Regulated Prices, Rent-Seeking, and Consumer Surplus

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www.paulklemperer.org

Jeremy Bulow
Graduate School of Business,Stanford University, CA 94305-5015, USA
jbulow@stanford.edu

and

Paul Klemperer
Nuffield College, Oxford University, Oxford OX1 1NF, England
+44 777 623 0123; paul.klemperer@economics.ox.ac.uk

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Price controls lead to misallocation of goods and encourage rent-seeking. The misallocation effect alone ensures that a price control always reduces consumer surplus in an otherwise-competitive market with convex demand if supply is more elastic than demand; or with log-convex demand (e.g., constant-elasticity) even if supply is inelastic. The same results apply whether rationed goods are allocated by costless lottery, or whether costly rent-seeking and/or partial decontrol mitigates the inefficiency. Our analysis exploits the observation that in **any** *market, consumer surplus equals the area between the demand curve and the industry marginal revenue curve.*

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I Introduction

Price controls in competitive markets can harm consumers in three ways. Textbook analyses typically, and sometimes exclusively, focus on the cost of reduced supply.[1] A second cost, discussed at least since Friedman and Stigler (1946) but rarely emphasized until Glaeser and Luttmer (1997, 2003), is that the available supply will not necessarily be allocated to those with the highest values.[2] A third cost is the cost of rent-seeking behavior, such as queueing, lobbying, and search costs. When do these costs outweigh the benefits of lower prices, so price controls reduce consumer surplus?

We show that if output is allocated randomly among those prepared to pay more than the controlled price and if supply is more elastic than demand, then a price control always hurts consumers if demand is convex (e.g., linear, log-linear, etc.). Even with completely inelastic supply, total consumer surplus falls whenever demand is log convex (constant elasticity is one example).

Furthermore, these results are unaffected if rent-seeking alters the allocation. Though rent-seeking leads to more-efficient-than-random allocation, the costs it dissipates mean a price control is guaranteed to hurt consumers under the identical conditions.[3]

Splitting the market between controlled and uncontrolled units also makes no difference to these results. Even though all the highest-value consumers can consume, the results are the same (although the magnitudes of consumers' losses are, of course, affected).[4]

[1]For example, the analyses in Taylor and Weerapana (2010, 177-78) and Boyes and Melvin (2010, 194–95), which are widely-used textbooks in US universities and colleges, simply assume efficient allocation without discussion.

[2]Welch (1974), Lott (1987, 1990), Palda (2000), and Luttmer (2007) discuss allocative costs in the context of minimum-wage legislation; and MacAvoy and Pindyck (1975), Braeutigam and Hubbard (1986), and Davis and Kilian (2011) analyse the costs of restricting new potential consumers' access to the natural gas market. A clear exposition of the standard theoretical analysis of these "allocative costs" is in Viscusi, Harrington, and Vernon (2005).

[3]But rent-seeking typically makes rationing (even) more likely to reduce consumer surplus if supply is elastic. These results also depend on rent-seeking costs being uncorrelated with valuations, as we also discuss later.

[4]Examples of partially controlled markets include Manhattan real estate (where some units are either rent-controlled or rent-stabilized while others are available to the highest bidder), and the British healthcare system (where a small private market coexists with a National Health Service which approximates random rationing since healthcare profes-

Finally, while there is always a windfall gain to incumbent consumers, these gains are often small. For example, below-market rents are typically phased in over time by rent freezes rather than cuts, and turnover in rentals is on average very high. So even when controls raise consumer surplus in the short run because of the incumbent effect, and even though a gradual implementation of rent control will also mean a slower decline in the value of the marginal rental seeker, the mis-allocation effect alone can quickly cause a net loss in consumer surplus.

Existing analyses fail to note that *consumer surplus equals the area between the demand curve and the industry marginal-revenue curve* up to the market quantity in an uncontrolled market–even in a competitive market–see figure 1. More generally, when goods are rationed, total consumer surplus equals the sum of the values to consumers of the units they receive less the sum of their marginal revenues.[5] *These facts are the key* to the development and interpretation of our results.

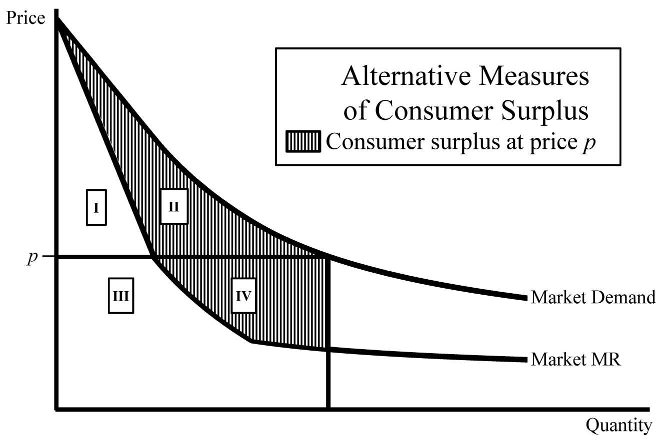

FIG. 1.—Alternative measures of Consumer Surplus.

In *any* market, *including a competitive one*, consumer surplus can be measured *as either* [area I + area II] *or* [area II + area IV], since elementary theory implies [area I + area III] = [area III + area IV].

sionals are roughly uniformly distributed across the population).

[5]For example, if the inverse demand curve were $p = 100 - q$ and so $MR = 100 - 2q$ then a consumer with a value of 70 (and so MR of 40) and a 50 percent chance of receiving a unit would account for $.5(70 - 40) = 15$ in consumer surplus, and aggregating across all consumers in this way correctly calculates total consumer surplus, even though the measure cannot be used for determining the amount of consumer surplus that goes to the individual consumer (which, of course, also depends on the market price).

One caveat is that our analysis ignores distributional issues. Even when price controls reduce aggregate consumer surplus, they redistribute it among consumers.[6]

We begin in Section II by assuming that output is allocated randomly among those willing to pay more than the controlled price, and then extend the model to address non-random allocation. Section III generalizes the model by incorporating rent-seeking and shows that secondary markets also fit easily into our analysis. Section IV considers partially-controlled markets. It shows that while splitting the market between uncontrolled and controlled (rationed) units can increase consumer surplus, it can never be consumer-optimal to use more than one control price. Section V concludes.

Appendix A illustrates our analysis by solving our model for a class of examples that includes the standard constant-elasticity, log-linear, and linear demand curves.

II The Basic Model: Rationing by Lottery

Consider a competitive industry with a demand curve $D(p)$ formed by a density of consumers $-D'(v) \geq 0$ with unit demand at value v,[7] and a supply curve $S(p)$.

We assume $S'(p) \geq 0$ (that is, no "backward-bending" supply). We also assume that demand is finite at all $p > 0$, and that its elasticity at all prices above some finite price is bounded strictly above 1. This condition ensures that total consumer surplus in an uncontrolled market that clears at price p (that is, $\int_p^\infty D(v)dv$) is finite. So, for example, inelastic constant-elasticity demand is ruled out.[8]

[6] We also ignore any effects on the provision of quality.

[7] We extend to consumers who individually have downward-sloping demand curves in Section III.C.

[8] Our assumptions also ensure that a monopolist's problem is well-defined. Although our model is a competitive one, demand conditions we develop also apply to a capacity-constrained monopolist and are related to the conditions that determine a monopolist's pass-through. See note 14.

We assume in Sections II.A-II.B that if a regulator sets a price p below the market clearing level then supply is randomly allocated among consumers with value $\geq p$.[9] This is the standard assumption made in, for example, Viscusi, Harrington, and Vernon (2005), but we will relax it in subsequent sections of our paper.

So consumer surplus at the controlled price, $CS(p)$, equals consumer surplus if the market cleared at p, times the ratio of supply to demand, $S(p)/D(p)$:

$$CS(p) = \frac{S(p)}{D(p)} \left[\int_{v=p}^{\infty} D(v) dv \right]. \tag{1}$$

A. Measuring Consumer Surplus using Marginal Revenues

For any quantity in any market, price times quantity equals total revenues equals the area under the monopolist marginal revenue (MR) curve. Therefore, if all consumers with values above p are served, *consumer surplus equals the area between the demand curve and the marginal revenue curve,* in ***any*** market, including our competitive one.

