

Rational Frenzies and Crashes

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Most markets clear through a sequence of sales rather than through a Walrasian auctioneer. Because buyers can decide whether to buy now or later, rather than only now or never, their current "willingness to pay" is much more sensitive to price than the demand curve is. A consequence is that markets will be extremely sensitive to new information, leading to both "frenzies," in which demand feeds on itself, and "crashes," in which price drops discontinuously. The paper also shows how a result from static auction theory, the revenue equivalence theorem, can be applied to solve for a dynamic price path.

I. Introduction

Asset markets are volatile, with both price and volume subject to wide fluctuations. The boom and bust in London, Texas, and Tokyo real estate, the silver bubble, the merger mania of the mid-1980s, the collapse of the junk bond market, the worldwide stock market crash of 1987, and the effective demise of Europe's exchange rate mechanism provide some recent examples. Common wisdom suggests that many of these events were caused by irrational behavior or other

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market imperfections. We suggest an alternative explanation. Rushes to trade and large changes in price may arise as a direct result of rational and strategic behavior in efficient markets.

Why is volatility often assumed to be a sign of a poorly performing market? Probably the reason is that our intuition is based on models in which the numbers of buyers and sellers change continuously at an exogenously given rate. For example, in a conventional partial-equilibrium supply and demand analysis with a Walrasian auctioneer, a small number of newcomers to the market have only a small effect, except in pathological cases. Similarly, in models in which customers arrive sequentially to trade with a specialist market maker, prices change from trade to trade only by the amount reflecting the private information held by one customer.

However, in the real world, buyers and sellers can choose *when* to trade: Our main point is that when agents make this choice strategically, a small event can trigger a very large volume of trade. A large volume of trade then reveals a large amount of information and can result in a large price change. Furthermore, in contrast to the recent models of "herding" (see, e.g., Banerjee 1992; Bikhchandani, Hirshleifer, and Welch 1992), our results occur even when all agents have independent values of the traded good.

We analyze a simple dynamic model of market clearing with a seller who has a fixed supply and a group of buyers who each have an independent value for a single unit. The seller calls out a price, and buyers then simultaneously announce whether they are willing to pay that price. If demand is positive but less than or equal to supply, the seller satisfies the demand and reoffers any remaining units at the same price. If demand is zero, the seller reduces price continuously until a buyer is found. If demand exceeds supply, then no transactions take place and the seller asks a higher price.

We show that this model yields "frenzies," in which a single purchase at a given price causes many other customers to come forward to offer the same price. A second prominent consequence is "crashes," in which it becomes common knowledge immediately after a frenzy that no further buyers will be willing to pay anything close to the price at which the frenzy took place. A further result is that the price path is highly sensitive to small changes in the underlying demand structure: transaction prices are neither continuous nor monotonic in buyers' reservation values.

The key to our results is the difference between a strategic buyer's valuation of the goods and his *willingness to pay* (WTP) at a given moment. His WTP is the amount he would be willing to pay to purchase the good at the current stage of the market game, and it will generally be less than his valuation. In plain terms, one may have a

value for a personal computer of \$15,000 but be unwilling to pay more than \$3,000 when entering a store, simply because one thinks that one should be able to find a personal computer for around that price.

A graph of buyers' WTPs at any stage of a game will be below the demand curve and will be much flatter at the top. For example, consider a model of supply and demand with no uncertainty. There are buyers who have reservation values ranging up to \$15,000, but the price at which supply equals demand is \$3,000. Then if a market maker calls out a price of \$4,000, no one will be willing to buy. The reason is obvious: everyone knows that the price will have to fall eventually, so there is no reason to buy immediately. In this case, the WTP of all buyers with valuations above \$3,000 would be \$3,000 and the WTPs of all buyers with lower valuations would be their private valuations. When we consider a more realistic model with uncertainty, the basic outline of the WTP curve remains the same: it is extremely flat for the highest-value buyers and then curves down and approaches the valuations of the lower-valuation buyers.

Now assume that a seller is asking a price that makes a consumer with the highest possible valuation indifferent to buying. Then because the WTP curve is almost flat for the highest-valuation buyers, any event that even slightly increases buyers' WTP can change a large number of customers from being unwilling to buy to willing to buy. For example, the purchase of a single unit slightly increases expectations of future trading prices. This slightly raises the WTP curve and triggers a "frenzy" at the current price. Furthermore, because buyers with valuations over a large range will therefore bid simultaneously, information will arrive in the market in large "chunks," revealing a great deal about overall demand. This leads to possibly large revisions in WTP and hence either crashes in price or further frenzies, depending on whether the news is good or bad for buyers.

The price paths of our model are consistent with a story of buyers holding off from entering a falling market until, when a little trading does take place, there is either a rush of "panic buying" by traders who are fearful of missing out on an upward move or a further fall if insufficient support develops at the current price.

Our model may also represent the sale of new securities by U.S. underwriters. An initial price is maintained or supported until either an issue sells out or it becomes apparent that there will be insufficient demand, in which case there may be a large fall in price.¹ Some

¹ For example, in the then-largest corporate bond issue in history, Salomon Brothers in 1979 was lead underwriter for, and sought to market, \$1 billion of International Business Machines securities. After several days, approximately \$300 million of bonds

commodity and currency markets have exhibited similar characteristics. For example, in the mid-1980s, tin producers attempted to maintain minimum prices in the face of falling demand. When it became apparent that the minimum was no longer viable, price had to be cut sharply.

Our main point is that flat WTP curves lead to frenzies and crashes in the trading of existing assets, but applying the same formal model to firms considering new investments can yield macroeconomic implications: A small change in the flat willingness-to-invest schedule may lead to a frenzy of investment activity that can then be followed either by several more "boom" periods of continued investment or by a "bust," that is, a period in which it is common knowledge that no investment will occur.²

Sections II–IV present our basic model and use the revenue equivalence theorem from auction theory to completely characterize equilibrium price paths. Section V highlights and illustrates the most important results. The remainder of the paper generalizes the model, showing that flat WTP curves and frenzies and crashes occur in a very wide range of dynamic models of market clearing if buyers have the freedom to choose when to trade.

II. The Model

A seller has K identical units of a good for sale, and $K + L$ risk-neutral potential buyers each wish to purchase a single unit. Each buyer can purchase at any moment by paying the current asking price. For simplicity, we assume that the buyers' valuations are independently and identically distributed, drawn from a distribution $F(v)$ that is strictly increasing and atomless on $[\underline{V}, \bar{V}]$, with density function $f(v)$.³ The seller and other buyers do not observe a buyer's v but know that it is drawn from $F(v)$. A buyer with value v obtains surplus

remained unsold. Finally, the asking price was reduced by roughly \$45 per \$1,000 bond—a huge drop for virtually default-free securities.

