

MULTI-PERIOD COMPETITION WITH SWITCHING COSTS

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We analyze the evolution of duopolists' prices and market shares in an infinite-period market with consumer switching costs, in which in every period new consumers arrive and a fraction of old consumers leaves. We show prices (and profits) are higher than without switching costs, and that this result does not depend importantly on our specific assumptions. We show switching costs make the market more attractive to a new entrant, even though an entrant must overcome the disadvantage that a large fraction of the market is already committed to the incumbent's product. We also examine the effects of market growth.

KEYWORDS: Switching costs, lock-in, oligopoly theory.

1. INTRODUCTION

IN MANY MARKETS, *switching costs* give each consumer a strong incentive to continue buying from the firm from which he has previously purchased, even if other firms are selling functionally identical products. Examples of switching costs include the transactions cost of closing an account with one bank and opening another with a competitor, the learning cost incurred by switching to a new make of computer after having learned to use one make, and the artificial switching costs created by frequent-flyer programs that reward customers for repeated travel on a single airline.²

Managers often seem more concerned with market shares than with short-run profits. Switching costs explain why this may be rational behavior: switching costs give firms a degree of monopoly power over their customers and so make current market shares an important determinant of future profits. Switching costs can thus help to explain phenomena such as the discounts firms offer to attract new purchasers, and the price wars that occur when there is new entry into a market.³ Switching costs may, moreover, have important implications at the macroeconomic level. The value firms place on their market shares can, for

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²Another kind of switching cost arises when uncertainty about product quality makes consumers reluctant to switch to untested products (see, for example, Schmalensee (1982)). Here however additional complications arise, due to the possibility of prices being used as signals of product quality and due to the existence of groups of customers who tried a brand and did not like it. We ignore these issues. For other examples of switching costs see Farrell and Shapiro (1989), Green and Scotchmer (1986), Klemperer (1987a, 1990), Wernerfelt (1989), and the references they cite.

³For a discussion of new entry when there are switching costs see Klemperer (1987c, 1989). See the discussion of two-period models below for an explanation of why firms offer discounts to new customers, and Klemperer (1987a) for further discussion and examples. The model we set out below applies when firms cannot offer discounts only to new customers (because old customers could successfully present themselves as "new" or because the good could be resold). See also note 17.

example, explain both why price-cost margins appear to vary over the business cycle and why trade balances respond so slowly to exchange rate changes.⁴

Several authors have used two-period models to analyze markets with switching costs.⁵ In the second period of such models firms' power over their existing customers leads to high prices and in the first period, therefore, firms typically set lower prices than if there were no switching costs, in order to capture market share that will be valuable in the future. Such models do not, however, tell us what to expect from competition over many periods when "old" locked-in customers and "new" uncommitted customers are intermingled and firms cannot discriminate between these groups of customers. Will firms' temptation to exploit their current customer bases lead to higher prices than in the absence of switching costs, or will firms' desire to attract new customers lead to lower prices? Similarly, two-period models cannot easily address whether consumer switching costs can reinforce a dominant firm's position, or protect an incumbent from new entry, when a flow of new uncommitted customers is arriving and replacing a fraction of old consumers in the market. Furthermore, two-period models may not be the most satisfactory ones for analyzing the effects of business-cycle fluctuations, exchange-rate changes, or other shocks, because of the special features of both the first and second periods of these models. This paper, therefore, develops and analyzes an infinite-horizon model of competition in a market with switching costs in which in every period new customers arrive and a fraction of old consumers leaves the market.

von Weizsäcker (1984) built a model of switching costs in continuous time but restricted firms to constant-price strategies and so abstracted from many of our interests.⁶ Farrell and Shapiro (1988) were the first to analyze an infinite-period model in which firms compete in spot prices in each period, but their model incorporates a number of unusual features. First, they assume consumers are myopic, that is, that every consumer always buys from the firm offering the lowest price in the current period, without regard to the future. In the presence of switching costs, however, rational consumers consider expected future prices when making today's purchase decision, since today's purchase decision locks the customer in to repeat purchasing from the same firm in the future. Second, in their equilibrium firms set prices sequentially, with the firms taking turns to be the first mover (i.e. each duopolist sets price first in every second period).⁷ Third, and perhaps most important, their model generates the extreme result that in each period one firm sells to all the repeat purchasers and no new customers, while the other has no repeat business but sells to all the new customers. Firms take turns in playing these two roles, and each firm always has

⁴ See Froot and Klemperer (1989), Klemperer (1990).

⁵ See Banerjee and Summers (1987), Beggs (1989), Caminal and Matutes (1990), Froot and Klemperer (1989), Klemperer (1987a, b, c), and Padilla (1992). A very early finite-period model of oligopolistic competition in the presence of "demand inertia" is in the classic paper of Selten (1965).

⁶ The working paper version of this paper sketches the extension of our (discrete time) model to continuous time and shows that our model's main results are unaffected.

⁷ This alternating price leadership emerges endogenously in Farrell and Shapiro's model.

a fifty percent share of the market.⁸ Our model overcomes these deficiencies. It makes more standard assumptions, and allows us to treat issues that the earlier models cannot satisfactorily address.