It follows that the increment in total consumer surplus in an uncontrolled market caused by a price reduction that leads to a one unit increase in quantity is $p - MR(p)$. So we can write *Marginal Consumer Surplus* as $MCS(p) \equiv [p - MR(p)]$, and also rewrite (1) as

$$CS(p) = \frac{S(p)}{D(p)} \int_{v=p}^{\infty} -D'(v)[v - MR(v)]dv = \frac{S(p)}{D(p)} \int_{v=p}^{\infty} -D'(v)[MCS(v)]dv \tag{2}$$

[9] For example, the 2012 British Olympic Committee distributed tickets using a random allocation procedure, and prevented resale by printing recipients' names on their tickets and requiring them to turn up with picture IDs. More significant examples of (approximately) random rationing without resale might include socialised health or education systems.

since $-D'(v)$ is the density of consumers with a value of v.[10] We will also write *Average Consumer Surplus* (per unit sold) as $ACS(p) \equiv \left[\frac{CS(p)}{S(p)}\right] = \left[\frac{1}{D(p)}\int_p^\infty -D'(v)[MCS(v)]dv\right]$.

Differentiating (2) with respect to price yields the change in consumer surplus due to a small price *cut*:

$$-CS'(p) = -D'(p)\frac{S(p)}{D(p)}[MCS(p)] + [D'(p)\frac{S(p)}{D(p)} - S'(p)][ACS(p)]. \quad (3)$$

Since $S'(p) \geq 0$, consumer surplus must decline if $ACS(p) > MCS(p)$.

The intuition is trivial: with random allocation of a fixed number of units, consumer surplus is proportional to *Average CS*, which of course declines if *Average CS* > *Marginal CS*. If supply falls with price, that only reduces consumer surplus further.

Figures $2A$ and $2B$ illustrate the result: figure $2A$ measures consumer surplus conventionally, as in equation (1). Consumer surplus at the market price p^{Market} is the heavily-shaded area, and the effect on consumer surplus of reducing price to a controlled level $p^{Control}$ would be the sum of areas X (the benefits to existing buyers) and Y (the benefits to new buyers), *if* supply could expand from $D(p^{Market})$ to meet the new level of demand $D(p^{Control})$. So with random allocation of supply, the average consumer surplus per consumer served, ACS, equals the average height of the whole area formed by both the shaded areas together.

So figure $2A$ suffices to show that if demand is sufficiently "fat-tailed", then average consumer surplus is decreasing in the price control, so rationing hurts consumers. But figure $2B$ tells us *how* fat-tailed.

[10] We can derive (2) directly from (1) using $MR(v) \equiv v + D(v)/D'(v)$ (the derivative of total industry revenue $vD(v)$ with respect to quantity $D(v)$ in an uncontrolled market with price v).

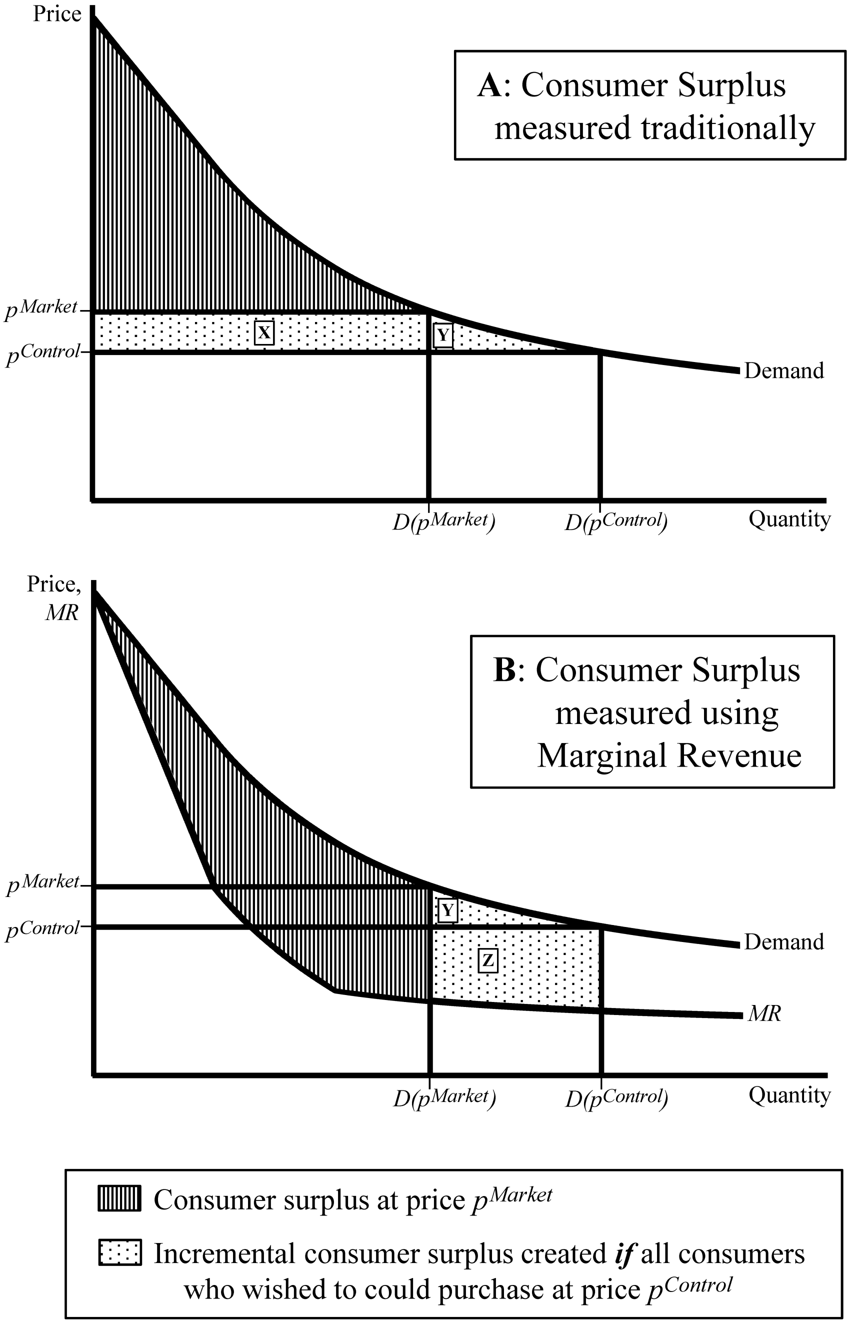

FIG. 2.—Effect of a Price Control on Consumer Surplus.
A, measured traditionally; B, measured using Marginal Revenue.

In figure $2B$ we have drawn the MR curve onto figure $2A$. Because, as we noted in figure 1, the area under the MR curve up to $D(p^{Market})$ equals total revenue at that quantity, the heavily-shaded areas in figs. $2A$ and $2B$ are equal and both represent consumer surplus at the market price, p^{Market}. Likewise, because the area under the MR curve up to $D(p^{Control})$ equals total revenue at that quantity, the sum of the heavily- and lightly-shaded areas in figs. $2A$ and $2B$ are also equal and would both represent consumer surplus at the controlled price, $p^{Control}$, *if* all the demand at that price could be satisfied. The lightly-shaded areas in figs. $2A$ and $2B$ are therefore equal as well, and represent the incremental consumer surplus from reducing the price *if* supply could expand to meet the incremental demand.[11] So if the average height of the heavily-shaded area exceeds that of the lightly-shaded area in figure $2B$, i.e., $ACS(p) > MCS(p)$, then *Average CS* falls, and therefore total consumer surplus also falls, even with no fall in supply.

Results about the effects of price controls on consumer surplus now follow straightforwardly:

PROPOSITION 1. When a rationed good is allocated randomly, consumer surplus is always reduced by a tighter price control if demand is log-convex.

Proof. If demand is log-convex, $D''(v)D(v) \geq (D'(v))^2$, so $MCS'(v) \geq 0$ (since $MCS(v) = v - MR(v) = -D(v)/D'(v)$), and therefore $ACS(v) > MCS(v)$.[12] **QED**

This result is also easy to see from figure $2B$, since $MCS(v) = v - MR(v) = constant$ for log-linear demand, but demand is steeper than MR (so consumers are hurt by price controls) for any log-convex demand, for

[11] Of course, only area Y of this incremental surplus would go to the new purchasers; the area $X + Y = Y + Z$ would be the incremental surplus gained by *all* consumers.

[12] The Proposition can, of course, be derived without measuring consumer surplus using marginal revenue (see our 2009 and 2011 working papers), but the MR approach allows easy extension to our subsequent analysis, especially in section III.

See, e.g., Prékopa (1971), An (1998), and Bagnoli and Bergstrom (2005) on log-convexity.

example, constant-elasticity demand.[13,14]

B. Elastic Supply

If $ACS(p) < MCS(p)$ (i.e., demand is log-concave, such as linear, or is a mixture of log-convex and log-concave), whether consumers are helped or hurt by a price control depends on the elasticity of supply:

PROPOSITION 2. When a rationed good is allocated randomly, consumer surplus is always reduced (increased) by a tighter price control if (i) demand is convex (concave) and (ii) demand is locally less (more) elastic than supply.

