HOLDING IDLE CAPACITY TO DETER ENTRY

Jeremy Bulow, John Geanakoplos and Paul Klemperer

In this Journal, Dixit (1980) carefully analysed the extent to which an established firm facing a prospective entrant might make an investment in capacity beyond that which is optimal for its pre-entry output. In his model, both the established firm and prospective entrants understand that a quantity setting Nash equilibrium would be established in any post-entry game.

Dixit showed that Spence (1977) had studied an imperfect equilibrium (that is, an equilibrium that depended upon a threat that was not credible) to obtain his result that firms might hold excess capacity to deter entry. In a perfect equilibrium, a firm would not wish to install capacity that would be left idle if entry did not occur. Dixit's conclusion, however, depends on his assumption that each firm's marginal revenue is always decreasing in the other's output. This is quite a restrictive assumption. For example it is never satisfied in the relevant range for economists' second-favourite demand curve—constant elasticity demand.

The basic point is unchanged if the firms anticipate price competition with differentiated products rather than quantity competition in any post-entry game: Under quite plausible conditions, rational firms facing rational potential entrants may install capacity that will certainly be left idle. We are thus able to rehabilitate Spence's original proposition in a perfect equilibrium setting.

1. DIXIT'S MODEL

Dixit's game, which is of complete information, is as follows. Firms \( i = 1, 2 \) have fixed costs \( f_i \) of competing in an industry, and produce with a Leontief technology in which capacity (capital) costs \( r_i \) and labour costs \( w_i \) per unit output. The established firm, 1, installs capacity \( k_1 \). This may subsequently be increased, but cannot be reduced. The prospective entrant, firm 2, observes this choice and either enters the industry or does not. No production occurs prior to firm 2's decision, but Nash equilibrium is established immediately afterwards.

If firm 1 produces output \( x_1 \) within the limit of the capacity it has installed (i.e. \( x_1 \leq k_1 \)) then its total costs are

\[
C_1 = f_1 + r_1 k_1 + w_1 x_1.
\]

If it wishes to produce \( x_1 > k_1 \), its costs are

\[
C_1 = f_1 + (r_1 + w_1) x_1.
\]

\(^1\) Spulber (1981) presents a model similar to Dixit's and also concludes that if firms follow Cournot strategies they will never add capacity that will always be idle. In Spulber's model the crucial assumption is that demand is concave, which is a sufficient condition for each firm's marginal revenue to be decreasing in the other's output.
Firm 2 has no prior commitment in capacity so that its costs are

\[ c_2 = f_2 + (r_2 + u_2) x_2. \]

The question is: Will firm 1 ever install capacity that it ends up not using? There cannot be any purpose in installing capacity which it would not use in any circumstance. Nor will the firm install more capacity than it needs after entry, if entry is certain. The only possibility is that it might install capacity which it would not use if no entry occurred, but which serves the purpose of deterring entry. If entry is to be deterred, it must be that the firm would use the capacity in the duopolistic post-entry Nash equilibrium. Thus we can only observe idle capacity if it would be optimal for the firm to use the excess capacity if entry did in fact occur.

II. NUMERICAL EXAMPLE

Assume that capacity costs 10 per unit, that variable costs are 20 per unit for both firms, and that fixed costs are 600, i.e. \( f_1 = f_2 = 600 \), \( r_1 = r_2 = 10 \), \( u_1 = u_2 = 20 \). Assume demand of constant elasticity \(-2 \), \( \beta = 1000(x_1 + x_2)^{-2} \).