Section 2 presents the model, Section 3 solves for the equilibrium, and Section 4 examines the nature of the equilibrium. We show that prices and profits are higher than in a market without switching costs and explain that while it is possible to construct models, such as von Weizsäcker's, in which the opposite is true, there are several reasons for believing that our result is the more general one. We also show that prices rise as firms discount the future more, fall as consumers discount the future more, fall as the turnover of consumers increases, and fall as the rate of growth of the market increases.⁹

We show that the higher profits in a market with switching costs mean that entry into the market may be more attractive than in their absence, even though an entrant must overcome the disadvantage that a fraction of the market is locked into the incumbent's product. After new entry or any shock, market shares converge monotonically to stable steady-state. If firms have equal costs the convergence is to equal shares, so the flow of new consumers into a market with switching costs results in the decreasing dominance of the initially larger firm.¹⁰

We believe that our model is a natural one in which to analyze other issues such as the effects of business cycles or of macroeconomic shocks in a market with switching costs, but we leave these extensions to future work.¹¹

Although our model formally represents a market with switching costs, similar results may also apply to markets in which a firm's future profitability depends on its current market share for other reasons. Such a dependence could arise not only from switching costs but also from consumers' search costs in discovering the existence of competing products,¹² from advertising generated by past sales, network externalities,¹³ or even "irrational" brand loyalty.

⁸ Farrell and Shapiro also consider a model with growing demand in which each period the number of new purchasers is larger by a factor γ than in the previous period, and in which the market shares are always $1/(1+\gamma)$ and $\gamma/(1+\gamma)$, and a model with fixed costs which sometimes yields complete monopoly. In an appendix they also set out a model in continuous time but their analysis of it makes non-primitive assumptions about value functions that are not satisfied by our model.

⁹ Farrell and Shapiro's model cannot address the second and third of these four questions and yields a less natural answer to the last (see note 32). von Weizsäcker does not directly address any of these questions.

¹⁰ Farrell and Shapiro's model, also, suggests switching costs facilitate entry, but their model cannot address how market shares evolve (see Section 4.2 and note 22). von Weizsäcker's model does not address new entry.

¹¹ Section 5.3 of the working paper version of our paper and Klempeter (1990) report early results about these issues. von Weizsäcker's model is unsuitable for studying them because it restricts firms to constant prices over time. Farrell and Shapiro's model results in firms always having the same market shares unless one firm completely drives the other out. (Our model, by contrast, results in market shares and prices varying smoothly in the model parameters.)

¹² See Phelps and Winter (1970), Sutton (1980).

¹³ See Katz and Shapiro (1985).

2. THE MODEL

Two firms A and B produce, at constant marginal costs c_A and c_B respectively, a good which cannot be stored. In each of infinitely many discrete periods every consumer has inelastic demand for a single unit of the product. In each period a cohort of ν "new" consumers enters the market. These consumers' tastes for different varieties of the product can be represented as uniformly distributed along a line segment $[0, 1]$, with the firms A and B at 0 and 1 respectively. A new consumer at y has a "transport" cost of ty of using A 's product or $t(1-y)$ of using B 's product in the period he arrives, and values consuming the product in this period at r less his transport cost (and net of any start-up cost of initially using the product). In each period, every "old" consumer (i.e. consumer who has previously purchased) values the product purchased at R and has no transport costs.¹⁴ We assume that it is too costly for a consumer to switch to buying from a firm other than the one from which he has previously bought.¹⁵ For simplicity, we assume that a new consumer who fails to buy must leave the market, but that this does not apply to old consumers.¹⁶ After each period a fraction $1-\rho$ of both new and old consumers (including any old consumers who failed to purchase) leaves the market ("dies"), where an individual's likelihood of dying is independent of his past history.

Firms and consumers have rational expectations and discount future revenues and costs using discount factors $\delta_F < 1$ and $\delta_C < 1$ per period respectively. We will be most interested in the case in which $\delta_F = \delta_C$, but our formulation allows us to show the effect of consumer expectations within the model. In particular, the case $\delta_C = 0$ represents myopic consumer decision-making.

Let the current stock of old consumers be S . We will be interested in the steady state where $\nu = (1-\rho)S/\rho$.

In each period each firm noncooperatively and simultaneously chooses a price to maximize its total expected future discounted profit.¹⁷ We write firm i 's price and profits in a given period as p_i and π_i . We will be interested in Markov perfect (closed loop) equilibria, i.e. equilibria in which each firm's strategy depends only on the state, that is the two firms' stocks of old consumers, x_A and x_B , and not otherwise on history.¹⁸

¹⁴ We discuss the case in which old as well as new customers have transport costs in Section 4.2.

¹⁵ We could explicitly include a start-up cost K of trying a brand and a cost K' of switching to the other brand (most naturally $K' \leq K$) and all that follows would hold for sufficiently large K, K' .

¹⁶ The solution we find remains valid if new consumers are allowed to wait. The lower bound for r is then in general higher than the one found in the Appendix as it is harder to guarantee that new consumers always wish to buy, though this can still be done by making r sufficiently large. The analysis is otherwise unaffected.

¹⁷ All a firm's customers pay the same price. Thus this model does not apply directly to contractually created switching costs (repeat-purchase discounts, frequent-flyer programs, etc.) in which firms receive lower net prices from repeat customers than from first-time buyers. We amend the model to analyze this case in Section 5.1 of the working paper version of our paper and show that when firms and consumers have a common discount factor this amendment makes no difference to firms' profits, the evolution of market shares, or the effective prices that consumers pay.

