Economics Letters 27 (1988) 111–115
North-Holland

CONSISTENT CONJECTURES EQUILIBRIA
A Reformulation Showing Non-uniqueness *

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Received 20 November 1987
Accepted 17 February 1988

We interpret both Bresnahan's original (1981) definition of consistent conjectures equilibria and his generalized (1983) definition as dominant strategy equilibria in reaction functions. We employ a simple constructive argument to show that *every* outcome satisfies the generalized definition.

1. Introduction

One of the difficulties with the 'consistent conjectures' approach to resolving the multiplicity of oligopoly equilibria was highlighted by Robson (1983): he showed that Bresnahan's (1981) consistent conjectures equilibrium (CCE) for a duopoly may not exist. Bresnahan (1983) responded by weakening the definition of the CCE to a 'generalized' CCE (GCCE), thus resolving the non-existence problem. In fact, Laitner (1980) had defined a rational conjectures equilibrium (RCE) that was essentially equivalent to Bresnahan's GCCE and had already shown that a multiplicity of RCEs exist, but his formulation was somewhat different and he provided no intuition for his multiplicity result and its (quite complex) proof.

This note interprets the CCE and the GCCE as dominant strategy equilibria in reaction functions, where the reaction functions are restricted to specific classes. This interpretation allows us to use a simple, constructive graphical argument to show that *every* outcome is a GCCE. Our construction also suggests that the problem of the multiplicity of oligopoly equilibria may be better tackled by the alternative approach, proposed by Robson (1981) and Turnbull (1983), of considering firms' responses to exogenous uncertainty.

2. Re-interpretation of the CCE

Consider a differentiated products duopoly with demand system $p_1 = f^1(q_1, q_2)$, $p_2 = f^2(q_2, q_1)$, and firms' cost curves $C^1(q_1)$ and $C^2(q_2)$. Bresnahan's (1983) reformulation of the CCE views the firms as competing using strategic variables α_1 and α_2, which determine quantities according to

$$q_1 = g^1(\alpha_1, \alpha_2) \quad \text{and} \quad q_2 = g^2(\alpha_2, \alpha_1). \tag{1}$$

* The authors are grateful to Tim Bresnahan for many helpful discussions.

0165-1765/88/$3.50

Hence i's profit, $\Pi^i(\alpha_i, \alpha_j) = g^i(\alpha_i, \alpha_j) f^i(g^i(\alpha_i, \alpha_j), g^j(\alpha_j, \alpha_i)) - C^i(g^i(\alpha_i, \alpha_j))$. A weakened, or generalized, CCE – henceforth GCCE – is defined as a pair of functions g^1 and g^2 such that in the Nash equilibrium in the corresponding strategic variables α_1 and α_2, the firms' Nash conjectures are correct. That is, if one firm were to deviate from its equilibrium level of α, the other's optimal choice would not be affected. The equilibrium outputs are determined from the equilibrium αs according to (1).

To re-interpret the GCCE, simply rewrite the equations in (1) as

$$q_1 = R^1(q_2, \alpha_1) \quad \text{and} \quad q_2 = R^2(q_1, \alpha_2). \tag{2}$$

(This transformation requires that g^i be strictly monotonic in α_i. Though this restriction is not imposed by Bresnahan, we show later that our multiplicity result is valid for GCCEs which satisfy this restriction and therefore for GCCE's of the more general form as well.) The expressions in (2) define, for each firm i, a class of allowable reaction function strategies, R^i, parameterized by α_i. Each firm's choice of α determines which member of its set of reaction fuctions it employs. A GCCE can thus be viewed as a pair of classes or reaction functions, R^1 and R^2, such that in the Nash equilibrium in reaction functions from these classes, the firms' Nash conjectures are correct. That is, each firms' equilibrium choice of reaction function is optimal against every member of its rival's class, so the Nash equilibrium is a dominant strategy equilibrium. The intersection of the equilibrium reaction functions determines the equilibrium outputs.

This consistency requirement could be motivated by regarding each firm as uncertain about which value of α or, equivalently, which specific reaction function, its rival will employ. Ideally, a firm would like to select a reaction fuction that traced through (q_1, q_2) pairs to achieve its profit-maximizing point on each of its rival's reaction functions; in a GCCE each firm can do this. Thus each firm is correct in conjecturing that even if it were to deviate within its class, its rival's optimal strategy would not change.