It is easy to check that firm 1’s marginal revenue is

\[ MR_1 = p \left( \frac{\frac{1}{2} x_1 + x_2}{x_1 + x_2} \right), \]

and firm 2’s marginal revenue is

\[ MR_2 = p \left( x_1 + \frac{1}{2} x_2 \right) / (x_1 + x_2). \]

If 1 builds no capacity in advance, both firms have marginal costs of 30, so the equilibrium is \( x_1 = x_2 = 312 \frac{1}{2} \), \( p = 40 \), \( MR_1 = MR_2 = 30 \), and both firms earn

\[ 312 \frac{1}{2} \times 40 - [600 + (20 + 10)312 \frac{1}{2}] = 2,525. \]

To deter 2’s entry, 1 has to install enough capacity in advance that 2 would earn zero in Nash equilibrium if it did decide to enter. It can be checked that 1 needs to install 720 units of capacity—if 2 did enter, the equilibrium would be

\( x_1 = 720, \quad x_2 = 180, \quad p = 33 \frac{1}{4}, \quad MR_1 = 20, \quad MR_2 = 30, \) and 2’s profits are

\[ 180 \times 33 \frac{1}{4} - [600 + (20 + 10)180] = 0. \]

However, if 1 installs 720 units of capacity (thus reducing its marginal costs to 20 for outputs less than 720), and 2 then rationally stays out of the market, 1’s monopoly output will be only 625. This gives \( p = 40 \), \( MR_1 = 20 \), and 1’s profits are

\[ 625 \times 40 - [600 + 720(10) + 625(20)] = 4,700. \]

(If it had used all its capacity the price would fall to 37.3 and its profits would be only 720(37.3) - [600 + (20 + 10)720] = 4,033.)

We can also easily check that if 1 installs capacity between 312 \( \frac{1}{2} \) and 720, thus reducing 2’s equilibrium output from the symmetric equilibrium output of 312 \( \frac{1}{2} \), but not completely deterring 2’s entry, then 1 always makes less than 4,700.\(^1\)

Thus 1’s optimal strategy is to deter entry by installing 720 units of capacity, but to use only 625 of these units to produce its monopoly output. 95 units will be left idle.

Fig. 1 illustrates the situation.

\(^1\) If \( k_1 \in [312 \frac{1}{2}, 720) \) then \( x_1 = k_1 \) and \( x_2 \) is 2’s optimal response to this choice of \( x_1 \) (i.e. \( x_2 \) satisfies \( MR_2 = 30 \)). It is not difficult to show that in our example the post-entry equilibrium price \( p \) is decreasing in \( k_1 \) (in this range), and also that for \( k_1 = 312 \frac{1}{2}, \quad p = (1,000/25) = 40 \) and for \( k_1 = 491 \frac{1}{2}, \quad p = (1,000/278) < 36 \). So for \( k_1 \in [312 \frac{1}{2}, 491 \frac{1}{2}], \quad x_1 < 492 \) and \( p < 40 \), so \( 95 (492 - 30) - 600 = 4,320 < 4,700 \). And for \( k_1 \in [492, 720], \quad x_1 < x_2 \) and \( p < 36 \), so \( 95 (36 - 30) - 600 = 4,080 < 4,700. \)
III. STRATEGIC SUBSTITUTES AND COMPLEMENTS

Note in Fig. 1 that it is critical that 1's reaction curve turns inwards near the $x_1$-axis. For 1 to hold idle capacity it must wish to produce more, given its marginal cost curve, for some $x_2 > 0$ than for $x_2 = 0$. In contrast, Dixit's analysis assumed that a firm's marginal revenue (or equivalently marginal profit) will always be decreasing in the other's output. Under Dixit's assumption, if 2 raises $x_2$, $MR_1$ is decreased for any given $x_1$ so that $x_1$ must be lowered to bring $MR_1$ back into balance with marginal cost. Thus the assumption that a firm's marginal revenue (and marginal profit) is always decreasing in the other's output is equivalent to the assumption that the firm's reaction curve is always downward sloping, and is enough to ensure that the firm will never hold idle capacity to deter entry. From Bulow et al. (1983) we take the following:

**Definition:** We call $x_2$ a strategic substitute for $x_1$ if $\partial \pi_1 / \partial x_2 \partial x_1 < 0$ and a strategic complement if $\partial \pi_1 / \partial x_2 \partial x_1 > 0$.