Proof. Dividing the right-hand side of (3) by $S(p)/p$ yields

$$\text{sign}[-CS'(p)] = \text{sign}[MCS(p)\,|\text{Elasticity of Demand}| - ACS(p)(\text{Elasticity of Supply} + |\text{Elasticity of Demand}|)]$$

So if (Elasticity of Supply) $\geq$ |Elasticity of Demand|, then $[-CS'(p)] < 0$ if $MCS(p) < 2ACS(p)$. But every linear demand satisfies $MCS(p) = 2ACS(p)$ (since marginal revenue is twice as steep as demand when demand is linear). Furthermore, the linear demand that is tangent to any convex demand curve at p has the same $MCS(p)$ as the convex demand (because $MCS(p) = -D(p)/D'(p)$), and has a lower $ACS(p)$ than it (because the convex demand is weakly higher everywhere). The results then follow (using

[13] The figures are drawn to scale for (inverse) demand which is linear (so $MCS(p) = p - MR(p)$ is increasing as price falls) for quantities below $0.36D(p^{Market})$; then log-linear (so $MCS(p) = p - MR(p)$ is constant) for quantities up to $0.72D(p^{Market})$; and finally constant-elasticity plus a constant (so $MCS(p) = p - MR(p)$ is decreasing as price falls) for larger quantities. For this demand, ACS is increasing for quantities up to $D(p^{Market})$, and then decreasing, so $MCS < ACS$ between p^{Market} and $p^{Control}$.

[14] Though the market we are modelling is competitive, these conditions for when consumers gain also apply when a monopolist with a vertical marginal cost curve is selling to capacity. The conditions also have other simple monopoly-theory interpretations. The condition for the constant-marginal-cost monopolist, that would set price p, to generate greater consumer surplus than profits is $ACS(p) > MCS(p)$ (because its per-customer profit $= p - AC = p - MC = p - MR(p) = MCS(p)$). The condition for such a monopolist to pass through $> 100\%$ of any (marginal) tax or cost increase is that demand is log-convex (because its pass-through rate $= \frac{dp}{dMC} = \frac{dp}{dMR} = \frac{slope\ of\ demand}{slope\ of\ MR}$, see Bulow and Pfleiderer (1982); this pass-through result extends to a broad class of Cournot oligopoly contexts, see Weyl and Fabinger (2009, 2011), and the 2008 version of our current paper).

a parallel logic for conditions that ensure *in*creased consumer surplus).[15]
QED

So a pass-through rate of 50% or more in a competitive industry with convex demand would imply consumers lose from a price control. (In a competitive market, pass-through = [*elasticity of supply*/(*elasticity of supply* + |*elasticity of demand*|)].) Campa and Goldberg's (2005) study based on exchange-rate changes estimates short-run and long-run pass-through for 23 countries at .46 and .64, respectively, and other studies using exchange-rate changes obtain similar results. Results such as these suggest that whether a small regulated price cut would benefit consumers is likely to vary from market to market.[16]

C. Incumbent Consumers vs. Newcomers

Thus far we have focused on the long-run distributional consequences of price controls. But for durables such as rental apartments the inefficiencies created by a price control will phase in gradually, so even if new tenants receive less consumer surplus on average after controls are implemented, there is a group of incumbents who receive a windfall transfer from the lower prices. Following Glaeser and Luttmer (1997), we can model this by assuming that the $S(p)$ buyers with the highest values buy with probability $\lambda + (1 - \lambda)S(p)/D(p)$, while the remaining $D(p) - S(p)$ buy with probability $(1 - \lambda)S(p)/D(p)$ (so if $\lambda = 1$, the rationing is perfectly efficient).

Clearly if enough of the supply is allocated efficiently, and without any reduction of supply, consumer surplus must rise. However, the conditions for

[15] This logic extends to give a more general result encompassing both Propositions 1 and 2. Let G_K be the class of demands satisfying $MCS(p) = K[ACS(p)]$, for any constant, K. Our logic implies a tighter control reduces (increases) consumer surplus if inverse demand is always larger (smaller) than any member of G_K to which demand is locally tangent, and supply is locally more (less) than $K - 1$ times as elastic as demand. The case $K = 1$ and $K = 2$ imply Propositions 1 and 2, respectively, since G_1 and G_2 are the sets of all log-linear demands, and all linear demands, respectively. (Any log-convex demand is always above any log-linear demand to which it is tangent; for $K \neq 1$, G_K includes the demands $p = \alpha - \beta(D(p))^{K-1}$, with α, β constants; see Appendix A.)

[16] Oligopolistic industries may have lower pass-through than competitive ones, so these results may understate average pass-through in competitive markets and so overstate consumers' expected benefit from a price control. (Estimates of pass-through based on price responses to excise taxes are generally higher than those based on exchange-rate changes.)

Economists tend to assume demand is convex, although relatively little is known about actual functional forms–see, e.g., Blundell, Browning, and Crawford (2008) and the references they cite.

consumer surplus to fall still do not seem onerous. An argument paralleling that of the previous subsection (see Appendix B, Section I) shows that *for any convex demand, consumers always lose from a small price cut below the uncontrolled market price if supply is at least* $\frac{1+\lambda}{1-\lambda}$ *as elastic as demand*; or *for any log-convex demand, if supply is at least* $\frac{\lambda}{1-\lambda}$ *as elastic as demand.* And we show in Section II of Appendix B that with demand of constant-elasticity η, and (any functional form of) supply with elasticity φ, consumers always lose from a small price cut below the market price if $\lambda > \frac{\varphi+1}{\varphi-\eta}$.[17]

Furthermore, this model assumes prices are immediately reduced when the control is announced. More commonly, price controls are phased in only gradually by restraining price increases to below-market rates. Also, turnover is on average high in markets such as that for rental accommodation. Both these things reduce the relative importance of the incumbents' windfall.[18] So even when controls raise consumer surplus in the short run because of the incumbent effect, the misallocation effect alone can quickly cause a net loss in consumer surplus. We illustrate this in Section III of Appendix B.

III A Model of Rationing with Rent-Seeking

Our basic model in which all consumers have an equal chance of being able to buy at the controlled price, with no additional search or rent-seeking costs, is a special case of a more general model in which consumers expend "effort" competing for the rationed good:

[17] We also generalise the allocation process further, by assuming an additional fraction of supply is allocated as inefficiently as possible above the controlled price–this case is obviously extreme, but Glaeser and Luttmer (1997) point out, for example, that long-time residents may have greater access to, but less desire for, rent-controlled apartments, than transients.

With linear demand, a tighter price control must increase consumer surplus, absent supply effects, however inefficiently supply is allocated. The reason is that linear demand always satisfies $p - MR(p)$ = vertical intercept of demand $-p$, so in this case the gain, $MCS(p)$ $(= p - MR(p))$, must exceed the loss, which is at most the loss (= vertical intercept $-p$), if the highest value consumer is displaced.

[18] A gradual implementation of price control does also mean a slower decline in the value of the marginal consumer, but some consumers with lower values than the current price jump in straight away to capture the expected gains from being an incumbent in the future.

Let each consumer have a marginal cost of effort drawn from an arbitrary distribution, independent of the consumer's value. A consumer's probability of purchase is proportional to its effort. Competition determines the probability of purchase per unit of effort expended: if E is the sum of all consumers' efforts, then $E/S(p)$ of effort earns one unit. So a consumer who has marginal cost of effort c chooses effort $E/S(p)$ if its value $v \geq p + cE/S(p)$, and expends no effort (and does not buy the good) otherwise. This condition determines the total effort, E, expended in equilibrium, and so the equilibrium allocation of the goods.[19]

Letting $n(v,p)$ be the expected quantity per consumer bought by consumers with value v when the price is p, equation (2) now generalises to:

$$CS(p) = \int_{v=p}^{\infty} -D'(v)\,[v - MR(v)]\,n(v,p)dv = \int_{v=p}^{\infty} -D'(v)[MCS(v)]n(v,p)dv \tag{4}$$

That is, (2) is the special case of (4) in which $n(v,p) = S(p)/D(p)$, $\forall v$. A direct way to obtain (4) uses an envelope-theorem argument *à la* Myerson (1981) to compute the total surplus of all consumers with value v, and then integrates by parts over all values v—see Appendix B, Section IV. But it is more instructive to observe that because (we assumed) consumers' effort costs are independent of their values, our *single* demand curve with consumers who face a *distribution* of effort costs is equivalent to a *set* of identical demand curves where consumers from any given demand curve face the *same* effort cost. Clearly the latter situation is equivalent to a set of identical demand curves each of which faces a different "effective price". (This "effective price" $= p + cE/S(p)$, that is, the controlled price *plus* the equilibrium cost of effort that consumers from that demand curve need to expend to obtain a unit.)