²Our basic model in Sec. II is formally equivalent to the following: A market will open at a fixed future date (e.g., a drug when its patent expires, or the new market opened by a free-trade agreement) and is large enough that K firms can compete profitably but $K + 1$ will be unprofitable. Sinking an investment at an earlier date, prior to the market opening, corresponds to paying a higher (present-value) price to participate in the market. The analysis thus explains that even if all the potential entrants have very different expected values of participating in the market, we should expect any first investment to be immediately followed by a frenzy of further investment activity, then, quite often, by a period in which it is common knowledge that no investment will occur, then by another frenzy of activity, and so on.

³All these assumptions can be relaxed. Atoms or "gaps" in $F(\cdot)$ make frenzies and crashes more likely. Nonindependent valuations increase the size of frenzies and are considered in Sec. VII.

$v - p$ if he buys a unit when the current asking price is p , regardless of the time at which he buys.

For concreteness, we focus mainly on a specific price-setting rule, where the seller begins with a very high price and then lowers price continuously until all units are sold, or supply exceeds demand: Specifically, the seller begins by asking a price of \bar{V} and then lowers the price continuously until a purchase occurs.⁴ When a purchase occurs, the seller then asks all remaining buyers (simultaneously and independently) if any of them has changed his mind and now wants to buy at the same price. If the number of customers who now wish to purchase does not exceed the remaining supply, then sales are made to these buyers also. As long as not all the goods are sold but purchases are occurring, the seller asks after each successive round of purchases whether anyone else is interested in buying. This goes on until (a) all the goods have been sold; (b) not all the goods are sold, but there is no one left who wants to pay the current price; or (c) more buyers simultaneously offer to buy than goods remain. If result *a* occurs, then the game ends. If result *b* occurs, the seller lowers the price continuously again until a purchase occurs.⁵ At this point the seller again asks all remaining buyers if they now want to buy at this purchaser's price, and so on, exactly as above. If result *c* occurs, with $k + l$ bidders simultaneously offering to buy k remaining units, then the auction begins over again at the initial (maximum) price \bar{V} , with the $k + l$ "tied" bidders competing for the remaining k units. (All previous trades, including those at the current price, remain valid.)⁶ Our analysis is restricted to symmetric equilibria in which bidders never bid more than their valuations.⁷

We call multiple sales at a single price a "frenzy."

A "crash" occurs if, after trade takes place at a price, it becomes common knowledge that no further purchases will be made until the price has fallen to some given strictly lower level.

⁴ Technical notes: In the (in equilibrium zero-probability) event that several buyers wish to purchase simultaneously at this stage of the game, all purchase offers are accepted. If supply is insufficient to meet demand, no units are sold immediately, all customers who did not bid are excluded, and the auction is restarted at a price of \bar{V} for those who bid. If all remaining bidders simultaneously offer to purchase a second time, then the seller holds a lottery among the bidders for the right to buy at the asking price. We assume that the seller lowers the price fast enough to reach a price of zero in finite time. These assumptions are made to ensure that the process terminates.

⁵ In the (in equilibrium zero-probability) event that there are simultaneous purchase offers at this stage, they are treated as in n. 4.

⁶ We show in Sec. VI that using an alternative tie-breaking mechanism, e.g., a lottery, would not importantly affect the results.

⁷ The only effect of this restriction is to rule out frenzies in which all bidders simultaneously offer to purchase, in the knowledge that therefore none will actually buy at this time.

Note that our basic model assumes a particular sequence of price offers.⁸ However, it is easy to generalize our results to any process in which the seller makes a sequence of price offers and satisfies all demand whenever possible; if demand after an offer ever exceeds supply, the seller asks higher prices to ration among those who bid. We solve and discuss this more general class of processes in the last part of Section III and in Section IX.

III. General Solution

At any point of the game, we write k for the number of units remaining, $k + l$ for the number of bidders remaining, and \underline{v} and \bar{v} for the lowest- and highest-possible-valuation bidders remaining conditional on all bidders having thus far followed their equilibrium strategies.⁹

At any point of the game, let $\omega(v)$ be the expected price a bidder with a value of v would pay, contingent on receiving an object, if the remaining goods were allocated according to a standard English auction. That is, when k units remain, $\omega(v)$ equals the bidder's expectation of the $(k + 1)$ st highest out of the $k + l$ remaining values, conditional on that value being below v .¹⁰ We shall see that this function has a crucial role in our model.

Let p be the current asking price.

Note that at any stage of our game a bidder has a higher probability of winning an object if he offers to buy than if he does not, so his optimal strategy is to offer to buy if and only if his value exceeds some cutoff level. (If a low-value "type" gets the same expected surplus from strategies with two different probabilities of receiving a unit, a higher-value type strictly prefers the high-probability strategy, and vice versa. So the higher-value type will not choose a strategy that wins the object with a lower probability than the low-value type.) It is straightforward that in a symmetric equilibrium the information publicly revealed about the remaining bidders is always just that their valuations all lie between some lowest and highest possible valuations \underline{v} and \bar{v} .¹¹ It therefore follows from elementary statistics that

⁸ This sequence was chosen to prevent a frenzy from developing immediately merely because the initial price was "too low."

⁹ Unless there has been excess demand, $l = L$.

¹⁰ In an English auction the price is raised continuously from zero until all but k bidders have dropped out, and these k winners therefore pay the actual $(k + 1)$ st value.

¹¹ If at any point there are fewer than k offers, then conditional on all bidders having followed equilibrium strategies, the remaining bidders are now all revealed to have values between the current \underline{v} and the current cutoff level for bidding, \bar{v} . Similarly, if

$$\omega(v) = \frac{\int_v^{\bar{v}} xf(x)[F(\bar{v}) - F(x)]^{k-1}[F(x) - F(\underline{v})]^{l-1} dx}{\int_v^{\bar{v}} f(x)[F(\bar{v}) - F(x)]^{k-1}[F(x) - F(\underline{v})]^{l-1} dx}. \quad (1)$$

Since it is also straightforward that the highest-valuation bidders receive the objects, we can apply the revenue equivalence theorem both to the whole game and to the continuation "subgame" that begins at any point, and so completely solve our model.

REVENUE EQUIVALENCE THEOREM. Assume that each of $k + l$ risk-neutral potential buyers has a privately known value independently drawn from a common distribution that is strictly increasing and atomless on the interval $[\underline{v}, \bar{v}]$, for one of k identical objects. Any mechanism that has the properties that in equilibrium (i) the objects always go to the buyers with the k highest values and (ii) any bidders who do not receive a unit get zero surplus yields the same expected revenue and results in a buyer with value v making expected payment $\omega(v)$ conditional on receiving an object.