A CCE according to Bresnahan's original (1981), stronger definition is just a GCCE in which

$$R^1(q_2, \alpha_1) = \psi^1(q_2) + \alpha_1 \quad \text{and} \quad R^2(q_1, \alpha_2) = \psi^2(q_1) + \alpha_2. \tag{3}$$

In other words, in a CCE each firm's uncertainty relates only to the level and not to the slope of its rival's reaction function. [1]

3. Non-uniqueness of the generalized CCE

Consider an arbitrary output pair $(\bar{q}_1, \bar{q}_2)$. Fig. 1 shows curves $q_1 = \psi^1(q_2)$ and $q_2 = \psi^2(q_1)$ that are tangent to firm 2's and firm 1's isoprofit curves respectively at $(\bar{q}_1, \bar{q}_2)$. At all other points on $\psi^1(q_2)$, 2's profit is lower than at $(\bar{q}_1, \bar{q}_2)$, and conversely. By construction, $(\bar{q}_1, \bar{q}_2)$ is an equilibrium in a quantity-setting game in which $q_2 = \psi^2(q_1)$ represents 1's conjecture about 2's reaction function, and conversely.

Now suppose firm 1 believes that 2 might deviate to any of the set of vertically-translated reaction functions $q_2 = \psi^2(q_1) + \alpha_2$. For each value of α_2, the new profit-maximizing point for firm 1 is at a tangency of an isoprofit curve with the reaction function corresponding to α_2. Let $q_1 = \phi^1(q_2)$ be the equation for the curve connecting 1's profit-maximizing points as α_2 varies. In general, $\psi^1(q_2)$ and $\phi^1(q_2)$ do not coincide: firm 2's conjecture about the way 1 would respond to a change in 2's

[1] Bresnahan's (1981) definition is, in fact, expressed as a local condition. This can be captured by considering only deviations to αs in ϵ-neighborhoods of the Nash equilibrium. Bresnahan is ambiguous about whether his (1983) condition is intended as a global or as a local condition. We interpret it as a global restriction and will exhibit a continuum of equilibria – a fortiori there are a continuum of equilibria if the restriction is local.

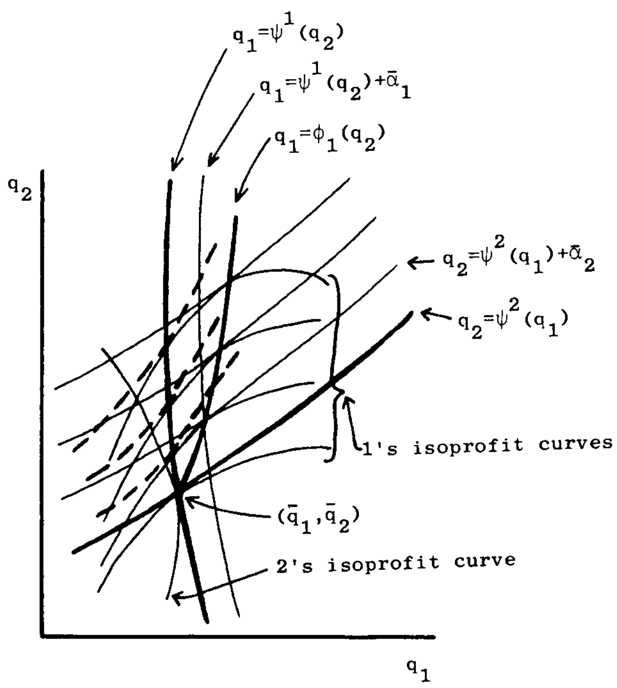

Fig. 1. Construction supporting an arbitrary $(\bar{q}_1, \bar{q}_2)$ as an equilibrium.

behavior does not match the way 1 would in fact wish to respond. Although the functions $\psi^1(q_2)$ and $\psi^2(q_1)$ form a Nash equilibrium when the firms' strategy sets are the classes of reaction functions $q_1 = \psi^1(q_2) + \alpha_1$ and $q_2 = \psi^2(q_1) + \alpha_2$, they do not form a dominant strategy equilibrium. (Against 2's strategy $q_2 = \psi^2(q_1) + \bar{\alpha}_2$, for example, 1's optimal response is $q_1 = \psi^1(q_2) + \bar{\alpha}_1$ – see fig. 1.) Hence the functions ψ^1 and ψ^2 do not form a CCE. [2]