As usual, the crucial point is that we can write consumer surplus as the integral of purchasing consumers' values less their marginal revenues. *And*, because the demand curves are identical scaled versions of the demand curve

[19] To see equilibrium is generally unique, observe a proportional increase in *anticipated* E yields the same proportional increase in effort for those consumers who still purchase, but reduces the number of purchasers, so yields a smaller than proportional increase in actual E.

Since our risk-neutral consumers want at most one unit each, nothing would change if we assume a single unit is allocated to each of the $S(p)$ consumers who make the greatest effort. (Technically, there is then no equilibrium if consumers make simultaneous effort choices, but the outcome in the text is the equilibrium if consumers make sequential choices; it is also the limit of equilibria of discrete versions of the simultaneous game.)

of our original problem, *the MR of a consumer with any given value is the same as that value's MR for the demand curve of our original problem.* By contrast, each consumer's effective price depends upon which demand curve it is from, so writing consumer surplus in the traditional way as the integral of consumers' values less their effective prices would be much less helpful.

So total consumer surplus is just the integral (over valuations) of consumers' values-minus-MRs (that is, their MCSs) times the total number of units bought by consumers who have the given value: this is equation (4).

The intuition is straightforward: from consumers' point of view, rent-seeking costs simply increase (by differing amounts) the "effective price"s they face. That the rent-seeking part of these effective prices is a social waste is irrelevant to them. So consumer surplus equals the integral of purchasers' values less their MRs, exactly as in our basic model without rent-seeking.[20]

Figure $2B$ illustrates this. Total consumer surplus is just the integral of the shaded area but with each strip of height $v - MR(v)$ (and of width $dq = -D'(v)dv$, since $-D'(v)$ is the density of consumers with value v) weighted by the total number of units, $n(v,p)$, that each bidder with value v gets. Thus fig. $2B$/equation (4) allows the computation of consumer surplus knowing only the probabilities with which different types of consumers receive units. (Of course, a consumer's own per-unit surplus is *not* equal to the height, $v - MR(v)$, of "its" strip–the calculation applies only in aggregate.)

By contrast, the corresponding integral of the shaded area in figure $2A$ measures consumer surplus only in our basic model (Sections II.A-II.B), and *not* in our current more-general model; the height, $v - p$, of type v's strip in fig. $2A$ shows its per-unit gross surplus *ignoring* the resources it expends on rent-seeking.[21] And, as we noted above, consumers' effective prices–and therefore their rent-seeking costs–differ across consumers who have the same

[20] The integral of MR alone (with the same weights) is the sum of consumers' pecuniary expenditure on goods *plus* the value of their rent-seeking efforts valued at the costs the rent-seekers themselves attribute to them.

[21] However, this integral, $\int_p^\infty -D'(v)\,[v-p]\,n(v,p)dv$, does show the consumer surplus that could be achieved by an informed principal who could allocate higher probabilities to favoured types unconstrained by any need to impose greater costs (either through the price charged, or through deadweight rent-seeking activities) on the favoured consumers. (In the same way, the revenue of an ordinary monopolist–including one that sets different prices at which consumers can buy goods with different probabilities–is the sum of the MRs of the consumers it sells to, but the revenue of a monopolist that can somehow price discriminate costlessly is the sum of the maximum willingnesses-to-pay of the consumers it sells to.)

value, v.

The implication is that if the demand curve is always steeper than the MR curve, so $(d/dv)\,[MCS(v)] > 0$ (and also $MCS(p) < ACS(p)$—see fig. 2B—so demand is log-convex), then any transfer of probability from a higher-v consumer to a lower-v consumer reduces consumer surplus.

Furthermore, a tighter price control always results in some high-v consumers being displaced by low-v consumers, and not vice versa (because the equilibrium amount of effort required to obtain a unit is increased, so high-v consumers who did not buy previously are more disadvantaged relative to any low-v consumers who did and who must therefore have lower rent-seeking costs). So, since any supply response only reduces consumer surplus further,[22] we can generalise Proposition 1:

PROPOSITION 3**.** When a rationed good is allocated to consumers who engage in rent-seeking, consumer surplus is always reduced by a tighter price control if demand is log-convex.

Of course, the converse also holds: if $(d/dv)\,[MCS(v)] < 0$ everywhere, so $MCS(p) > ACS(p)$, then a price control that does not cause a supply cut must increase consumer surplus. In this case, because rent-seeking costs that are independent of values must lead to a more-efficient-than-random allocation (because there is less substitution of high-v by low-v consumers than in a random allocation), the fact that $MCS(v)$ is decreasing in v also implies that no distribution of rent-seeking costs can yield greater consumer surplus than a random allocation. The consumer surplus from a random allocation will be equalled when at least a fraction $S(p)/D(p)$ of consumers have zero costs of rent-seeking; since no rent-seeking costs are then actually incurred, this corresponds exactly to our basic model of random rationing in Sections II.A-II.B.

Note that because rent-seeking yields a more efficient allocation than random rationing (less substitution of high-v by low-v consumers), rent-seeking must result in lower consumers' losses from a tighter price control when demand is log-convex, but also result in lower consumers' gains from a tighter price control when demand is log-concave and supply is inelastic.

[22]The effect of a price cut can be divided into the effect which would occur were supply inelastic, and a supply effect. The latter effect always reduces consumer surplus (by the sum of $MCS = v - MR > 0$ across all the consumers it displaces, since no consumers buy as a result of the supply effect who would not buy in its absence).

In the extreme case, if all consumers have identical costs of rent-seeking activities, the available supply is efficiently allocated to the highest-value consumers, exactly as in an uncontrolled market, but (with inelastic supply) the entire price reduction is eaten up by the rent-seeking activity–so consumer surplus is unaffected. And if there is any supply response at all, the "effective price" to consumers rises–that is, more than the entire price reduction is eaten up by the rent-seeking activity, and consumer surplus is reduced.

So with log-convex demand a price-control is always bad news for consumers, though less so with rent-seeking, while with log-concave demand rent-seeking reduces any benefits to consumers.[23]

A. Elastic Supply with Rent-Seeking

With inelastic supply, and either log-convex or log-concave demand, rent-seeking always dampens but never reverses the effect on consumers of imposing a price-control. But the supply response to a price control (which always hurts consumers[24]) is, of course, independent of whether or not there is rent-seeking. So, when demand is log-concave and supply responds to price, the overall effect on consumer surplus may turn from positive with random rationing to negative with rent seeking.

We therefore now generalise Proposition 2 about any convex demand (including mixtures of *log*-concave and *log*-convex) when supply is elastic.

PROPOSITION 4. When a rationed good is allocated to consumers who engage in rent-seeking, consumer surplus is always less than in an uncontrolled market if (i) demand is convex and (ii) demand is sufficiently less elastic than supply that $D(p)/D(p^M) \leq S(p^M)/S(p)$, in which p^M is the uncontrolled market price and p is the controlled price.

Proof. See Section V of Appendix B.

[23] If demand has both a log-concave section *and* a log-convex section at higher prices, then rationing with rent-seeking *may* help consumers even when random rationing would hurt them, if new consumers displace low-v low-MCS consumers, but would displace a mixture of these and higher-v higher-MCS consumers with random rationing. For example, with sufficiently inelastic constant-elasticity demand at high prices, linear demand at lower prices, and sufficiently inelastic supply, we can find market- and controlled-prices on the linear part of demand that lower consumer surplus with random allocation, but raise it if half of consumers have no rent-seeking costs while the other half have identical (positive) costs.

[24] See note 22.

Condition (ii) of Proposition 4 (i.e., $D(p)/D(p^M) \leq S(p^M)/S(p) \iff D(p)S(p) \leq D(p^M)S(p^M)$) is precisely the extension to a discrete price change of condition (ii) of Proposition 2 (i.e., $-pD(p)/D'(p) \leq pS(p)/S'(p) \iff d(D(p)S(p))/dp \geq 0$). However, Proposition 4 discusses only the *total* effect of a price reduction from the market-clearing price–so it is only a partial generalisation of Proposition 2. The reason is that both the amount spent on rent-seeking, and the extent to which it increases the allocation efficiency, can vary substantially as the controlled price changes–indeed they can *fall* as the price falls.[25] So the rate of change of consumers' non-supply benefits from a tighter price control, and therefore the extent to which they can outweigh supply effects, can also vary substantially as the price control changes.[26] So it is *not* true that any *marginal* tightening of an existing price control necessarily makes consumers worse off under the conditions of Proposition 4.[27]

Also unlike Proposition 2, Proposition 4 has no "converse" that a price control benefits consumers if demand is concave and more elastic than supply. For any finite ratio of the elasticity of demand to that of supply, rent-seeking activities can consume enough of any benefits from rationing that the supply effect of the price control (which is unaffected by rent-seeking) dominates. Indeed, we have already noted that if all consumers have identical rent-seeking costs then consumer surplus must fall unless supply is perfectly inelastic.