Proof. For any mechanism, write $\pi(v)$ for the probability that a buyer with value v receives a unit, $S(v)$ for his expected surplus, and $E(v)$ for his expected payment conditional on receiving a unit. Since no "type" of buyer who is behaving optimally can gain by mimicking another type's strategy,

$$\begin{aligned} S(v^a) &= \pi(v^a)[v^a - E(v^a)] \\ &\geq \pi(v^b)[v^a - E(v^b)] \\ &= S(v^b) + \pi(v^b)(v^a - v^b), \quad \text{for all } v^a, v^b \in [\underline{V}, \bar{V}]. \end{aligned}$$

So $S(\cdot)$ has derivative $dS/dv = \pi(v)$, and therefore $S(v) = S(\underline{V}) + \int_{\underline{V}}^v \pi(x) dx$.

But property i implies $S(\underline{V}) = 0$ and property ii implies that $\pi(x)$ equals the probability that x exceeds the k th highest of the $k + l - 1$ other valuations. Therefore, $S(v)$ and hence also $E(v) = v - [S(v)/\pi(v)]$, and hence also the seller's expected revenue ($= [k + l] \int_{\underline{V}}^{\bar{V}} E(x) f(x) \pi(x) dx$) is constant across all mechanisms satisfying properties i and ii. But the English auction satisfies properties i and ii and has $E(v) = \omega(v)$. Q.E.D.

This theorem was first developed in different forms by Vickrey (1961), Myerson (1981), and Riley and Samuelson (1981). Our statement follows Myerson (lemma 3 and corollary, pp. 64–66), special-

there are more than k offers, then all nonbidders are eliminated and the remaining eligible bidders all have values between \bar{v} and the current \bar{v} . Therefore, the remaining bidders' valuations always remain independently drawn from $F(\cdot)$ conditional on being between some values \underline{v} and \bar{v} .

ized to the case of symmetric bidders but straightforwardly generalized to $k \geq 1$ objects.¹²

By telling us what each buyer's expected payment must be in equilibrium, the revenue equivalence theorem allows us to compute directly at what price each type of buyer must bid in equilibrium.

PROPOSITION. *Characterization of equilibrium bidding strategies.*—At any point in the game, a bidder with valuation v offers to purchase if and only if $\omega(v) \geq p$.

Proof. Let \tilde{v} be the valuation of the marginal bidder, who is indifferent to bidding p . Such a bidder either would receive a unit immediately at price p or, if there is excess demand at p , would surely be outbid subsequently. So the bidder with value \tilde{v} makes expected payment p conditional on receiving a unit. Therefore, by the revenue equivalence theorem, $\omega(\tilde{v}) = p$. Thus $v \geq \tilde{v}$ if and only if $\omega(v) \geq p$. Q.E.D.

We call $\omega(v)$ the willingness to pay function since the proposition tells us that this is the price at which a buyer with value v is actually willing to trade at a given time. The distinction between this curve and the standard demand curve—which represents only buyers' willingness to accept take-it-or-leave-it final offers—is crucial for us.

Seller Optimality

The revenue equivalence theorem proves that the seller's expected revenue is the same with our mechanism as with any standard auction, for example, an ascending (English) auction or a sealed-bid auction in which the K highest bidders win and pay the $(K + 1)$ st bid or the K highest bidders win and pay their own bids. This also implies that, provided $F(v)$ is "regular"—that is, $v - \{[1 - F(v)]/f(v)\}$ is strictly increasing in v —the trading process maximizes the seller's expected revenue over all possible mechanisms that require the sale of all K units.¹³

¹² This generalization has been considered by Milgrom and Weber (1982), Bulow and Roberts (1989), and Maskin and Riley (1989). The theorem (and therefore our results) holds under even weaker assumptions on the distribution.

¹³ See Myerson (1981, sec. 5), Maskin and Riley (1989, sec. 3), or Bulow and Roberts (1989, sec. 5). Bulow and Roberts explain that assuming regularity is equivalent to making the common assumption that a monopolist's marginal revenue is downward sloping. If $F(v)$ is not regular, the monopolist cannot maximize revenue by simply selling to the bidders with the highest values, so no standard auction maximizes revenue. In this case the monopolist could gain by precommitting to "crash" the price and then to hold lotteries, even when crashes do not arise endogenously, so frenzies and crashes seem even more likely to arise. (For the theory of optimal monopoly pricing with demand uncertainty, see Harris and Raviv [1981]. For discussion of the case of irregular $F(\cdot)$, see especially Mussa and Rosen [1978].)

If the seller were able to precommit to a reservation price, r , then with r chosen optimally (ex ante), the game maximizes the seller's expected revenue over all mecha-

Therefore, with regularity, the sales process is consistent with rational behavior by a risk-neutral seller.¹⁴

Other Efficient Sales Mechanisms

The logic above applies to all efficient (and therefore, under the conditions described above, seller-optimal) sales mechanisms that involve the seller's calling out a sequence of prices and meeting all demand whenever it is less than supply: the lowest-valuation type of bidder at any price p will receive a unit either immediately or never. Therefore, the expected payment contingent on receiving a unit will be p for this buyer. Thus, from the revenue equivalence theorem, the valuation of this type, \bar{v} , must be such that $\omega(\bar{v}) = p$. The implication is that the proposition applies to the general class of mechanisms described in the last paragraph of Section II. That is, all buyers always have a WTP of $\omega(v)$, regardless of how future asking prices will be determined.

IV. Characterization of Equilibrium Price Paths

This section provides a complete characterization of the equilibrium price paths of the trading process. Section V highlights the most significant features and can be read independently of this section.

The evolution of the trading process is charted in figure 1. We use the proposition to compute, at each stage, those bidders whose current WTP, $\omega(v)$, exceeds the current asking price, p . At the same time we keep track of how the function $\omega(v)$ changes as information is revealed about the parameters k , l , \underline{v} , and \bar{v} by the trading to date.

Strictly, the seller begins by asking the price \bar{V} . However, no bidder will be willing to pay more than $\text{WTP}(\bar{V}) = \omega(\bar{V})$, so we can think of the seller as setting $p = \omega(\bar{v})$ (step 1; initially $\bar{v} = \bar{V}$).

Price is then lowered continuously, and as long as there is no sale, \bar{v} is continuously revised downward to $\omega^{-1}(p)$ (step 2). The first sale will be made to the bidder with the highest actual valuation. Since this bidder knows that if he bids first his valuation must be the highest, the

nisms that allow the seller to precommit to a reservation price, provided $F(\cdot)$ is regular. In this case, our proposition is amended so that a bidder offers to purchase if and only if p is less than or equal to the bidder's expectation, conditional on receiving a unit, of the maximum of r and the $(k + 1)$ st highest remaining value. Frenzies and crashes arise exactly as in our original mechanism.