It is simple, however, to construct classes of reaction functions R^1 and R^2 which support the arbitrary point $(\bar{q}_1, \bar{q}_2)$ as a GCCE outcome. [3] At every point along $\psi^1(q_2)$, draw a curve tangent to 1's isoprofit curve passing through that point. These tangents (the dashed curves in fig. 1) form a set of reaction functions for 2 which includes $\psi^2(q_1)$ and can be parameterized by α_2: $q_2 = R^2(q_1, \alpha_2)$. An analogous construction along $\psi^2(q_1)$ gives a set of reaction functions for 1, $q_1 = R^1(q_2, \alpha_1)$, parameterized by α_1. By construction, the function $q_1 = \psi^1(q_2)$ passes through all of the points that are profit-maximizing for 1 against $q_2 = R^2(q_1, \alpha_2)$ as α_2 varies, and conversely. Thus, for these strategy sets R^1 and R^2, $\psi^1(q_2)$ and $\psi^2(q_1)$ form both a Nash equilibrium and a dominant strategy equilibrium, and so support $(\bar{q}_1, \bar{q}_2)$ as a GCCE outcome. Note that we can make R^1 and R^2 continuous and differentiable in q and α. [4] Also, since we can ensure that the elements of each firm's

[2] This does not show that $(\bar{q}_1, \bar{q}_2)$ cannot be supported as a CCE outcome, merely that the pair of reaction functions ψ^1 and ψ^2 cannot form part of a CCE.

[3] If firms have the option of producing no output, then only outcomes in which both firms make non-negative profits can be supported by GCCEs.

[4] The outcome can be supported as a GCCE by merely having, in the case of substitutes, $g^i(0, \alpha_j) = \bar{q}_i$ for all α_j and, if $\alpha_i \neq 0$, $g^i(\alpha_i, \alpha_j)$ greater than $\bar{q}_i$ and large enough to ensure that i's price is below its minimum marginal cost, for all α_j (in Bresnahan's terminology). Alternatively, in our reinterpretation, we would have $R^i(q_j, 0) = \bar{q}_i$ for all q_j and, if $\alpha_i \neq 0$, $R^i(q_j, \alpha_i)$ greater than $\bar{q}_i$ and sufficiently large to yield negative profits to i, for all q_j. [In these (and other) degenerate cases, a firm does not achieve its profit-maximizing points on its rival's reaction functions.] However, our constructive argument demonstrates the much stronger result that there are functions R^1, R^2 (and g^1, g^2) that are *continuous* and *differentiable* in the firm's own value of α and that support any output pair as a GCCE outcome. Thus our argument also applies to Laitner's formulation – see below.

strategy set never intersect, we can make each R^i stricly monotonic in α_i simply by choosing the values of these parameters appropriately. [5]

In summary, weakening the conditions on strategy sets from (3) to (2) allows *any* outcome to be supported by a GCCE. [6] In Bresnahan's terminology, allowing complete freedom in the choice of firms' strategic variables, represented by any (invertible) functions g^1 and g^2 in (1), makes every output pair a GCCE outcome. [7]

4. Comment

The GCCE's consistency requirement can be motivated by implicit uncertainty on the part of each firm about its rival's strategy – this uncertainty makes a firm want to choose a reaction function as a strategy, rather than choose the single point which would be optimal in the absence of uncertainty. The definition of a GCCE, though, places no restrictions on the manner in which the implicit uncertainty affects the firms. Consequently, as we have shown, it is possible to invent forms of uncertainty, and corresponding strategy sets, to support *any* outcome in equilibrium.

The GCCE can be viewed as a game-theoretic formulation of Laitner's (1980) RCE. In Laitner's model of a homogeneous good duopoly, each firm chooses a quantity on the basis of conjectures about its rival's response to a deviation from any starting point. Given its conjectured response functions (one through each point), each firm i can determine a set M^i of points which it believes are profit-maximizing. A RCE is an output pair that lies in both M^1 and M^2 and a set of conjectured response functions for each firm, such that for each i, M^i coincides with one of j's conjectured response functions. The conjectured response functions play the same role in a firm's decision-making as the other firm's reaction function strategies play in our formulation; that is, the sets M^1 and M^2 contain the points traced through by the functions $\psi^1(q_2)$ and $\psi^2(q_1)$.

Our constructive argument demonstrates more simply than Laitner's complex proof that an infinite number of points are RCE's, and our approach is valid for differentiated products as well. Our construction exploits the fact that ψ^i must be a dominant strategy for i but that the other elements of i's strategy set are arbitrary: they can be chosen to make ψ^j a dominant strategy for j. Similarly, in Laitner's model, firm j must base its behavior on a correct conjecture about M^i, but no rationality requirement is placed on j's other conjectured response functions: they can therefore be constructed to make i's conjecture about M^j correct.

Contrasting with these multiplicity results is the example of Robson (1983), which showed the possibility of non-existence of a CCE, that is, of non-existence of a GCCE in which the strategy sets R^1 and R^2 are restricted to be additively separable, as in (3). Taken together, these results suggest that the consistent conjectures approach of seeking classes of reaction functions within which dominant strategy equilibria exist, will be unable to achieve both uniqueness and existence of an oligopoly equilibrium.