¹⁸ We thus rule out the kind of punishment strategies first studied by Abreu (1988), which might allow more collusive equilibria to be supported.

3. SOLVING THE MODEL

Our method of solution is constructive. We look for an equilibrium in which consumers always buy, so $x_A + x_B = S$, firms' strategies, $(p_A(x_A), p_B(x_B))$, are affine, that is

$$(1) \quad p_i(x) = d_i + e_i x,$$

and old consumers' value functions (the sums of their expected discounted utilities), $W_i(x_i)$, are also affine. Note that, since $x_A = S - x_B$, the state-space is one-dimensional and we could write all these expressions as functions of a single variable, but the current form keeps our equations symmetric.

We derive certain necessary conditions for an equilibrium to be of this kind and show that there is only one possible solution. In order for the solution we have identified to be an equilibrium, certain other conditions, such as sales always being nonnegative, must be satisfied, and we show that for a range of the parameters this is indeed the case.

The marginal consumer, who receives the same payoff from buying from either firm, is in equilibrium located at a distance $z_i(x)$ from i , which depends both on current prices and expected future values to consumers. If W_A and W_B are affine it follows that $z_i(x)$ is affine and that firm i 's stock of old consumers next period, $f_i(x)$, is determined by

$$(2) \quad f_i(x) = \eta_i + \mu x.$$

Since value functions are the sum of current payoffs and expected future payoffs, this in turn implies that W_A and W_B are indeed affine and firms' value functions, V_A and V_B , are quadratic, and that these functions are uniquely determined by $p_A(\cdot)$ and $p_B(\cdot)$.

A condition for firms' optimal behavior can be obtained by calculating $Z_i(p_i, p_j, x)$, the location of the marginal consumer when firms charge p_i and p_j in the current period but are expected to follow equilibrium strategies thereafter. The affineness of W_A and W_B implies that Z_i is affine and this plus the fact that V_i is quadratic implies, using the principle of optimality, that the first-order conditions for firms' optimal behavior are affine in x . This in turn implies that firms' (current-period) optimal strategies $p_A(\cdot)$ and $p_B(\cdot)$ are indeed affine, and are uniquely determined by $V_A, V_B, W_A,$ and W_B .

Combining the arguments of the last two paragraphs, we can therefore solve for $p_A(\cdot)$ and $p_B(\cdot)$ by the method of undetermined coefficients. The necessary algebra is performed in the Appendix, as is the verification of the other sufficient conditions mentioned above. We thus obtain the following theorem:

THEOREM 1: *If $\rho \leq 4/7$ and $|c_A - c_B| \leq \bar{c}(\rho)$, then there is a range of R and r for which there exists a Markov perfect equilibrium in which firms' equilibrium strategies are of the form (1).*

This is the unique equilibrium in which agents pursue affine strategies and all consumers buy in equilibrium.

The Appendix contains expressions for firms' equilibrium prices and market shares as functions of the state and the parameters of the model. (Note that they are independent of R and r .)

The bound on the permissible cost difference, $\bar{c}(\rho)$, is continuous and decreasing in ρ . As ρ tends to zero, $\bar{c}(\rho)$ tends to $3t$, the cost difference up to which both firms have positive sales in a standard model where all consumers are new and so have no switching costs. At $\rho = 1/3$, $\bar{c}(\rho)$ is greater than $1.5t$, and at $\rho = 1/2$ it is greater than $0.5t$.¹⁹ When firms' costs are very different (large $|c_A - c_B|$) no equilibrium in affine strategies exists, because for some range of x one firm would prefer to set price R to sell to only its old customers and leave all the new consumers to its rival, given the rival's conjectured strategy. With either a very small flow of new consumers (large ρ) or too high consumer reservation prices (large R) we conjecture that an equilibrium involves mixed strategies. This is because a firm whose rival was charging a relatively low price would set the (high) price R to fully exploit its own old customers, but a firm whose rival was charging a high price at or close to R would undercut its rival slightly to attract the new consumers without giving up too much revenue on its old customers. On the other hand, if consumers' reservation prices are too low some consumers will not buy so we no longer have $x_A + x_B = S$ and our solution is not valid.

We have not shown that there are not other Markov perfect equilibria which do not have an affine form. However if ρ and $|c_A - c_B|$ are sufficiently small, the unique equilibria of the finite-period versions of this model are of this form and converge to the equilibrium of our model as the number of periods becomes large.

For the remainder of this paper we assume the parameters are such that an equilibrium in affine strategies exists, and consider only this (unique) equilibrium.

4. THE NATURE OF COMPETITION

4.1. *The Evolution of Market Share*

PROPOSITION 1:²⁰ *Firm i 's period- T market share $\sigma_{i,T}$ evolves as*

$$(3) \quad \sigma_{i,T} = \sigma_i + \mu^T(\sigma_{i,0} - \sigma_i)$$

in which σ_i is i 's steady-state market share and $\mu \in (0.26\rho, 0.36\rho)$.

If firms have equal costs, their market shares converge to equality from any starting point ($\mu < 1$), so the dominance of the initially larger firm decreases.

¹⁹ By restricting δ_C and δ_F one can improve the bounds for ρ and $|c_A - c_B|$, but the improvement is slight. Imposing $r = R$ is possible but requires tighter conditions on ρ and $|c_A - c_B|$. We do not wish to restrict ourselves to $r = R$, since first-time users may obtain higher utility from the product but on the other hand may have to pay a start-up cost.