¹⁴ Furthermore, because these remarks apply to every continuation "subgame," and because if $F(\cdot)$ is initially regular the distribution of values always remains regular, the sales process is also *sequentially* rational, i.e., time consistent if $F(\cdot)$ is regular.

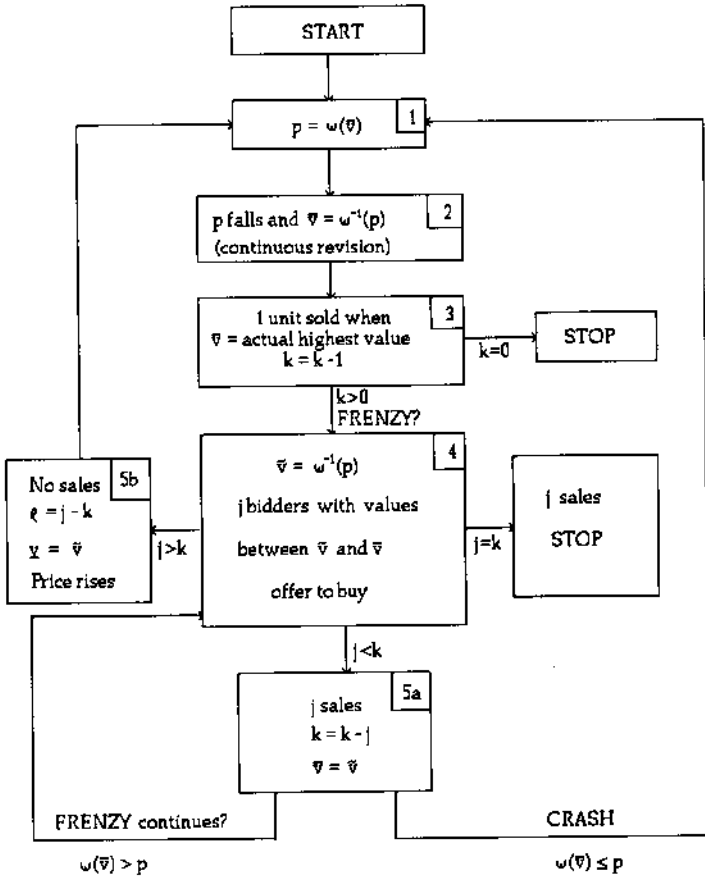


FIG. 1.—Equilibrium of trading process

first sale is made at $p = \omega(\bar{v})$ when \bar{v} equals this bidder's valuation, and k is then revised to $k - 1$ (step 3).¹⁵

Removing one bidder and one unit by the first sale must increase the price each remaining bidder expects to pay; that is, setting $k = k - 1$ increases $\omega(v)$ for all v .¹⁶ Since immediately previously we had

¹⁵ That is, if v_1 is the actual highest valuation, then from (1), the first sale occurs at

$$p = \frac{\int_y^{v_1} x f(x) [F(v_1) - F(x)]^{k-1} [F(x) - F(y)]^{l-1} dx}{\int_y^{v_1} f(x) [F(v_1) - F(x)]^{k-1} [F(x) - F(y)]^{l-1} dx}$$

¹⁶ That is, the expected k th highest of $k + l - 1$ values exceeds the expected $(k + 1)$ st highest of $k + l$ values, or, equivalently, the expected value of the l th from the bottom of $k + l - 1$ values exceeds the expected value of the l th from the bottom of $k + l$ values.

$\omega(\bar{v}) = p$, there is now some value $\bar{v} < \bar{v}$ for which $\omega(\bar{v}) = p$. Our proposition shows that at this time all bidders with values v between \bar{v} and \bar{v} participate in a frenzy (step 4). We shall show in the next section (fig. 4) that this may be a large number of bidders.

If the number of bidders j who now offer to buy is less than k , then all participants in the frenzy are allocated a unit. The number of units remaining is revised to $k = k - j$, but the maximum value of any remaining bidder is revised to $\bar{v} = \bar{v}$ (step 5a).¹⁷ The first of these revisions raises $\omega(v)$ for all v , and the second reduces it. If j is sufficiently low, we then have $\omega(\bar{v}) \leq p$ and price must fall to at least $p = \omega(\bar{v})$ before there is any chance that another buyer can be attracted; that is, we have a crash (return to step 1). If j is sufficiently large, on the other hand, we have $\omega(\bar{v}) > p$ and there is the prospect of a continued frenzy (return to step 4).¹⁸ Note that the larger j is, the lower the new cutoff valuation $\bar{v} = \omega^{-1}(p)$ will now be, that is, the greater the range of valuations that will participate in the next round of the frenzy. In this sense the frenzy "feeds on itself," and a frenzy may run on for several rounds before ending in either excess demand or a crash.¹⁹

If $j > k$ at step 4, then there is excess demand, so all players who did not offer to buy are now eliminated from the game. The total number of remaining bidders $k + l$ is thus revised to j , that is, $l = j - k$. Since all these bidders have values $v \geq \bar{v}$ (in equilibrium), we reset $\bar{v} = \bar{v}$ (step 5b). On the basis of these revisions, a new, higher, $\omega(\bar{v})$ is calculated, and we return to step 1 to allocate the remaining units among the remaining bidders.

Illustration: The Uniform Distribution

We can illustrate our analysis by considering the case in which buyer values are drawn from a uniform distribution. When the distribution

¹⁷ Strictly, \bar{v} is now the supremum of the remaining values, since we have arbitrarily assumed that indifferent buyers do bid.

¹⁸ It is straightforward to show that if $j = k - 1$, then $\omega(\bar{v}) \geq p$, and if $j = 0$, then $\omega(\bar{v}) \leq p$. So if a frenzy ends with $k = 1$ and $j = 0$, then $\omega(\bar{v}) = p$, and the price falls continuously. For all $k > 1$, both these inequalities are strict, so for $k > 1$ there is both a positive probability that the current round of bidding will be immediately followed by a crash and a positive probability that it will be followed by a further frenzy.