Typically, therefore, if supply is elastic, rent-seeking makes a price control (even) more likely to reduce consumer surplus.

B. Resale

Our analysis can be generalized to account for secondary markets. In this case, total *final*-consumers' surplus is the sum of all their MCSs *plus* the surplus that they could have earned by *not* consuming.

To see this, we can repeat our trick of dividing demand up into a set of demand curves; now each curve consists of consumers who face the same

[25] For example, with the distribution of rent-seeking costs described at the end of note 23, rent-seeking makes the allocation substantially more efficient than random when supply is 3/4 of demand, but both the expenditures on rent-seeking and its effects on the allocation fall to zero as price falls to where supply is 1/2 of demand.

[26] This does not affect Proposition 3, since a tighter price control then always reduces consumer surplus, independent of supply effects.

[27] For example, with linear demand and supply as elastic as demand, a small tightening of the price control strictly benefits consumers at the price at which supply is 2/3 of demand (but would be neutral with random rationing), if the distribution of rent-seeking costs is as described at the end of note 23.

rent-seeking costs *and* same resale-market participation costs. Consider the demand curve whose consumers can obtain a unit (using rent-seeking and/or the secondary market) for a total "effective price" p^e in equilibrium, and who can receive a total "effective sale price" s^e by reselling a unit. If $s^e > p^e$, only consumers with $v \geq s^e$ are final-consumers. *If* they had actually paid s^e each, their total surplus would have been the sum of their MCSs as usual, so their actual surplus is $s^e - p^e$ per consumer greater than this.[28] In effect, each of them bought at p^e, and each then made a profit of $s^e - p^e$ reselling to itself, before making an additional consumption surplus of $v - s^e$. Assuming rent-seeking and resale costs are independent of consumers' values allows us to re-aggregate the MCSs across demands with different p^es and s^es. So the total surplus of final consumers equals the sum of their MCSs (exactly as computed in equation (4) except that $n(v, p)$ now depends on the distributions of resale as well as rent-seeking costs) *plus* the surplus that these consumers could have earned by acting as middlemen and not consuming. Additionally there is some middleman (as opposed to consumer) surplus received by those who do not consume but for whom $s^e > \max\{v, p^e\}$.

So with inelastic supply, since final consumers' surplus (weakly) exceeds the sum of their MCSs, their surplus is always higher in a controlled than in an uncontrolled market if demand is log-concave, but a price control has an ambiguous effect on them if demand is log-convex. Indeed with costless resale, final consumers always weakly benefit from a price control, because the secondary market price then equals the original market price and some of them get units more cheaply.

More generally, in addition to raising consumers' surpluses by their potential trading gains, permitting resale is equivalent to lowering each consumer's cost of rent-seeking effort. However, since it is consumers' *relative* costs of rent-seeking effort that matter in equilibrium, the latter effect can *damage* consumer surplus.[29] In particular, the entry of sufficiently many low-cost

[28] This is just as in standard incentive problems, in which the expected surplus of a given type of agent is the integral of the incentive constraint (this corresponds to the MCS) *plus* the surplus of the lowest participant (zero absent resale, but $s^e - p^e$ here) cf. the mathematical development of our basic result in App. B, Sec. IV.

[29] It is not hard to show, for example, that if rent-seeking costs are high and identical for most consumers, but low for a few, and the price control is tight enough that only those with low costs consume absent resale, costless resale can lower final-consumer surplus. *Costly* resale can also lower total non-producer surplus (if middlemen dissipate most of their earnings in costs), and can *reduce* the efficiency of allocation (with the same distribution of rent-seeking costs, very dispersed resale-market participation costs, and a much

middlemen would equalise all consumers' acquisition costs, so completely undo the non-supply effects of any price control and, with elastic supply, result in lower consumer surplus than in an uncontrolled market.

C. Further Extensions

Several other extensions are straightforward.

Consumers with Downward-Sloping Demand—We modelled demand as comprised of consumers who have different values for a single unit each, as might be appropriate, for example, for applications such as rental housing, healthcare, and minimum wages. However, all our previous results apply to a model in which demand consists of consumers who have downward-sloping demands that are identical, or proportional to each other: because any given consumer can make independent decisions about whether or not to purchase each individual unit, this model is identical to the model of a set of demand curves where the (multiple) consumers on each demand curve all face the same effort cost, and we noted above that the latter model is equivalent to our main model.

If a fraction $S(p)/D(p)$ of consumers has no rent-seeking costs (or alternatively the complementary fraction has infinite costs), then we exactly replicate Section II's lottery model. In this case each consumer is either fully served at the controlled price or not served at all, so the inefficiencies and consumer surplus losses result from overconsumption by those lucky enough to be served. Supplies of natural gas are an example (see, Davis and Kilian, 2011).[30]

More General Rent-Seeking Cost Functions—We can easily relax our assumption that each consumer has a constant cost of rent-seeking effort, drawn from some distribution. Our results are unaffected if each consumer has a cost-of-effort *function*, drawn from some set. The reason is that a *single consumer* with a given marginal cost-of-effort *function* for an increasing probability of winning a single unit is equivalent to (the limit of) a large *set of*

less tight price control, the allocation may be close to efficient without resale, and less efficient with resale).

[30] If consumers have *decreasing* average costs of rent-seeking, then their surplus losses are *even greater* than in our model with constant marginal and average rent-seeking costs. For example, the rent-seeking cost of queueing for tickets might be independent of the number bought. On the other hand, consumers with increasing rent-seeking costs will have lower losses; for example, limiting the number of tickets any consumer can buy creates an infinite marginal cost at the limit–the allocation of food during wartime might be an example of rationing that is more-efficient than in our model.

consumers each of whom has a (different) *constant* marginal cost of effort for a small probability of winning a unit.[31]

Rent-Seeking Costs Correlated with Values—If consumers with higher values for the good have higher costs of effort, this obviously reduces consumer surplus.[32] If the distributions of effort costs for consumers with higher values for the good first-order stochastically dominate the distributions for lower-value consumers, then consumer surplus must be lower than if all consumers' costs were drawn from a common distribution (independent of value) that yields the same allocation of the good.[33] Of course, if effort costs are proportional to $(v - p)$, then consumer surplus at the rationed price, p, is zero.

Conversely, if higher-value consumers have lower costs of effort this increases consumer surplus. However, only if all consumers whose values exceed the uncontrolled market-clearing price can acquire units with zero effort costs will we obtain the traditional textbook outcome with neither misallocation nor rent-seeking costs.

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[31] The limit of the increasing marginal cost function created by piecing together the per-unit costs of this large set of consumers (each piece corresponding to the small amount of probability that is the maximum that the corresponding consumer can win) replicates the original (single) cost-of-effort function if that function is increasing, and otherwise replicates a version of it that is modified to be weakly increasing. (Specifically, it replicates the maximum of the original marginal cost-of-effort function and the largest weakly-increasing function that is never above the original average cost-of-effort.) As before, an alternative demonstration of the equivalence follows the logic of App. B, Sec. IV—defining $cs(v)$ as there, $dcs/dv = \partial cs/\partial v = n(v, p)$ for the same reasons as there, etc.

To check uniqueness of equilibrium, write E for the anticipated sum of *all* consumers' efforts, as before. A consumer with value v, and cost-of-effort function $c(e)$, chooses an effort, e, that maximises $(eS(p)/E)(v - p) - c(e)$ s.t. $eS(p)/E \leq 1$. A larger E yields an e/E that is lower for all consumers, and strictly lower for some, so there is a unique E for which the sum of all consumers' efforts equals E, and hence a unique equilibrium allocation, $n(v, p)$.

[32] *Welfare* might be less affected if those consumers with higher values and effort-costs are higher-income consumers whose gains have lower welfare weights.

[33] Let the distribution of type v's effort cost, $F(\cdot\,; v)$, first-order stochastically dominate $F(\cdot\,; v')$, $\forall v > v'$. Assume the type, $\widehat{v}(c)$, which purchases if and only if its effort cost $\leq c$, is strictly increasing (otherwise there in general no common distribution that yields the same allocation of the good). Then substituting $F(\cdot\,; \widehat{v}(\cdot))$ for $F(\cdot\,; v), \forall v$, changes neither the allocation, nor the aggregate effort expended by the set of consumers of any type, but reduces the aggregate *costs* of the effort expended by every such set of consumers.