²⁰ Proofs of all propositions can be found in the Appendix.

The intuition is that the larger firm gains relatively more from charging a high price to exploit its current customer base than from charging a low price to gain a greater share of the new consumers. Thus the larger firm charges the higher price, sells to fewer new consumers and so loses market share to its smaller rival.²¹

More generally, even with unequal costs, a firm with more than its steady-state share of old customers sells at a higher price-cost margin and wins fewer new consumers than in steady state, thus losing market share. However it sells to more than its steady-state share of consumers in all, so convergence to the steady state is monotonic ($\mu > 0$).²² Note that convergence is also very rapid ($\mu \approx \rho/3$).

4.2. Comparison with Price Levels in the Absence of Switching Costs

In this and the following subsection we examine how switching costs affect prices and profits. We compare the latter to those in a market without switching costs, by which we mean a market of the same size in which all consumers are new and so have no switching costs.²³

PROPOSITION 2: *In symmetric steady-state equilibrium, firms' common price exceeds the price in a market without switching costs.*

We now discuss several reasons for believing that the result that a market with switching costs has a higher price than a market without is a fairly general one.²⁴

The naive argument why prices will be higher in the presence of switching costs is that firms will exploit their monopoly power over their locked-in customers. If customers and firms were myopic ($\delta_C = \delta_F = 0$), this would be the only force at work and prices would be $c + (t/(1 - \rho))$. If however firms care about the future then they will compete more fiercely for new customers, since

²¹ Both joint profits and the sum of the firms' value functions are maximized at market shares of 0 and 1, so this result contrasts with Budd, Harris, and Vickers (1989), in whose model one firm becomes increasingly dominant when joint flow profits are maximized on the boundary. The reasons are straightforward. First, in their model, unlike ours, profits are additively separable from the costs of attracting customers. Second, we explicitly model consumer behavior and consumers prefer, *ceteris paribus*, to be attached to the smaller firm (it is expected to offer lower prices in the future). Third, in our model dominance is further eroded by the turnover of customers.

²² This result seems natural but contrasts with Farrell and Shapiro's model in which firms' shares of new and of old customers alternate between zero and one, because in their model consumers live only two periods.

²³ Note that r and R do not affect prices (within the range in which our equilibrium is valid), so that it is unimportant that we are comparing our model, in which old consumers have reservation price R , with a model in which all consumers have reservation price r : our results hold for all r and R for which our equilibrium is valid. Likewise, we explain below that the results in our propositions remain true if we give old as well as new consumers "transport costs."

²⁴ The result of the proposition remains true for asymmetric costs unless cost differences are very large. Always, the lower-cost firm's steady-state price and profits, and industry steady-state profits, are higher than in the absence of switching costs.

these customers will become valuable repeat purchasers in the future.²⁵ Suppose initially, therefore, that firms have to commit themselves once and for all to a single price (and customers are myopic). It is easy to check that the symmetric equilibrium price will be $c + t((1 - \rho\delta_F)/(1 - \rho))$. This exceeds $c + t$, the price without switching costs. Thus the fact that firms discount the future is sufficient to imply that the desire to exploit old customers outweighs the desire to attract new ones.

This conclusion is reinforced when firms cannot commit themselves to a single price. First, each firm will be less tempted to cut price since it will realize that a smaller market share for its rival today means that the rival will be more aggressive tomorrow. Second, if customers are rational they will recognize that a firm that cuts price today will charge higher prices in the future. This reduces consumers' responsiveness to price cuts and lowers still further the incentive to cut price.

In fact, it is arguable that even this underestimates the extent to which prices exceed those in a market without switching costs. For we have compared our model in which old consumers have no "transport costs" to a model in which all consumers have "transport costs."²⁶ If, in our model of switching costs, all consumers have transport costs in every period and each consumer remains at the same position on the line in every period (unchanged tastes), then a new consumer takes into account the entire expected discounted sum of the transport costs he will pay and acts as if he had a one-time transport cost

$$t(1 + \rho\delta_C + (\rho\delta_C)^2 + \dots) = \frac{t}{1 - \rho\delta_C}.$$

This multiplies equilibrium prices by $1/(1 - \rho\delta_C)$.²⁷

This last result reflects the fact that because switching costs lock consumers into buying from a firm for more than one period, the importance of the current values of economic variables in consumers' decision-making depends on the extent to which these values are likely to persist. Thus consumers' choices between firms are relatively insensitive to current prices, and so extremely high equilibrium prices obtain, when tastes for real product differentiation are constant but firms cannot commit themselves to hold prices constant (so differences in prices will not persist).

At the other extreme, if we return to our assumption that consumers pay transport costs only when new (that is tastes for real product differentiation are

²⁵ In the two-period models of Klemperer (1987a, b) this is sufficient to imply that prices are generally lower in the first period than in the absence of switching costs.

²⁶ However, our model gives the same equilibrium prices as one in which all consumers have transport costs, but consumers' future positions on the line are independent of their current positions and, in expectation, equidistant between the two firms (because then future transport costs have no effect on consumers' current preferences between the two firms, so whether or not old consumers have transport costs has no effect on prices). (We cannot easily compare our model to a model without switching costs in which some consumers have no transport costs, as the latter has no pure strategy equilibrium.)