¹⁹ To see that a frenzy will commonly run on into several rounds, note that after the first unit is sold at any price, the price equals the seller's expectation of the $(k + 1)$ st price (which equals the seller's expectation of its revenue per remaining unit). However, the condition for a crash is that the price be above the expectation of the $(k + 1)$ st price that would be held by a buyer with the highest possible remaining value. Thus a frenzy runs on into further rounds as long as the aggregate information that is revealed by the number of bidders jumping in in the earlier rounds is either good news for the seller or bad but not awful news.

is on $[0, 1]$, it is easy to perform the integrations in (1) to obtain

$$\omega(v) = \frac{\sum_{j=0}^{k-1} v \binom{k-1}{j} \left[\frac{(-v)^j}{j+l+1} \right]}{\sum_{j=0}^{k-1} \binom{k-1}{j} \left[\frac{(-v)^j}{j+l} \right]}, \quad (2)$$

and since, conditional on the bidding to date, the distribution always remains uniform on $[\underline{v}, \bar{v}]$, $\omega(v)$ always remains an affine transformation of (2).²⁰ For $v = \bar{v}$, $\omega(v)$ always has the particularly simple form

$$\omega(\bar{v}) = \frac{k\underline{v} + l\bar{v}}{k+l}, \quad (3)$$

so that, in a crash or at the beginning of the game, the price crashes to this level and then falls continuously until the first sale actually takes place at $(k\underline{v} + lv_1)/(k+l)$, in which v_1 is the actual highest valuation.

For example, figure 2 shows how the trading process evolves when $K+L=5$ bidders with values drawn from a uniform distribution on $[0, \bar{V}]$ compete for $K=3$ units. From (3), the first unit is sold at $p = .4v_1$. From (2), all bidders with values of at least $2v_1/3$ now jump in.²¹ If none does, the price crashes $16\frac{2}{3}$ percent and on average falls by $33\frac{1}{3}$ percent before another buyer can be found. However, even if only one more buyer does jump in at $p = .4v_1$, there is a 27 percent chance that this second sale will generate demand from at least one more buyer. The overall probability that a frenzy will occur at some point in the game is over 90 percent.

²⁰ The general form of (2) is

$$\omega(v) = \frac{\underline{v} + \sum_{j=0}^{k-1} \binom{k-1}{j} \left[-\left(\frac{v-\underline{v}}{\bar{v}-\underline{v}}\right) \right]^j \left(\frac{v-\underline{v}}{j+l+1}\right)}{\sum_{j=0}^{k-1} \binom{k-1}{j} \left[-\left(\frac{v-\underline{v}}{\bar{v}-\underline{v}}\right) \right]^j \left(\frac{1}{j+l}\right)},$$

but it is easier to renormalize units and work with the version in the text (see, e.g., n. 21).

²¹ The easiest way to check this is to renormalize v_1 to one, so the distribution is now on $[0, 1]$ with $k=2$ and $l=2$; so (2) implies $\omega(v) = (1/3v - 1/4v^2)/(1/2 - 1/3v)$. The (normalized) price is $p = .4$, so $p = \omega(v)$ implies $(3v-2)(6-5v) = 0$, and the relevant root is $v = 2/3$, i.e., in the original units, $v = 2/3v_1$.

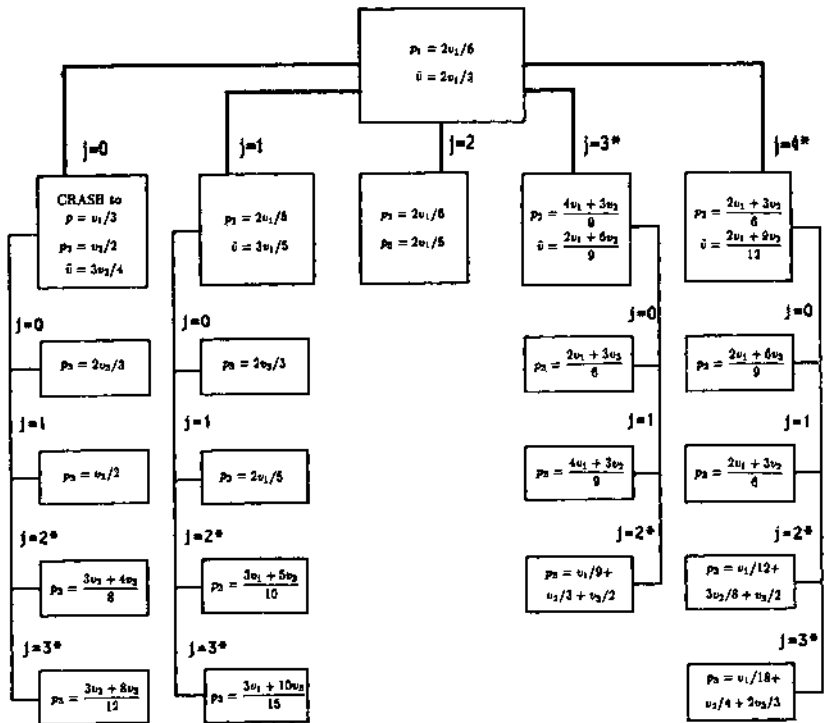


FIG. 2.—Evolution of trading with five bidders and three goods: v_i is the i th highest actual value; p_i is the sale price of the i th unit; \bar{v} is the cutoff value for participating in a frenzy; j is the number of additional bidders at the current price; $j > 0$ implies frenzy; and an asterisk implies excess demand.

V. Frenzies and Crashes

The surprising feature of our trading process is that frenzies and crashes are likely to be very big.

Frenzies

Large frenzies occur because the WTP curve $\omega(v)$ is very flat; for $k > 1$, it is perfectly elastic at \bar{v} , regardless of the slope of the demand curve. Therefore, a small upward shift in $\omega(v)$, such as that caused by a single purchase, turns bidders with a wide range of reservation values from bystanders into buyers.

Why is $\omega(v)$ so flat? Formally, observe that $\omega(v)$ is the expectation for a bidder with value v of the $(k + 1)$ st highest value, conditional on that value being below v ; provided v is sufficiently high that the $(k + 1)$ st value is almost certainly below v , this expectation is almost

independent of v . Consider, for example, $k = 10$ units, $k + l = 20$ bidders, and values drawn from a uniform distribution on $[0, \bar{v}]$. A bidder with value \bar{v} would know that his value is highest, so his estimate of the $(k + 1)$ st, that is, the eleventh, value of the 20 bidders is just his estimate of the tenth value of the other 19 bidders, that is $.5\bar{v}$. However, a bidder with value $.8\bar{v}$ would know that his value also exceeds the actual eleventh value with probability .998, so his $\omega(v)$ is also very close to the estimate of the tenth value of the other 19 bidders; in fact $\omega(.8\bar{v}) = .499\bar{v}$. Figure 3 graphs $\omega(v)$ for this example (to scale).

The general point is that when a large number of units remain for sale, a large number of bidders are fairly sure that they are all inframarginal. Since, conditional on their being inframarginal, these bidders' expectations of the market-clearing price are independent of their own exact values, all these bidders will have almost identical WTPs.

