with density $f(.)$, and independently makes a single bid $b(x_i)$. The highest bidder wins an object whose value is a symmetric function $v(x_1, \ldots, x_n)$ which is strictly increasing in its arguments, but *all* the bidders pay an equal share, $(1/n)$th, of the winning bid. Assuming $b(0) = 0$, and that second-order conditions are satisfied for any solution that satisfies the first-order condition,

(i) by writing down a first-order condition for optimal bidding, show that, if $n = 2$, the symmetric equilibrium bidding function is

$b(x) = \int_{y=0}^{x} [2v(y, y,)f(y)/F(y)]dy$

(ii) find the symmetric equilibrium bidding function, $b(x_i)$, assuming $F(.)$ is the uniform distribution on [0,1], and $v(x_1, \ldots, x_n) = (\sum_{i=1}^{n} x_i)/n$ (without making any assumption about n).

Hint: write y for the maximum of the other signals and think about how the bidding function you found in part (i) is modified.

(iii) Do you expect risk-averse bidders to bid more or less than risk-neutral ones? Explain your answer.

[Oxford, 1st Year Micro, 2021]

A28. Each of two risk-neutral bidders has a private value for an object independently drawn from a uniform distribution on (0,1). They compete in an auction in which the price rises continuously at a constant rate from 0 to 1. The bidder who drops out last wins the object and pays the price at which she drops out. The other bidder pays nothing. (Ties are resolved randomly.) At each price p each bidder finds out whether or not its competitor has dropped out prior to price kp, $0 \leq k \leq 1$. That is, if $0 < k < 1$, bidders receive information about whether the other bidder has quit only after a time delay, and if $k = 0$ bidders receive no information about the other bidder's behaviour until after the auction. A bidder who finds out that its competitor has dropped out prior to price kp quits immediately at price p.

a. For what values of k, if any, is this auction strategically equivalent to (i) a standard ascending auction? (ii) a standard first-price sealed-bid auction? (iii) a standard second-price sealed-bid auction? (iv) a standard Dutch (i.e., descending) auction?

b. Assume each bidder bids up to the same constant fraction, y, of its value, if it does not learn earlier that the other bidder has quit. That is, a bidder with value v bids up to price $p = yv$ unless it finds out earlier that the other bidder has already quit. As a function of k, v, and y, what is the expected payment made by a bidder with value v?

c. Assuming bidders bid as in part b, (i) explain how to use the result of part b to find equilibrium bidding strategies (ii) Do this to find y.

d. For what values of k, if any, would the auctioneer's revenue be higher if bidders were risk-averse.

[Oxford, 1st Year Micro, 2022 (both core & advanced sections)]

A29.(i) Under what assumptions can, and will, every rational bidder in a standard product-mix auction which permits both positive and negative bids, make a set of bids that expresses her true valuation?

(ii) Consider a bidder who places bids at coordinates (5, 0), (0,3), (5, 3), and (20, 18), for 2 units, 2 units, minus 1 unit, and 1 unit, respectively, in such a product-mix auction.

Draw a diagram showing the demands that the bidder is expressing, as a function of the prices p_1 and p_2 of goods 1 and 2, respectively.

(iii) What is the valuation, $V(i, j)$, for$i \geq 0, j \geq 0$ units of goods 1,2, respectively, with i, j integer, that is being expressed by the bidder in (ii).

(iv) Very briefly, explain ONE example of how you might modify the standard product-mix auction design if different, or weaker, assumptions than you assumed in (i) apply.

[Oxford, 1st Year Micro, 2023 (advanced section)]

A30. When there is a single unit for sale, are bidders likely to prefer an ascending auction, or a (first-price) sealed-bid auction?

[Oxford, 1st Year Micro, 2024 (core section–1st sitting)]

A31. How should the design of an auction depend upon whether the auctioneer wants to maximise expected revenue, or expected efficiency?

[Oxford, 1st Year Micro, 2023 (core section–2nd sitting)]

A32. (a) Each of two bidders, A and B, compete in an auction for an object.

A's value is $100 + 3z_A + 2z_B$ and B's value is $100 + 2z_A + 3z_B$. Bidder A observes the value of z_A, but *not* z_B; conversely, bidder B observes the value of z_B, but *not* z_A; the bidders are risk-neutral; and it is common knowledge that z_A and z_B are each independently drawn from a uniform distribution on [0, 120].

(i) How would you expect A and B to bid in an ascending auction? Explain your answer.

(ii) Very briefly explain what is meant by the "winner's curse" in this context.

(iii) How would you expect A and B to bid in a second-price sealed-bid auction? Explain your answer.

(iv) How, qualitatively, would your answer to part (i) be affected if A and B were risk-averse? Explain your answer.

(v) How, qualitatively, would your answer to part (i) be affected if z_A and z_B were positively correlated? Explain your answer.

(vi) What is A's expected profit from the auction as a function of z_A.

(b) Now consider a problem with 3 bidders: assume A's value is $100 + 3z_A + z_B + z_C$; B's value is $100 + z_A + 3z_B + z_C$; and C's value is $100 + z_A + z_B + 3z_C$. As before, bidder i observes only the value of z_i, but *not* z_j for $j \neq i$; the bidders are risk-neutral; and it is common knowledge that z_A, z_B, and z_C are each independently drawn from a uniform distribution on [0, 120].

(i) How would you expect A , B, and C to bid in an ascending auction? (Assume that if a bidder quits the auction, this is observed by the other bidders.)

(ii) What is the auctioneer's expected revenue from an ascending auction if all of A, B, and C are bidding in this ascending auction?

(iii) What is the auctioneer's expected revenue from an ascending auction if only A and B are permitted to bid in the ascending auction (but their values are as described in section (b))?

(iv) How, qualitatively, would your answer to part (ii) be affected if the bidders (A , B, and C) were risk-averse? Explain your answer.

(v) How, qualitatively, would your answer to part (iii) be affected if the bidders (A and B) were risk-averse? Explain your answer.

Note: you may if you wish use the fact that the expected value of the kth highest of n signals independently drawn from a uniform distribution on [0,1] is $(n+1-k)/(n+1)$.

[Oxford, 1st Year Micro, 2024 (advanced section–this question was also originally set for the core section, but then substituted by questions A30 and A31, above)]