²⁷ When old consumers have transport costs, the values of r and R for which the equilibrium is of the form we have examined are slightly altered.

transient) but assume that each firm chooses a price that cannot be altered in the future (that is prices are persistent), then consumers pay more attention to current prices and less to product differentiation and so equilibrium prices fall (to $c + t((1 - \rho\delta_F)/(1 - \rho))(1 - \rho\delta_C)$). Thus in this extreme case, the market may be either more or less competitive than in the absence of switching costs. However, making any one or more of the changes (a) giving consumers transport costs in every period, or (b) looking for a closed loop equilibrium (in which firms cannot precommit themselves to future prices), or (c) making consumers myopic, gives the unambiguous result that the market with switching costs has the higher prices. (In fact, in our model, when firms have the same costs, even a new entrant with no customer base to exploit charges a higher price than in the absence of switching costs.) Thus our analysis suggests a strong presumption that prices are higher in the presence of switching costs than in their absence.

This discussion explains why von Weizsäcker (1984) found that switching costs are likely to lower prices: he assumed both that firms commit themselves not to alter their prices in the future and that consumers' tastes for real product differentiation change over time.

One advantage of our model is that it can address whether the existence of switching costs tends to raise or lower prices when firms must balance the temptation to exploit old customers with the desire to attract new ones.²⁸ A drawback of the steady-state approach is that firms always start with an "installed base." If we add a "first period" in which neither firm has any old customers, pricing is more aggressive in this first period than in a market without switching costs, as is typically found in two-period models (e.g., Klemperer (1987a, b)). However, even in this case, firms' present discounted profits remain higher than in the absence of switching costs.^{29,30}

4.3. New Entry

PROPOSITION 3: *If firms have the same marginal costs, then each firm's value function is greater than in the absence of switching costs, even when it has no customer base.*

²⁸ Farrell and Shapiro's model would yield price competition with undifferentiated products, hence zero profits, in the absence of switching costs, so switching costs cannot do other than raise prices.

²⁹ We cannot solve our model easily if we simply add on an initial period with ν new consumers and no old ones, because the number of consumers in the market then increases only gradually to its steady-state level, $\nu + S$. However if we add a first period with $\nu + S$ new consumers and no old ones, the continuation in every subsequent period will be determined by our steady-state solution, so it is routine to check that with equal costs the price in this first period is less than in a market without switching costs with either myopic or rational ($\delta_C = \delta_F$) consumer expectations. It remains true, for the reasons discussed above, that firms' present discounted profits exceed those in the corresponding market without switching costs.

³⁰ The assumption that the proportion of new customers is always the same is also responsible for the result in our model that an incumbent's price falls monotonically towards that of a new entrant. In Klemperer (1989) the incumbent and entrants both charge lower prices in the entry period and subsequently raise price, because in that model the new entrants serve a pool of consumers with low reservation prices who had not been served by the incumbent, so that there is a very large proportion of new consumers in the entry period.

It follows that if A were a monopolist and B were a potential entrant with equal marginal costs but facing a fixed cost of entry, B would enter in the face of larger fixed costs than would be the case in a market without switching costs.

The intuition is simply that although a new entrant is disadvantaged by the fact that a fraction of the market is locked into the incumbent's product, this deterrent to entry is outweighed (at least for survival rates such that our equilibrium is valid) by the higher markups in the presence of switching costs.

The conclusion of the Proposition remains true if firms' costs are different, unless cost differences are very large, so the presence of switching costs generally facilitates new entry of either lower- or higher-cost firms.³¹

4.4. Comparative Statics

PROPOSITION 4: (i) *Each firm's price increases in its own cost, in its rival's cost, and in its market share.*

(ii) *In symmetric, steady-state equilibrium, firms' common price increases in consumers' repeat-purchase rate, decreases in firms' discount factor, increases in consumers' discount factor and, when firms and consumers have a common discount factor, decreases in the common discount factor.*

Appendix C gives expressions for firms' prices.

Firms' prices increase in both firms' costs for the same reasons as if there were no switching costs, while the result that a firm's price increases in its market share reflects a larger firm's greater incentive to exploit its old customer base.

The consumer repeat-purchase rate, ρ , is also the fraction of consumers who have switching costs in any period, so it follows naturally from the discussion in Section 4.2 that prices increase in the consumer repeat-purchase rate.

Price decreases in the firms' discount factor, δ_F , because market share becomes more valuable to firms as they care more about the future and so they compete more fiercely for it. (This effect is only partly mitigated by firms knowing that fiercer behavior today results in a fiercer rival tomorrow.)

We also saw in Section 4.2 that price increases in the consumers' discount factor, δ_C , since customers who value the future more are less sensitive to price cuts.

When firms' and consumers' discount factors are held equal, the effect of the former on price dominates that of the latter. Thus an increase in the discount factor lowers prices both in the natural model with rational expectations (holding $\delta_F = \delta_C$) and with myopic expectations (holding $\delta_C = 0$).

Beggs and Klemperer (1989) give additional comparative statics results for prices, profits, and market share when firms have unequal costs. With minor exceptions the results are as for the symmetric steady-state case.

³¹ Proposition 3 also holds when old as well as new consumers have transport costs (see Section 4.2).

4.5. Pricing in a Growing Market

In this section we ask how competition in the presence of switching costs is affected by steady growth in the market size. We find that although the form of the equilibrium is similar, prices and possibly even discounted profits are lower than in a static market.