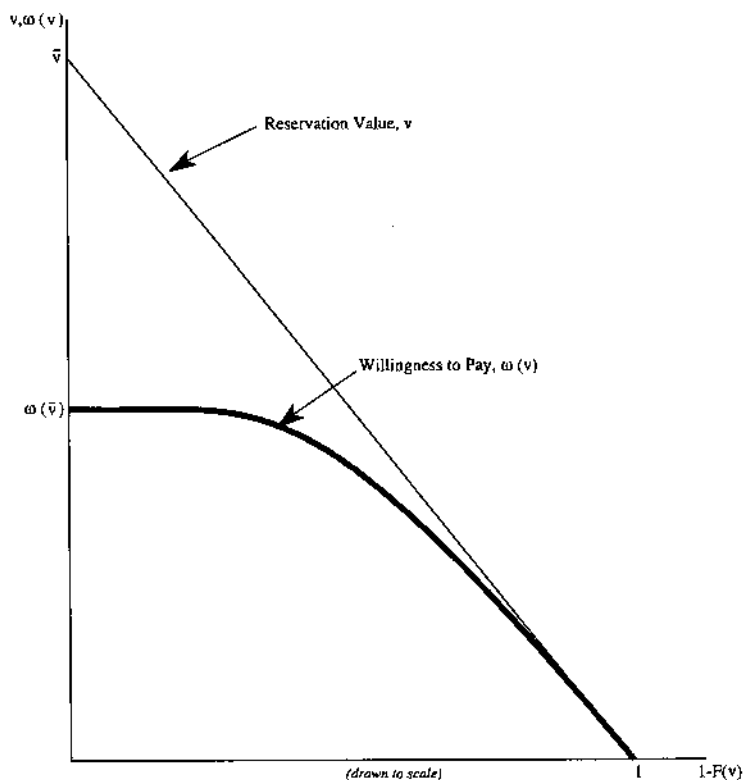


FIG. 3.—Willingness to pay: 20 bidders compete for 10 objects; values are drawn from a uniform distribution.

In the example, before any sale is made, the asking price $p = \omega(\bar{v}) = .5\bar{v}$. As price falls, so does \bar{v} , and the bidders always remain uniformly distributed on $[0, \bar{v}]$, so figure 3 remains unchanged but with the units on the y-axis scaled appropriately. While the price is falling, it always equals $\omega(\bar{v})$, that is, equals the intercept of the WTP curve and the y-axis.²²

When a sale does occur (at $\omega(\bar{v})$), it causes a small upward shift in $\omega(v)$ for all remaining bidders (see fig. 4). In the example, the estimated market-clearing price is now the estimated tenth value of the remaining 19. Thus another bidder with value \bar{v} would now pay the estimated ninth value of the other 18 bidders or $.526\bar{v}$, and a bidder

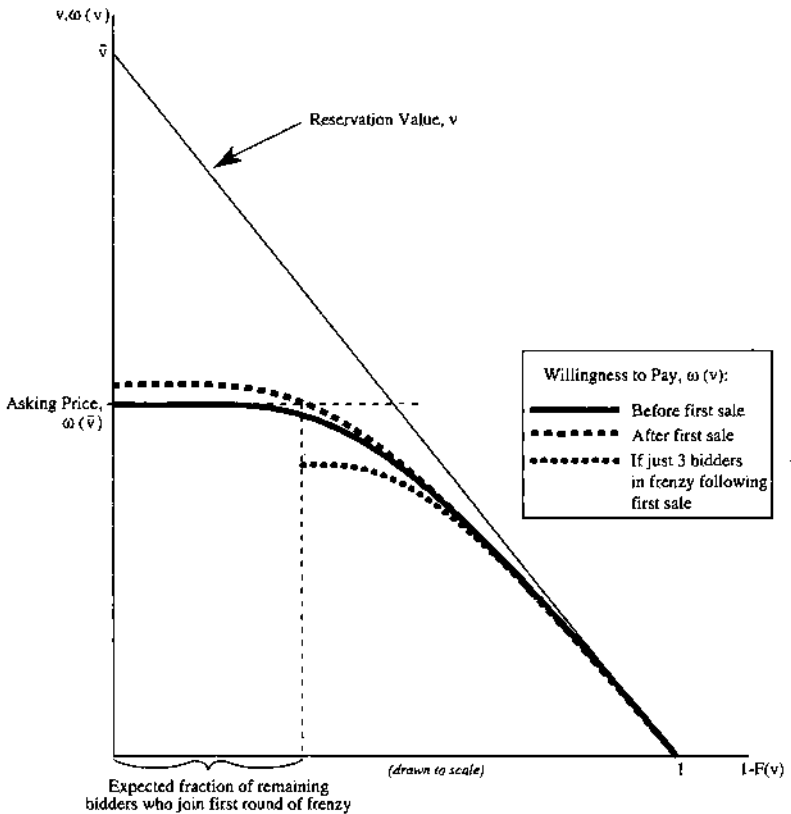


FIG. 4.—Size of the first frenzy: 20 bidders compete for 10 objects; values are drawn from a uniform distribution.

²² For general $F(\cdot)$, the important features of fig. 3 remain unchanged; i.e., $\omega(v)$ is always flat at the y-axis for $k > 1$ and price always equals the intercept of $\omega(v)$ with the y-axis. Other details of the figure will change as price falls.

with value $.8\bar{v}$ would, as before, pay almost the same as one with value \bar{v} , actually $.525\bar{v}$. Since in fact all bidders with values exceeding $.65\bar{v}$ would now pay more than the current asking price of $.5\bar{v}$, on average 6.6 bidders join the first round of the frenzy.

With larger numbers of units, $\omega(v)$ is even flatter; with $K = 50$ and $K + L = 100$, on average 40 bidders join the first round of the first frenzy.

Finally, another way to see that frenzies must be large is just to ask, Why will any bidder jump in first? Since a bidder pays the price he bids, why would he not always gain by holding off and bidding second, after someone else has bid first?²³ The reason must be that there is a nontrivial probability that as soon as someone does bid first, there is then immediately excess demand, so no one can guarantee being the second bidder and paying the first bidder's price. And for the probability of immediate excess demand to be nontrivial, the expected number of simultaneous bidders must, of course, be large.

Crashes

The large expected size of the frenzy is precisely what makes big crashes possible. In the example illustrated, roughly $6\frac{1}{2}$ buyers are expected immediately on average, but the standard deviation is about two. Finding out that there are only three bidders with relatively high values instead of six or seven would substantially reduce $\omega(v)$ for all remaining bidders (see fig. 4) and lead to a crash of more than 18 percent. Even with $K = 50$ and $K + L = 100$, the probability that the price will fall by more than 10 percent (15 percent) after buying stops at the first price is greater than 10 percent (2 percent).

The key is that the frenzy brings a large block of information into the market at one time, and if that information is unfavorable to the seller, then prices must crash.