IV Partially-Controlled Markets

A. Partially-Controlled Markets with *Rent-Seeking*

The results of our rent-seeking model are unaffected if only a fraction of goods are sold at a controlled price, while the remainder are sold on the free market—because allowing consumers to pay a price premium for an uncontrolled unit is equivalent, from their point of view, to selling all the units at the controlled price but capping their rent-seeking costs. (The cap would be such that in equilibrium a consumer's total rent-seeking costs of obtaining a unit would be not more than the (equilibrium) difference between the controlled price and the free-market price.)

Likewise, our results generalize further to cases where, as in cities such as New York, price controls vary by unit, with some units at the minimum controlled price, some at higher but still constrained prices, and some at unconstrained prices. One can think of the units being sold off for varying packages of money and search effort, with each consumer acquiring whatever is cheapest for him given his cost of effort (see Section VI of Appendix B).

In all these cases, consumer surplus can still be calculated as in equation (4) (with the appropriate adjustment if resale is possible).

B. Maximising Consumer Surplus: Partially-Controlled Markets without *Rent-Seeking and* without *Resale*

A special case of the discussion above is a market in which a fraction of goods is sold at a controlled price *without any* rent-seeking costs, i.e., using a costless lottery, but with resale impossible, while the remainder is sold at an uncontrolled price on the free market.

From consumers' point of view, this corresponds exactly to a fully-controlled market in which all units will be sold at the same controlled price, but in which a randomly selected group of consumers will have no rent-seeking costs, while all the others will have equal, positive costs. Consumers who succeed in the lottery in the partially-controlled market correspond to those who will have no rent-seeking costs in the fully-controlled market. Consumers who fail in the lottery, but then buy the free-market units in the partially-controlled market, correspond to consumers who will have positive rent-seeking costs in the fully-controlled market; these consumers' total equilibrium costs of rent-seeking will equal the equilibrium price difference between uncontrolled and controlled units in the partially-controlled market.

More generally, we show in Section VII of Appendix B that a market in which different numbers of units are sold at different controlled prices, using costless lotteries and without resale, and perhaps also with some uncontrolled units, is equivalent to a fully-controlled market with a single control price, but in which different groups of consumers have different rent-seeking costs.[34]

Using this equivalence, it is not hard to show that if supply is inelastic, consumer surplus can be maximised by setting at most one controlled price:

PROPOSITION 5. With inelastic supply, and without rent-seeking, consumer surplus is maximised by one of (i) a pure market solution, (ii) a pure lottery, or (iii) allocating some units by lottery at a fixed price, and selling the remaining units on the free market.

Proof. See Section VII of Appendix B.

We give the recipe for finding the consumer-optimal price(s) in Section VII of Appendix B. We have already seen, for example, that with inelastic supply and log-concave demand (final-)consumer surplus is maximised by a pure lottery, but with log-convex demand it is maximised by the pure market solution. With elastic supply, the market allocation is (even) more often the consumer-optimum, of course, but the details are sensitive to how supply depends on the vector of control prices.

V Conclusion

Price controls lead to inefficient allocation and rent-seeking, in addition to reduced supply. Even absent any supply effect, inefficient allocation may cost consumers all the surplus gains they receive from a lower price and more. The results apply whether the good is allocated randomly through a lottery without rent-seeking costs, or whether greater search and other rent-seeking activities undertaken by higher-value consumers results in a more-efficient-than-random allocation. The results also apply when only some units are allocated at below-market prices, while other are sold on the free market.

In short, and especially if supply is fairly elastic, it is unlikely we can be confident that consumer surplus is enhanced by any price control.

[34]Note that rent-seeking and partial decontrol may have very different long-run supply effects. For example, reducing rents on existing housing, but credibly committing to never interfering with new housing, must help consumers if the supply of new housing is perfectly elastic, since they can always rent a new unit at the market price.

Appendix A

Example

We illustrate our results for the standard distributions of demand–including linear, log-linear, constant-elasticity, etc.–in the class of Generalized Pareto distributions (GPDs). For GPDs

$$D(p) = k\left(1 + \frac{\xi(p-\mu)}{\sigma}\right)^{-1/\xi}$$

($\xi = -1$ gives linear demand, $\xi \to 0$ gives log-linear demand, and $\xi = \sigma/\mu > 0$ gives constant elasticity demand with elasticity $-1/\xi$.[35]) We write η ($= \frac{pD'(p)}{D(p)} = \frac{-p}{\sigma+\xi(p-\mu)}$) for the elasticity of demand at p. We do not restrict the functional form of supply, but write φ for its elasticity at p.

Demands in the GPD class have the useful property that $MR(p)$ is affine in p, since $MR(p) = p + D(p)/D'(p) = \xi\mu - \sigma + (1-\xi)p$. In particular, therefore, $E\{MR(x)\} = MR(E\{x\})$, for any distribution of x.

Effect of a Price Control: From equation (4) total consumer surplus is $CS(p) = \int_{v=p}^{\infty} -D'(v)n(v,p)\left[v - MR(v)\right] dv$. Equivalently, writing $\overline{v}$ and $\overline{MR}(v)$ for the expected value and the expected MR, respectively, of consumers who get units, $CS(p) = S(p)\left[\overline{v} - \overline{MR}(v)\right]$ (since, of course, $S(p) = \int_{v=p}^{\infty} -D'(v)n(v,p)dv$, so the probability density that a consumer who gets a unit has value v is $(-D'(v)n(v,p)/S(p))$). Therefore, $CS(p) = S(p)\left[\overline{v} - MR(\overline{v})\right]$ (since $MR(v)$ is affine in v for GPDs).

But, writing $\overline{c}$ for the expected amount per-unit spent on rent-seeking (priced at the cost to the consumers who expend the effort), we can also write $CS(p) = S(p)\left[\overline{v} - (p + \overline{c})\right]$. So we have $p + \overline{c} = MR(\overline{v}) = \xi\mu - \sigma + (1-\xi)\overline{v}$, so also $\overline{v} = \frac{1}{1-\xi}\left[\sigma + p + \overline{c} - \xi\mu\right]$. Substituting this expression for $\overline{v}$ in $CS(p) = S(p)\left[\overline{v} - (p + \overline{c})\right]$ yields

$$CS(p) = \frac{S(p)}{1-\xi}\left[\sigma + \xi(p + \overline{c} - \mu)\right] \qquad (5)$$

[35] Our model requires $\xi < 1$ so that consumer surplus is finite.

For $\xi \neq 0$, we can rewrite the GPD as $p = \alpha - \beta(D(p))^{-\xi}$ in which $\alpha = \mu - (\sigma/\xi)$ and $\beta = -k^{\xi}(\sigma/\xi)$ (so $\xi = -1$ gives linear demand, and $\xi > 0$ gives (inverse) demand that is constant-elasticity plus a constant). As $\xi \to 0$ the GPD becomes $D(p) = ke^{(\mu-p)/\sigma}$ with $\sigma > 0$ (or $p = (\mu + \sigma \log k) - \sigma(\log D(p))$, that is, log-linear demand).

So the effect of a small *tightening* of a price control on aggregate consumer surplus is

$$-CS'(p) = \frac{-S'(p)}{1-\xi}\left[\sigma + \xi(p + \overline{c} - \mu)\right] - \frac{S(p)}{1-\xi}\xi(1 + \frac{d\overline{c}}{dp})$$

Noting $\sigma + \xi(p - \mu) = -p/\eta$ and $\varphi = pS'(p)/S(p)$, gives

$$-CS'(p) = \frac{S(p)}{1-\xi}\left[\frac{\varphi}{\eta}(1 - \frac{\eta\xi\overline{c}}{p}) - \xi(1 + \frac{d\overline{c}}{dp})\right] \quad (6)$$

Consumer Surplus Effects with No Rent-Seeking: With random allocation without rent-seeking $\overline{c} = \frac{d\overline{c}}{dp} = 0$, so consumers gain from a tighter price control if and only if $\xi\eta > \varphi$.

Consumer Surplus Effects with Rent-Seeking: With rent-seeking we know from Proposition 3 that consumers must lose from any tighter control if $\xi \geq 0$, and it is clear from (5) that consumers' total surplus is always lower with rent-seeking than without if $\xi < 0$. So the conditions for consumers to gain from *any* price cut from the market price are *always* tighter with rent-seeking than without, in this class of demands.