We assume that in every period the number of consumers is larger than in the previous period by a factor γ . We again look for Markov perfect equilibria. Now however the natural choice of state variable is a firm's current share of old customers, rather than its absolute number of old customers. Provided $\gamma\delta_F$ is less than unity, so value functions are bounded, we obtain the following proposition:

PROPOSITION 5: *If $\rho/\gamma \leq 4/7$, $|c_A - c_B| \leq \bar{c}(\rho/\gamma)$, and $\gamma\delta_F < 1$, then there is a range of R and r for which there is a unique Markov perfect equilibrium in which agents' equilibrium strategies are affine and consumers always buy in equilibrium.*

Firms' equilibrium prices are the same functions of their current shares of old customers as would apply in Theorem 1 if there were no growth but the survival rate were ρ/γ and discount factors were $\gamma\delta_C$ and $\gamma\delta_F$.

To understand this, first consider the problem in which the survival rate, ρ , is also multiplied by γ . In this case the ratio of new to old consumers is unchanged, so for any given current sales, the numbers both of old consumers attached to each firm next period and of new consumers next period would be γ times the numbers in the original static model. For given consumer demand, therefore, we have simply multiplied the value of each successive period by γ , so the effect on firms is exactly as if we had multiplied their discount factors by γ and left the survival rate unchanged. For consumers, given this behavior, the fact that their probability of reaching each successive period is multiplied by γ is equivalent to multiplying their discount factor by γ and leaving the survival rate unchanged. Thus the solution to the original static problem when both discount factors are multiplied by γ is the solution to the problem in which both the survival rate and the growth rate is multiplied by γ .

The solution to the problem in which only the growth rate is multiplied by γ is, of course, the solution to the above problem with the survival rate divided by γ , hence our result.

PROPOSITION 6: *In symmetric steady-state equilibrium, prices are decreasing in the rate of growth of the market, but are always higher than in a market without switching costs.*

There are two reasons why prices are decreasing in the rate of growth: first, more rapid growth reduces the proportion of locked-in old consumers and so lowers prices. Second, the increased relative importance of the future means

that firms would compete more vigorously for future profits, which also lowers prices.³² In fact, more rapid growth may reduce prices so much that both current-period profits and expected total discounted profits may be lower in a more rapidly growing market (holding constant the number of repeat-purchasers in the current period).

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APPENDIX

A. PROOF OF THEOREM 1

Let

$$V_i(x) = k_i + l_i x + m_i x^2$$

and

$$W_i(x) = g_i + h_i x.$$

Let the new consumer who is just indifferent between purchasing from A and B be located at a distance $z_i(x)$ from i . Then

$$(A1) \quad f_i(x) = \eta_i + \mu x = (1 - \rho)S z_i(x) + \rho x.$$

Equating the payoffs to the marginal consumer from buying from the two firms, we have

$$\begin{aligned} R - t z_i(x) - p_i(x) + \rho \delta_C W_i(f_i(x)) \\ = R - t(1 - z_i(S - x)) - p_j(S - x) + \rho \delta_C W_j(S - f_i(x)). \end{aligned}$$

Using this and (A1) we can solve for $z_i(x)$. Substituting back in (A1) yields

$$(A2a) \quad \eta_i = \frac{(1 - \rho)S(t - (d_i - d_j) + e_j S + \rho \delta_C(g_i - g_j - h_j S))}{2t - \rho(1 - \rho)\delta_C S(h_A + h_B)},$$

$$(A2b) \quad \mu = \frac{2\rho t - (1 - \rho)S(e_A + e_B)}{2t - \rho(1 - \rho)\delta_C S(h_A + h_B)}.$$

(Additional algebraic details for this Appendix can be found in Beggs and Klempere (1989): we can show that none of our expressions involves dividing by zero.)

Using (1) and (2)

$$W_i(x) = R - d_i - e_i x + \rho \delta_C W_i(\eta_i + \mu x)$$

holds for all x so we can equate the coefficients on x and equate the constant terms to obtain

$$(A3a) \quad h_i = -e_i + \rho \delta_C h_i \mu,$$

$$(A3b) \quad g_i = R - d_i + \rho \delta_C (g_i + h_i \eta_i).$$

³² The second effect would arise even if the proportion of old consumers were held constant as the growth rate increased (provided $\delta_C \leq \delta_F$). Farrell and Shapiro obtain the opposite result, that prices are higher in a growing market. Their result is due to the timing of moves in their game; they do not consider it intuitive.

We can solve these to obtain h_i and g_i as functions of $d_A, d_B, e_A,$ and e_B and the parameters $R, \rho, \delta_C, S,$ and t . (Determining h_A and h_B in terms of e_A and e_B involves solving a quadratic equation—we can show that only the negative root is relevant.)

Using (1) and (2) again (and noting that i 's total current sales are $(\eta_i + \mu x)/\rho$ in equilibrium),

$$V_i(x) = (d_i + e_i x - c_i)(\eta_i + \mu x)/\rho + \delta_F V_i(\eta_i + \mu x).$$

So we can equate coefficients to obtain

$$(A4a) \quad m_i = e_i \mu / \rho + \delta_F m_i \mu^2,$$

$$(A4b) \quad l_i = (d_i - c_i) \mu / \rho + e_i \eta_i / \rho + \delta_F l_i \mu + 2 \delta_F m_i \mu,$$

$$(A4c) \quad k_i = (d_i - c_i) \eta_i / \rho + \delta_F (k_i + l_i \eta_i + m_i \eta_i^2).$$

We can solve these for each of $k_i, l_i,$ and m_i in terms of $d_A, d_B, e_A, e_B,$ and the parameters of the problem.