(Note the analogy with the definition that products are substitutes if $\partial \pi_2 / \partial x_2 < 0$ and complements if $\partial \pi_2 / \partial x_2 > 0$. Producing more of a substitute reduces the total profit of an opponent, and consumers buy less for any given price. Producing more of a strategic substitute reduces an opponent's marginal profit so that it produces less for any given demand.)

It may seem most natural that if an increase in $x_2$ lowers 2's revenue (and profit) it should lower 2's marginal revenue (and marginal profit) also. This is always the case, for example, with concave or linear demand. However, such a relationship is by no means necessary, as the numerical example shows. In-
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For any constant elasticity demand curve \( p = \epsilon(x_1 + x_2)^{1/\eta} \), \( x_2 \) is a strategic complement for \( x_1 \) for all \( 0 \leq x_2 \leq (-1/\eta) x_1 \).

If \( x_2 \) is always a strategic substitute for \( x_1 \), we can be sure that the firm will not install capacity that will then be left idle. However, the slopes of two marginal profit curves can be quite different even where their two demand curves may look locally quite similar. Thus it is most unlikely, as a matter of empirical practicality, that we will ever be able to rule out the possibility that \( x_2 \) is a strategic complement for \( x_1 \) around \( x_2 = 0 \).

IV. PRICE COMPETITION

With undifferentiated products, and the entrant’s (constant) marginal costs at least as great as the incumbent’s, and a fixed cost of entry, we would never expect to see entry followed by price competition. With price competition no entrant can ever recoup its fixed cost. A fortiori, we should certainly not expect to see monopolists holding idle capacity to deter this kind of entry. More interesting is the case in which entrants anticipate differentiated products price competition.

With price competition the condition of ‘strategic substitutes’ \( \frac{\partial^2 \pi_i}{\partial p_i \partial p_j} < 0 \) often does not hold. If one player in a duopoly plays more aggressively by lowering its price, we expect that the other player will respond more aggressively by lowering its price also. The question is whether the incumbent might lower its price so far in response to entry that it actually produces more in a post-entry Nash equilibrium than as a monopolist.

As with quantity competition, it is true that in the ‘benchmark’ case of linear demand and constant marginal costs an incumbent produces less in any post-entry game than as a monopolist. Also as with quantity competition, however, there are quite natural demand and cost structures in which the incumbent produces more in the post-entry game. Thus firms anticipating price competition, as well as those anticipating quantity competition, may hold idle capacity to deter entry.

\[ \frac{w_2}{r_2 + w_2} = \frac{MC_2}{MC_1} \leq \frac{\eta^2}{\eta^2 - \eta - 1} \]

then \( x_2 \) will be a strategic complement for \( x_1 \). Note that for \( \eta < -1 \) (elastic industry demand) \( \frac{\eta^2}{\eta^2 - \eta - 1} \) has a minimum value of \( \frac{1}{2} \), so if firm 1’s marginal costs (after investing in capacity) are 20% or more below firm 2’s marginal costs, then \( x_2 \) will be a strategic complement for \( x_1 \) in the post-entry equilibrium, regardless of industry demand elasticity.

Consider, for example, the demand \( x_1 = Ap_1 - Bp_1 \), \( x_2 = Ap_2 - Bp_2 \), where \( A > B \), which is just a generalisation of our previous numerical example. With constant marginal costs, an incumbent may produce more in a post-entry Nash equilibrium than as a monopolist, and may rationally install idle capacity.
V. CONCLUSION

By relaxing an assumption that is difficult to justify theoretically, and perhaps impossible to test empirically, we have confirmed Spence's original intuition that firms might in some circumstances rationally hold idle capacity to deter entry.

Graduate School of Business, Stanford University
Cooles Foundation, Yale University
St Catherine's College, Oxford, and Graduate School of Business, Stanford University

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