Consumer Surplus Effects of Partial Decontrol: Because $MR(v)$ is affine in v for GPDs, the mathematics of partial decontrol are the same: in this case, p is the *average* cash price paid for units, including both those controlled and de-controlled. So from (5), when supply is inelastic, the average "effective total price" to consumers, $p + \overline{c}(p)$, is a sufficient statistic for the effect of a price control on them, that is, the effect of any change in cost to purchasers is independent of whether it is due to a partial control, or a change in rent-seeking, or both.[36]

Appendix B

Omitted Proofs, etc.

I. More-Efficient-than-Random Rationing with No rent-seeking in the General Case

At the market-clearing price, a \$1 price cut increases consumer surplus by \$1 for each of the $\lambda S(p)$ efficiently-allocated units, so (3) becomes

[36] However, the amount of rent-seeking generally depends on the *distribution* of controlled prices, not just on the average price, p, and supply may do so too. So S, S', $\overline{c}'$, and hence $CS'(p)$, generally depend on how this distribution changes.

$$-CS'(p) = (1-\lambda)\{-D'(p)\tfrac{S(p)}{D(p)}[MCS(p)] + [D'(p)\tfrac{S(p)}{D(p)} - S'(p)][ACS(p)]\}$$
$$+\lambda\left\{S(p)\right\}.$$

Dividing the right hand side by $D(p) = S(p)$ (and reorganising, recalling $MCS(p) = -D(p)/D'(p)$), yields

$$\text{sign}\left[-CS'(p)\right] = \text{sign}\left[1 - (1-\lambda)\left(\left|\frac{\text{Elasticity of Supply}}{\text{Elasticity of Demand}}\right| + 1\right)\frac{ACS(p)}{MCS(p)}\right]$$

So if demand is convex, then since (as we noted in the main text) $\frac{ACS(p)}{MCS(p)} > \frac{1}{2}$ we have $-CS'(p) < 0$ if $\left|\frac{\text{Elasticity of Supply}}{\text{Elasticity of Demand}}\right| > \frac{1+\lambda}{1-\lambda}$, while for any log-convex demand $\frac{ACS(p)}{MCS(p)} > 1$ so $-CS'(p) < 0$ if $\left|\frac{\text{Elasticity of Supply}}{\text{Elasticity of Demand}}\right| > \frac{\lambda}{1-\lambda}$.

II. Non-random Rationing with No rent-seeking in the GPD Case

Continuing the GPD example of Appendix A, if fraction λ of the supply is allocated perfectly efficiently among fraction λ of the market, (6) together with the fact that a \$1 price cut increases consumer surplus by \$1 per efficiently-allocated unit at market-clearing, implies

$$-CS'(p) = S(p)\left[\frac{(1-\lambda)}{(1-\xi)}\left[\frac{\varphi}{\eta} - \xi\right] + \lambda\right] = \frac{S(p)}{1-\xi}\left[(\frac{\varphi}{\eta} - \xi) + \lambda(1 - \frac{\varphi}{\eta})\right]$$

So consumers gain from a price reduction iff $\lambda > \frac{\varphi - \xi\eta}{\varphi - \eta}$. For example, with constant-elasticity demand this requires $\lambda > \frac{\varphi+1}{\varphi-\eta}$.

If also fraction θ of supply is allocated as *inefficiently* as possible above the controlled price among fraction θ of the total market, then for this fraction, a \$1 price cut from the market-clearing price increases consumer surplus by \$1 per customer (\$$\theta S(p)$ in all), but removes $\theta(S'(p) - D'(p)) = \theta(\varphi - \eta)S(p)/p$ units from the highest-value consumers, costing $[(\mu\xi - \sigma)/\xi] - p = p/\eta\xi$ each if $\xi < 0$. (If $\xi \geq 0$, the highest-value consumer has $v = \infty$, so $-CS'(p) \to \infty$ for any $\theta > 0$.) So

$$-CS'(p) = S(p)\left[\frac{(1-(\lambda+\theta))}{(1-\xi)}\left[\frac{\varphi}{\eta} - \xi\right] + \lambda + \theta(1 + \frac{1}{\xi}(1 - \frac{\varphi}{\eta})\right] \quad \text{if } \xi < 0$$

which simplifies to

$$-CS'(p) = \frac{S(p)}{1-\xi}\left[(\frac{\varphi}{\eta} - \xi) + (\lambda + \frac{\theta}{\xi})(1 - \frac{\varphi}{\eta})\right] \quad \text{if} \;\; \xi < 0.$$

When $\theta = 1$, $-CS'(p) = \frac{S(p)}{\xi(1-\xi)}\left[1 + \xi - \frac{\varphi}{\eta}\right] > 0$ for all $\xi < -1$ if $\varphi = 0$. So, for example, with linear demand consumer surplus is enhanced by a price control however inefficiently supply is allocated, if there are neither supply effects nor rent-seeking costs.

III. Dynamic Model of Incumbents and Newcomers

Assume the price falls gradually from the market level, p^M, asymptoting to $p^M - \Delta$, so the controlled price at time t is $p(t) = (p^M - \Delta) + \Delta e^{-zt}$. Consumers leave at rate δ, and are replaced by new consumers with values drawn from the distribution corresponding to demand $D(\cdot)$, using random rationing (without rent-seeking costs) among all potential consumers who wish to purchase at that time. The continuous interest rate is r. Assume GPD demand.

The consumer surplus gain per time-0 incumbent equals the present value (to infinity) of an immediate rent reduction of Δ less the present value of the excess above $p^M - \Delta$ that is paid as prices gradually fall, that is, $\frac{\Delta}{r+\delta} - \frac{\Delta}{r+\delta+z} = \frac{z\Delta}{(r+\delta)(r+\delta+z)}$.

Relative to paying p^M, the present value of the price cuts, as of time t, to a newcomer who buys for the first time at time t, is $\frac{\Delta}{r+\delta} - \frac{\Delta e^{-zt}}{r+\delta+z}$. Furthermore, the present value of the stream of prices that will be paid by (any) time-t newcomer equals the present value of the *marginal* time-t newcomer's stream of consumption, and so also equals the present value of the *average* over all time-t newcomers of their stream of MRs. Also, for the GPD, a \$1 increase in the average of consumers' MRs implies a corresponding $\$1/(1-\xi)$ increase in the average of their values, and therefore a corresponding $\$1/(1-\xi) - 1 = \$\xi/(1-\xi)$ increase in their average surplus. It therefore follows that the average present value of the surplus, as of time t, to newcomers who buy for the first time at time t, is $\frac{-\xi}{1-\xi}\left(\frac{\Delta}{r+\delta} - \frac{\Delta e^{-zt}}{r+\delta+z}\right)$. So the present value of surplus gained by all newcomers is $\int_0^\infty \delta\left[\frac{-\xi}{1-\xi}\left(\frac{\Delta}{r+\delta} - \frac{\Delta e^{-zt}}{r+\delta+z}\right)\right] e^{-rt}dt = \frac{-\xi}{1-\xi}\left(\frac{\delta z\Delta}{(r+\delta)(r+z)}\right)\left(\frac{2r+\delta+z}{r(r+\delta+z)}\right)$.

The ratio of surplus gained by future consumers to that gained by incumbents is therefore $\frac{-\xi}{1-\xi}\frac{\delta}{r}\left(\frac{2r+\delta+z}{r+z}\right):1$.

For calibration, the 2007 American Housing Survey (e.g., Table 4-12) estimates that 12.4 million out of 35.0 million renters moved in the previous year, which would correspond to a continuous hazard rate of $\delta = -ln(\frac{35-12.4}{35}) = .43$. So, for example, with demand of constant elasticity, η, $r = .02$ (real interest rate of 2%) and $z = .2$ (so half the ultimate price reduction takes place in $(-ln(1/2)/.2) \approx 3.5$ years), then the ratio of newcomers' surplus loss to incumbents' gain $\approx 65 : -(\eta+1)$.[37]

IV. Direct Derivation of (4)

In equilibrium, each consumer chooses its effort, and so purchase quantity, optimally, so the set of consumers with values $v + dv$ obtains $n(v,p)dv$ more surplus per consumer than the otherwise-identical set with value v. (The total surplus of the additional $dn(-D'(v+dv))$ consumers with value $v+dv$ who would not have purchased units if their value were just v is second order.) So, letting $cs(v)$ be the expected per-consumer surplus of consumers with value v, $dcs/dv = \partial cs/\partial v = n(v,p)$. Also $cs(p) = 0$, so $cs(v) = \int_p^v n(x,p)dx$.