Finally, optimal behavior by firm i requires

$$(A5) \quad p_i(x) \in \operatorname{argmax}_{p_i} \left\{ (p_i - c_i) \left(x + ((1 - \rho) / \rho) S Z_i(p_i, p_j, x) \right) + \delta_F V_i(\rho x + (1 - \rho) S Z_i(p_i, p_j, x)) \right\}$$

in which the marginal new consumer's distance from $i, Z_i(p_i, p_j, x),$ satisfies

$$(A6) \quad -p_i + \rho \delta_C W_i((1 - \rho) S Z_i(p_i, p_j, x) + \rho x) - t Z_i(p_i, p_j, x) \\ = -p_j + \rho \delta_C W_j(S - ((1 - \rho) S Z_i(p_i, p_j, x) + \rho x)) - t(1 - Z_i(p_i, p_j, x)).$$

Differentiating (A5) with respect to p_i we obtain the first-order conditions for i 's equilibrium strategy and can remove their dependence on p_j by noting that in equilibrium $Z_i(p_i, p_j, x) = z_i(x),$ which latter function is defined by (A1) and (A2). Since the first-order conditions hold for all $x,$ we can equate the constant term and the coefficient with those in (1) to obtain

$$(A7a) \quad e_i = \frac{2t^* \mu}{S(1 - \rho)} - 2\rho \delta_F m_i \mu,$$

$$(A7b) \quad d_i = c_i + \frac{2t^* \eta_i}{S(1 - \rho)} - 2\rho \delta_F m_i \eta_i - \rho \delta_F l_i,$$

in which $t^* = t - [\rho(1 - \rho)\delta_C S(h_A + h_B)]/2.$

The right-hand sides of (A7a) and (A7b) are known functions of $d_A, d_B, e_A,$ and $e_B,$ and the parameters of the problem, so these four simultaneous equations can be solved to determine these four variables and hence all the other unknowns.

In fact, the problem can be reduced to solving a single equation: eliminating m_i from (A7a) yields a linear equation for e_i in terms of t^* and $\mu,$ so, since exactly the same equation holds for A and $B,$ we have $e_A = e_B = e,$ say. Furthermore, eliminating $h_A + h_B$ from (A2b) gives an equation for μ in terms of $e,$ so we can now eliminate μ (and also $h_A + h_B$ and t^*) from (A7a) to obtain an equation for e that is independent of d_A and $d_B.$ (It follows that $e,$ hence also $\mu,$ hence (from (A7a)) also $m_A = m_B = m$ and therefore also $h_A = h_B = h$ are independent of the firms' costs.)

Having solved the equation for $e,$ we can straightforwardly solve (A7b) for d_A and $d_B,$ and hence all the other variables. It remains, therefore, to show that the equation for e does indeed have a solution. It is easiest to examine the equivalent equation for $\mu.$ Using (A4a) we can solve for m in terms of e and $\mu,$ and substituting in (A7a) solve for e in terms of μ and $t^*.$ Substituting for e in (A2b) we obtain an equation linking μ and $t^*:$

$$\mu \left[\frac{3 - \delta_F \mu^2}{1 + \delta_F \mu^2} \right] = \frac{\rho t}{t^*}.$$

Using (A3a) we can solve for e in terms of $h.$ Substituting into (A2b) we can solve for h in terms of

μ . Substituting for h in the definition of t^* , we obtain

$$t^* = t(1 + (\rho - \mu)\rho\delta_C).$$

Hence, using this to substitute for t^* in the previous equation, we obtain

$$(A8) \quad \rho(\delta_F\mu^2 + 1) - \mu(\delta_F\mu^2 - 3)(\rho\delta_C\mu - (1 + \rho^2\delta_C)) = 0.$$

This is a quartic with four real roots in the intervals $(-\infty, -\sqrt{3/\delta_F})$, $(0, \rho)$, $(1, (1 + \rho^2\delta_C)/\rho\delta_C)$, and $((1 + \rho^2\delta_C)/\rho\delta_C, \infty)$, but we will show that the only admissible solution for μ is the root in the interval $(0, \rho)$.

Sufficient conditions for the above equations to be a solution to the model are that in every period (a) all new consumers prefer to purchase from either firm rather than not buy at all (it is not enough to guarantee that every consumer is always willing to buy from his preferred firm in equilibrium because our analysis assumes that if a firm raised its price then any new customers it lost would buy from the other firm), (b) all old consumers wish to purchase, (c) equation (A5) specifies a concave problem for each firm so that the first-order conditions are yielding maxima, (d) no firm would prefer to raise its price above the level specified by (A7) and serve only old consumers (the mathematics (incorrectly) assumed that at sufficiently high prices a firm sells to a negative number of new customers and so the profitability of very high prices was underestimated—we must therefore consider such strategies separately), (e) the equations do not require a firm to serve more than 100% of the new consumers (as noted above, this constraint was not explicitly included in the mathematics).