"Chaos"

In contrast to conventional models, seller revenue in our mechanism is neither continuous nor monotonically increasing in bidders' reservation values. The reason is that information about demand is revealed in blocks, with bidders with a wide range of values revealing themselves simultaneously, and a change in one bidder's value can dramatically alter the flow of this information. Slightly *higher* bidder value(s) can lead to significantly *lower* revenues.

²³ Of course, it cannot be an equilibrium for no one to bid first. If all bidders entered simultaneously at a price of zero, most of them would have done better to enter a fraction earlier.

Consider a small decrease in v_1 , the reservation value of the first buyer. If this buyer pays a little less, then the cutoff value above which bidders will now jump in will also be slightly lower, and an extra bidder may be induced to buy. This extra bidder could make the difference between a continuing frenzy and a crash, so the lower value of the first bidder could benefit the seller. Returning to our example of five bidders uniformly distributed on $[0, 1]$ and competing for three units (see fig. 2), imagine that actual bidder values are .95, .62, .60, .45, and .40. The first unit would be sold for .38, a price that is too high to generate any further sales. After a crash, the second and third units would be sold for .31 each. Total seller revenues would equal 1.00. Now reduce all bidder values to .90, .61, .55, .30, and .10. In this case the first sale at .36 will lead to a second, and then a third sale at the same price. Revenues would be $3(.36) = 1.08$.

VI. Rationing Excess Demand

While our mechanism generally requires the seller to meet all demand at the offering price, we do allow the offer to be retracted and the price to rise when there are more immediate bidders than units remaining. This assumption allows us to meet the requirements of the revenue equivalence theorem and to guarantee seller rationality.

An alternative to raising price would be to hold a lottery among the remaining bidders. Making this assumption would preclude our using the revenue equivalence theorem. However, if $F(\cdot)$ is uniform, then the first sale takes place at the same price as in the original mechanism,²⁴ and it is followed by a larger frenzy.²⁵ The intuition is

²⁴ Define $\pi(v)$ and $S(v)$ as we did in the proof of the revenue equivalence theorem, and note that the argument given there that $S(v) = S(\underline{V}) + \int_{\underline{V}}^v \pi(x) dx$ is unaffected. Furthermore, since K units are sold to $K + L$ potential buyers,

$$\frac{K}{K + L} = \int_{\underline{V}}^{\bar{V}} f(x) \pi(x) dx$$

in any equilibrium. So if $F(\cdot)$ is uniform and $S(\underline{V}) = 0$, which is guaranteed if the price never falls below \underline{V} , we have $S(\bar{V}) = [K/(K + L)](\bar{V} - \underline{V})$. Since, conditional on the trading to date, the remaining valuations are always uniformly distributed, it follows that at any point in the game a bidder who knows that he has the highest possible remaining valuation, \bar{v} , expects surplus of $[k/(k + l)](\bar{v} - \underline{v})$, independent of the allocation mechanism. Therefore, the price always crashes to $(k\underline{v} + l\bar{v})/(k + l)$, and this is the price when the first sale takes place, as in the original mechanism; see eq. (3).

²⁵ Assume (for contradiction) that the marginal bidder in a frenzy has the same valuation as in the original process. His utility from not bidding would then be the same as before, because each possible number of bidders, j , that may now bid would be as likely as before and would give him the same surplus as before (since if there is remaining stock after these bidders' purchases, this buyer has the highest valuation and therefore receives the same surplus as before; see n. 24). However, his utility from

that the marginal buyer has a greater incentive to bid at the current price, because by bidding he may earn some surplus even if there is excess demand.

In our example with five bidders with values drawn from a uniform distribution competing for three units, on average 47 percent of remaining bidders participate in the frenzy after the first sale, compared with 33 percent in the original model. The larger expected frenzies also imply larger expected crashes.

Resale

In fact the discussion above probably understates the magnitude of frenzies and crashes when lotteries are used as tiebreakers. We have assumed, as is standard in the auctions literature, that buyers are unable to resell among themselves. This becomes relevant in mechanisms that sometimes allocate units to lower-valuation bidders. If resale is in fact possible, then we expect frenzies to be even larger on average because the marginal bidder has the added incentive that if he wins he may be able to resell for more than his value and he therefore has a higher WTP. The more surplus that goes to resellers, the larger we expect frenzies to be.²⁶

VII. Common Values

The polar alternative to our "private-values" assumption is to consider a pure "common-values" model, in which each bidder, if endowed with the same information, would value a unit at a common price. We consider a common-values model attributable to Myerson (1981). Each of $K + L$ risk-neutral symmetric bidders, $j = 1, \dots, K + L$, obtains a signal, v_j , independently and identically distributed between \underline{V} and \bar{V} according to the common strictly increasing and atomless distribution $F(v)$. The true value of a unit to any bidder is $\sum_{j=1}^{K+L} v_j / (K + L)$. There are K units available.

Because bidder signals are still independent, a version of the reve-

bidding is higher with the lottery since it allows him the chance to receive a unit even if there is excess demand. Therefore, this bidder would strictly prefer to bid if he thought he was marginal, so the marginal value must be lower and the frenzy must be larger.

²⁶ In general, resale will not be efficient if bidders retain private information about their values (see Myerson and Satterthwaite 1983). Efficient resale is possible if, when there is excess demand for $j > k$ units, ownership of k/j of an object is assigned to each bidder (see Cramton, Gibbons, and Klemperer 1987). In this case (and when bids that exceed valuations are permitted), it is an equilibrium for the highest-valuation bidder to enter at the same price as under our original mechanism, for any $F(\cdot)$, and for all remaining bidders to enter in the first frenzy.

nue equivalence theorem still applies.²⁷ However, because any buyer's decision to purchase raises the valuation of all other bidders, frenzies are even more extensive.

As a parallel to our earlier example, with five bidders with signals drawn from a uniform distribution competing for three units, on average 52 percent of all remaining bidders jump in immediately after the first sale, compared with 33 percent in the private-values case.

The intuitive reason for these larger frenzies is that any new information causes a larger shift in the WTP curve. Not only does a new transaction raise remaining bidders' WTP by slightly worsening the balance of demand and supply, but it also provides information about the remaining bidders' valuations, further raising each one's WTP.

This latter effect makes frenzies and crashes much less surprising in the common-values case, and most of the current work on frenzies and crashes depends heavily on a common-values assumption.²⁸

VIII. Elastic Supply

Although it makes the algebra more cumbersome, it is easy to extend our analysis to the case of a seller with an elastic supply curve, because our mechanism remains revenue equivalent to an English auction. Each bidder's expected payment contingent on winning now equals the expectation of the maximum of the valuation of the highest non-winning valuation and the cost to the seller of the last unit supplied, conditional on this bidder winning.