Integrating across all consumers, total consumer surplus at price, p, is

$$CS(p) = \int_p^\infty -D'(v)cs(v)dv = \int_p^\infty -D'(v)\int_p^v n(x,p)dxdv$$

so, integrating by parts and observing $\left[D(v)\int_p^v n(x,p)dx\right]_p^\infty=0$,[38] we have

$$CS(p) = \int_p^\infty D(v)n(v,p)dv \qquad \text{(cf. equation(1))}$$

[37] The calculation is purely illustrative! Issues include: using declining hazard rates with the same average tenure would reduce the ratio. We have also not accounted for any surplus that current incumbents may expect to receive in future roles as newcomers. And if it is difficult to re-enter the market to obtain a new apartment, turnover rates will be lower than in an uncontrolled market. On the other hand, these lower turnover rates are caused by tenants whose values are at least below the market price, and may be below the controlled price if they are uncertain about their future values. Furthermore, if expected turnover rates differ, consumers with longer expected residence will jump in to the market sooner, reducing the efficiency of rationing among newcomers, and further reducing their surplus.

[38] $\left[D(v)\int_p^v n(x,p)dx\right]_p^y=D(y)cs(y) < D(y)y$ and $lim_{y\to\infty}D(y)y = 0$ by our earlier assumption that the elasticity of demand is bounded strictly above 1 at all sufficiently high prices.

so, noting $MR(v) \equiv v + \frac{D(v)}{D'(v)}$, we can write $D(v) = -D'(v)(v - MR(v))$ to obtain equation (4).

V. Proof of Proposition 4

Since $MCS(v) = -D(v)/D'(v)$, we have $(\frac{-D(v)}{MCS(v)})' = D''(v) \geq 0$ if demand is convex, so $\frac{MCS(p)}{MCS(p^M)} \geq (\leq) \frac{D(p)}{D(p^M)}$ if $p > (<) p^M$. So the fact that the price control removes some consumers with higher values than the market-clearing price, p^M, has a more negative impact on consumer surplus than would removing the same consumers from the linear demand that is tangent to our demand at p^M, and the fact that it adds some consumers with lower values than p^M has a less positive impact. (By "the same consumers", we mean those whose rank-order in the distribution of values is the same, i.e, those for whom $D(v)$ is the same.)

Furthermore, rent-seeking costs that are uncorrelated with values lead to less substitution of high-v by low-v consumers than in a random allocation. So, in the linear demand case, the specified substitutions would have a less beneficial impact on consumer surplus, than if the consumers to be added and removed were selected randomly from those above the controlled price (since for linear demand $MCS(v)$ is decreasing in v). But it is easy to check that the random selection/linear demand case hurts consumers if $D(p)/D(p^M) \leq S(p^M)/S(p)$. **QED**

VI. Partially-Controlled Markets

Let q_i units be rationed at price p_i, with $p_N < p_{N-1} < ... < p_1$. Let the equilibrium uncontrolled market price be p_0, and the equilibrium effort required to obtain a unit at price p_i be e_i. (For simplicity, we ignore resale.) Clearly $e_N > e_{N-1} > ... > e_1 > e_0 = 0$, and consumers sort themselves so that those with costs of effort $c \in (c_{i+1}, c_i)$ buy a unit at price p_i, where $c_i = (p_{i-1} - p_i)/(e_i - e_{i-1})$ (defining $c_{N+1} = 0$), iff their value also exceeds $p_i + e_i c$; those with costs of effort above c_1 buy an uncontrolled unit iff their value exceeds p_0.

To see that there is generally a unique equilibrium for any given $\{p_1, .., p_N, q_1, .., q_N\}$, observe that the values of all of the c_i and e_i can be determined sequentially from c_N, that a lower c_N implies that all of the c_i and e_i are lower, and so also $p_0 (= p_1 + e_1 c_1)$ is lower, and so (since p_0 and c_1 are both lower) increases demand for uncontrolled units but must (weakly) reduce their supply; so there is generally a unique c_N for which demand equals supply and which is therefore consistent with equilibrium.

VII. Maximising Consumer Surplus: Partial Control without *Rent-Seeking*

Assume l_i units are allocated at price p_i, with $p_i < p_{i-1}$, in the ith of N lotteries that are run in sequence, starting with the Nth, that is the *lowest*-price one. (If desired, the final (highest) price, p_1, can be set at what would be the equilibrium uncontrolled price for the l_1 units in the final "lottery", so that all so-far-unsuccessful consumers who have values above p_1 then receive units. We can perform similar analyses for differently-ordered lotteries.)

Call the $q_i = D(p_i) - D(p_{i-1})$ consumers whose value is between p_i and p_{i-1} "group" i (defining $p_0 = \infty$), and write $Q_i = \sum_{j=1}^{i} q_j = D(p_i)$. Clearly, each "group" i consumer will enter each lottery in turn until it either wins the right to buy a unit or loses in the ith lottery, and will obtain a unit with probability $\pi_i = 1 - \prod_{j=i}^{N}(1 - \gamma_j)$, in which γ_i is the probability with which a participant in the ith lottery is a winner. (The values of all the γ_i can easily be determined recursively, starting with γ_N, using the fact that $l_i = \gamma_i Q_i \prod_{j=i+1}^{N}(1 - \gamma_j)$.) The situation is equivalent, from consumers' viewpoint, to our rent-seeking model with a single control price of p_N, in which a (random) fraction l_i/Q_i of consumers have total rent-seeking cost, in equilibrium, of $p_i - p_N$ (since l_i/Q_i is the fraction of those with values exceeding p_i who buy at p_i).

Proof of Proposition 5: Let $\overline{MCS}_i = \frac{1}{q_i}\left[\int_{Q_{i-1}}^{Q_i} MCS(D^{-1}(x))dx\right]$ (= the average MCS of group i's consumers). So total consumer surplus = $\sum_{j=1}^{N} \pi_j q_j \overline{MCS}_j$, and since supply, S, is fixed, and $\sum_{j=1}^{N} \pi_j q_j = S$, maximising total consumer surplus requires $\overline{MCS}_i \geq \overline{MCS}_{i+1}, \forall i$. (If not, total consumer surplus could be increased by reducing π_i and increasing π_{i+1}.) So $\forall i$, consumer surplus can be weakly increased by reallocating probability from groups $i+1, i+2, ...$ to groups $i, i-1, ...$, until either $\pi_{i+1} = 0$, or $\pi_i = ... = \pi_1 = 1$. So total consumer surplus can always be maximized by one of (i) the market solution, in which all consumers whose values exceed the market price buy with probability $\pi_1 = 1$, or (ii) a pure lottery, in which all consumers whose values exceed some cutoff-level buy with some probability $\pi_1 < 1$, or (iii) a combination, in which some consumers buy with probability $\pi_1 = 1$, while some others buy with some probability $\pi_2 < 1$. **QED**

Case (iii) can be implemented by first randomly allocating $\pi_2(q_1 + q_2)$ units at the "controlled" price $p_2 = D^{-1}(q_1 + q_2)$, and then selling $(1 - \pi_2)q_1$ "free-market" units to the highest bidders at the equilibrium price for them, $p_1 = D^{-1}(q_1)$.[39]

[39] Case (iii) can alternatively be implemented by selling the "free-market" units first.

The consumer-surplus-maximising allocation of supply S follows straightforwardly. Write $TS(q)$ for total consumer surplus when quantity q is sold at price $D^{-1}(q)$, and write $\widetilde{TS}(q)$ for the minimum weakly-concave function satisfying $\widetilde{TS}(q) \geq TS(q)$. If $TS(S) = \widetilde{TS}(S)$, the market outcome maximises consumer surplus. If $TS(S) < \widetilde{TS}(S)$ let $Q_1 = \sup_{q<S}\{q \text{ s.t. } TS(q) = \widetilde{TS}(q)\}$, and $Q_2 = \inf_{q>S}\{q \text{ s.t. } TS(q) = \widetilde{TS}(q)\}$, then run a lottery for $(S - Q_1)(1 + \frac{Q_1}{Q_2 - Q_1})$ units at price $D^{-1}(Q_2)$, and finally sell the remaining supply to the highest bidders at $D^{-1}(Q_1)$.[40] (The case $Q_1 = 0$ corresponds to the pure lottery.)

To achieve equivalence, the number of free-market units sold at market clearing would be increased by the number of consumers with values above p_1 who would win the lottery at p_2 in the implementation of the text. The market-clearing price would fall by $\pi_2(p_1 - p_2)$ so that a consumer with value p_1 obtains the same expected surplus from buying for sure at the market-clearing price as from waiting for the lottery and buying with probability π_2.

[40] The identical outcome can alternatively be implemented by *first* selling Q_1 units to the highest bidders, and then running a lottery for $S - Q_1$ units at price $D^{-1}(Q_2)$. See note 39.

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