[5] Strict monotonicity of each R^i in α_i, coupled with uniqueness of the intersection of R^1 and R^2 for all (α_1, α_2), implies that the corresponding g^i function is strictly monotonic in α_i.

[6] Note that we can support $(\bar{q}_1, \bar{q}_2)$ as a GCCE outcome by starting with *any* pair of reaction functions $\psi^i(q_j)$ such that $\psi^i(\cdot)$ is tangent to j's isoprofit curve at $(\bar{q}_1, \bar{q}_2)$ and gives j lower profit at every other point, and conversely – the $\psi^i(q_j)$ need not be linear in q_j, though they can be if firms' isoprofit curves are concave. Thus any outcome can be supported by and *infinite* number of GCCEs

[7] We doubt that any simple application of perfection ideas would select a unique equilibrium: we conjecture that if 1's 'trembles' are sufficiently concentrated on the set $R^1(q_2, \alpha_1)$, then $\psi^2(q_1)$ in 2's strict best response and conversely, so that any outcome is a trembling-hand perfect Nash equilibrium [in the spirit of Selten's (1975) definition] when each firm's strategy set includes all possible reaction functions.

An alternative, and perhaps more natural, way to predict what strategies firms will use is to consider explicitly firms' optimal responses to exogenous uncertainty. With exogenous uncertainty about, say, market demand, a firm again has a set of profit-maximizing points, rather than a single optimal point. Rather than plotting this set in (q_1, q_2) space to derive an optimal reaction function strategy, suppose we plot it in (p_i, q_i) space to determine i's optimal 'supply schedule' strategy. Given the uncertainty about demand and given a choice of supply schedule by its rival, each firm has a best response supply schedule, and we can focus on Nash equilibria in supply schedules. [8] Because the manner in which uncertainty affects the firms is now not arbitrary but rather is determined by the economic environment, this framework does not allow the type of construction we used to generate the multiplicity of GCCE's. Robson (1981) and Turnbull (1983) have shown that for linear demand and marginal costs, with an exogenous, unobservable, additive shock to demand, there exists a unique Nash equilibrium when strategy sets are the sets of all linear supply functions. [9,10] In a more general model which allows firms to choose *any* supply schedule, the incorporation of exogenous uncertainty can again lead to a unique equilibrium [Klemperer and Meyer (1987)]. This approach, therefore, may hold more promise for predicting which strategic variables firms will employ, and hence how intense competition will be, in different oligopolistic environments.

References

Bresnahan, Timothy F., 1981, Duopoly models with consistent conjectures, American Economic Review 71, 934–945.

Bresnahan, Timothy F., 1983, Existence of consistent conjectures: Reply, American Economic Review 73, 457–458.

Klemperer Paul D. and Margaret A. Meyer, 1986, Price Competition vs. quantity competition: The role of uncertainty, Rand Journal of Economics 17, 618–638.

Klemperer, Paul D. and Margaret A. Meyer, 1987, Supply function equilibria in oligopoly under uncertainty, Mimeo. (Oxford University, Oxford).

Laitner, John, 1980, 'Rational' duopoly equilibria, Quarterly Journal of Economics 95, 641–662.

Robson, Arthur J., 1981, Implicit oligopolistic collusion is destroyed by uncertainty, Economics Letters 7, 144–148.

Robson, Arthur J., 1983, Existence of consistent conjectures: Comment, American Economic Review 73, 454–456.

Selten, Reinhard, 1975, Reexamination of the perfectness concept for equilibrium points in extensive games, International Journal of Game Theory 4, 25–55.

Turnbull, Stephen J., 1983, Choosing duopoly solutions by consistent conjectures and by uncertainty, Economics Letters 13 253–258.

8 We argue in Klemperer and Meyer (1987) that in the presence of exogenous uncertainty, a supply schedule is a more natural notion of strategic variable than a reaction function, because a firm can implement a supply schedule with reference only to its own market, without direct knowledge of its opponent's behavior, and because organizational form and management decision rules provide a natural way of committing to a supply schedule. A vertical supply schedule corresponds to choosing quantity as strategic variable, and a horizontal one corresponds to choosing price.

9 In the linear/additive case, the equilibrium outputs are the same as in Bresnahan's original CCE, because the uncertainty (explicit or implicit) affects the firms in exactly the same manner in both equilibria. This equivalence does not, however, hold generally, even when the CCE exists.

10 In the same spirit, Klemperer and Meyer (1986) show, under more general cost and demand conditions, that when firms can choose to set either quantities of prices, the incorporation of exogenous demand uncertainty can lead to unique equilibrium choices of strategic variables.