There are two effects that make WTP curves even flatter than in the inelastic supply case. First, for given $F(\cdot)$, more elastic supply makes higher-valuation bidders even more certain of receiving a unit. Second, more elastic supply reduces the variance of the "market-

²⁷ Specifically, any mechanisms that award units to the bidders with the highest signals and give no surplus to any bidder who does not receive a unit are equivalent in expected revenue and in the expected payment, conditional on receiving an object, of every type of bidder. Therefore, we can solve the common-values model in a manner similar to the private-values model. Our mechanism still maximizes expected revenue, subject to the constraint that the units must be allocated to the K bidders with the most optimistic signals.

²⁸ Interesting papers that generate "herd behavior" in a common-value setting include Scharfstein and Stein (1990), Banerjee (1992), Bikhchandani et al. (1992), Caplin and Leahy (1992), Welch (1992), Zwiebel (1992), Brandenburger and Polak (1993), Chamley and Gale (1993), and Romer (1993). Gennotte and Leland (1990) and Jacklin, Kleidon, and Pfeiderer (1992) explain the 1987 crash by a model in which agents may misinterpret portfolio-insurance trades as containing new information about fundamentals. Madrigal and Scheinkman (1992) use a common-values model like the one in Sec. VII. While our seller has no private information, Madrigal and Scheinkman's market maker may cause crashes through his strategic use of information.

clearing" price. Both effects reduce the option value to high-valuation bidders of not paying the expected market-clearing price, and so make the WTP of these high-valuation bidders closer to the WTP of a bidder with the highest possible valuation.²⁹

Thus we conjecture frenzies to be larger with more elastic supply.³⁰ However, for any given $F(\cdot)$, more elastic supply will mean less variation in the Walrasian price and very likely smaller crashes. Of course, in the extreme case of perfectly elastic supply, the WTP curve becomes perfectly flat and all transactions take place simultaneously.

IX. Are Frenzies and Crashes Inevitable?

Our main results are independent of the specific way in which the seller chooses prices: The reason is that any efficient sales mechanism in which the seller calls out prices to which the buyers simultaneously say "yes" or "no" yields exactly the WTP curve we have derived (see the last paragraph of Sec. III). Because the WTP curve is so flat, all except a very small subset of prices that could possibly attract any buyers will probably attract many buyers.

The only way to almost surely avoid multiple simultaneous offers is to start at a sufficiently high price (at least $\omega(\bar{v})$) and then lower the price continuously until an initial single sale is made, as in our mechanism, and then without allowing further sales at this price, to immediately raise the price (to at least the revised $\omega(\bar{v})$) and repeat this process after every individual sale. Any other efficient mechanism in which the seller calls out prices results in a positive probability of multiple bids at the same price.

For example, if price cannot be varied continuously, then it will be almost impossible to avoid frenzies. Consider the example with 20 bidders whose values are drawn independently from a uniform distribution between zero and 100, and with a supply of 10 units. At the beginning of the game the seller will make no sales if he asks a price of 50.00.³¹ However, if the price is reduced discontinuously from 50.00 to 49.99, then the probability of at least one sale at 49.99 will be over 97 percent, and expected demand would be $3\frac{1}{4}$.³²

²⁹ Recall that the WTP of the highest-possible-valuation bidder is exactly his expectation of the market-clearing price and is also the transaction price when a frenzy is triggered.

³⁰ In our uniform distribution example, the expected size of the first frenzy is larger than in the inelastic supply case, for any linear supply curve that now yields the same expected total sales.

³¹ Except in the zero-probability event that there is a bidder valuation of exactly 100.

³² To maintain our efficiency assumption, we assume for the purposes of this calcula-

Finally, note that if in any mechanism a large number of bidders are expected at a certain time, then there is a substantial chance that demand will be sufficiently below expectations that there will be a crash.³³ In the example above, if only one bidder is willing to pay 49.99, then the price will have to crash to 44.09.

While these results indicate the near inevitability of frenzies and crashes in our framework, we have made several restrictive assumptions. We have only one seller. The seller's entire supply is available at any quoted price. There is private information on only one side of the market. These simplifications imply that we must be cautious in applying our analysis to, say, the stock market. Still, our general intuition seems robust. Agents with very different valuations should be willing to trade at nearly identical prices, and the information generated by traders' responses to any given price should therefore be sufficient to produce substantial drops in prices when demand is weak. One caveat is that if not all agents can act at a given time or if some traders are irrational, then the market response to a price may yield less information. Therefore, inefficiencies of these sorts may reduce the scale and frequency of frenzies and crashes.

X. Conclusion

We have presented a simple market-clearing model in which almost every first sale at a new price triggers a frenzy of buying. This extra demand may "feed on itself," attracting more buyers, until demand exceeds supply. Alternatively, the frenzy will end with a crash in which price falls discontinuously. Furthermore, small changes in bidder values can dramatically change outcomes, implying that successive uses of trading processes like ours in similar environments can yield strikingly different results.

The key to these results is that while expected demand may be relatively inelastic, the WTP curve, which represents the prices buyers are willing to pay at any given moment, is almost perfectly elastic for higher-valuation buyers. Intuitively, these buyers are all almost certainly inframarginal in terms of the decision of *whether* to buy and are therefore all solving the virtually identical optimization problem

tion that the seller can quote prices in arbitrarily fine increments when necessary to break ties.

By contrast, if price is reduced continuously from 50.00, the expected price of the first sale would be just 47.62. A bidder with an initial WTP of 49.99, through constant revision in WTP caused by a lack of any sales, will have a WTP below 47.62 if the price falls to 47.62 with no sales.

³³ Unless the price chosen is so low that with very high probability there will be excess demand.

of *when* to buy. All these buyers will be willing to pay amounts that are just under their estimates of the Walrasian market-clearing price.

Elastic WTP curves imply that if an asking price can attract any buyers, it should attract many buyers. It follows that the existence of frenzies and crashes does not depend on our specific process. Although in our model each frenzy is begun by a single sale, any other small change in market conditions that leads to a small change in buyers' expectations about the market-clearing price could be the trigger for a new frenzy. If the number of buyers in a frenzy is below expectations, then each remaining market participant will reduce his willingness to pay to fully reflect the new information. No rational buyer will be willing to pay an amount close to the previous asking price, and price must crash.

We showed in Sections VI–IX that introducing other elements of reality into our basic model such as common values, resale, elastic supply, and alternative methods of dealing with excess demand all appear to further accentuate frenzies. In models with rational bidders, frenzies are generally common. Ironically, it is precisely because bidders are rational and strategic that they are so sensitive to market information and adopt behavior that leads to frenzies and crashes.

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