A condition that ensures (a) holds is $r \geq \max_{i=A,B} (d_i + eS + t - \rho\delta_C W_i(f_i(S)))$, that is, every new consumer would prefer to pay the highest price ever charged in equilibrium rather than not buy at all. Condition (b) requires $R \geq \max_{i=A,B} (d_i + eS)$. Straightforward algebra shows that (c) is always satisfied provided $-1 < \mu < 1$, i.e. the system is stable, which is in any case required for (e) (see below).

Condition (d) requires $(R - c_i)x + \delta_F V_i(\rho x) \leq V_i(x)$ for all x . We now show that this is certainly satisfied if $R \leq \max_{i=A,B} (d_i + eS)$, so long as (e) holds:

Without loss of generality, let $c_i \geq c_j$, so $d_i \geq d_j$. Given (e), i sells to a non-negative number of new customers if it sets a price $d_i + eS$ when it has S old customers and j charges d_j . It follows that i would sell to a non-negative number of new customers at any smaller share of customers if i charged $d_i + eS$ and j followed the strategies specified by (A7) (this follows from the equation for $Z_i(p_i, p_j, x)$ that we can derive from (A6)). Since j has lower costs, customers prefer, ceteris paribus, to buy from j , so j would sell to a nonnegative number of new customers if j deviated to $d_i + eS$. (Note that i 's equilibrium price is higher than j 's equilibrium price.) It follows that, since the previous analysis has evaluated the profitability of either firm charging any price up to $d_i + eS$ correctly, neither firm will deviate. (This argument proves only that there is a single value of R consistent with (b) and (d). A slightly harder argument shows that if $c_i = c_j$, (d) is certainly satisfied if $R \leq d_i + eS + t(4 - 7\rho)/(4 - 4\rho)$, so that an interval of values of R is consistent with (b) and (d). Numerical computations show that the range of R for which our solution is valid is generally much larger.)

The final condition to be checked is (e). An equivalent condition is that our solution specifies that each firm will always sell to a non-negative number of new customers, or that each firm has at least as many old customers next period as it would if it had sold to no new customers, i.e.

$$(A9) \quad \eta_i + \mu x \geq \rho x, \quad \forall x.$$

Now $\eta_i + \mu x + \eta_j + \mu(S - x) = S$. Therefore $\mu > 1$ would imply $\min\{\eta_i, \eta_j\} < 0$ so the conditions would fail at $x = 0$, while $\mu < -1$ would imply $\min\{\eta_i, \eta_j\} < -\mu S$ so the conditions would fail at $x = S$. Thus we require $|\mu| \leq 1$, so we must select the unique root for μ in the interval $(0, \rho)$. In the symmetric case $\eta_i = (1 - \mu)S/2$, so (A9) becomes $(1 - \mu)S/2 + \mu S \geq \rho S$ which is satisfied for all $\rho \leq 4/7$, since $\mu > 2\rho/(6 + 5\rho^2\delta_C)$ —see Appendix B. (It is hard to improve on this bound; for $\delta_C = \delta_F = 0$, (A9) is violated for $\rho > 0.6$.) Furthermore, since η_i is a linear function of costs (see Appendices B and C) condition (e) will hold so long as the two firms are not too asymmetric.

B. PROOF OF PROPOSITION 1

$\sigma_{i,T} = x_{i,T+1}/S$ (firms' current shares of old customers are their previous-period market shares), so $\sigma_{i,T} = \eta_i/S + \mu\sigma_{i,T-1}$, from (2), which implies (3), in which $\sigma_i = \eta_i/((1 - \mu)S)$.

Furthermore, from (A8),

$$\mu = \left((\mu^2 + 1/\delta_F) / (3/\delta_F - \mu^2) \right) \left(\rho / (1 + \rho^2\delta_C - \rho\delta_C\mu) \right)$$

so, for the root of interest ($\mu \in (0, \rho)$), we have $\mu > \rho / (3(1 + \rho^2\delta_C)) > \rho/6$. Substituting $\mu > \rho/6$ in the equality yields $\mu > 2\rho / (6 + 5\rho^2\delta_C)$, so for $\rho \leq 4/7$ we have $\mu > 0.26\rho$. Also, the equality yields $\mu < ((\mu^2 + 1)/(3 - \mu^2))\rho$, so $\rho \leq 4/7 \Rightarrow \mu < 0.36\rho$.

C. EXPRESSIONS FOR PRICES AND MARKET SHARES

Straightforward, but tedious, algebra yields

$$(A10) \quad p_i(x) = c_i + 2\sigma_i\omega + \left(\frac{x}{S} - \sigma_i \right) v$$

in which i 's steady-state market share is

$$\sigma_i = \frac{1}{2} + \frac{c_j - c_i}{4\omega + 2(1 - \rho\delta_C)t},$$

and

$$\omega = \frac{t}{1 - \rho} (1 + \rho^2\delta_C - \rho\mu\delta_C - (2\rho\delta_F / (3 - \delta_F\mu^2))) \approx \frac{t}{1 - \rho} \left(1 + \frac{2}{3}\rho^2\delta_C - \frac{2}{3}\rho\delta_F \right),$$

$$v = \frac{t}{1 - \rho} (2\rho(1 - \delta_F\mu^2) / (3 - \delta_F\mu^2)) \approx \frac{t}{1 - \rho} \left(\frac{2\rho}{3} \right),$$

where μ is the root of (A8) in $(0.26\rho, 0.36\rho)$.

D. PROOFS OF PROPOSITIONS 2-6

Propositions 2 and 4 are easily obtained from (A10) using our earlier result that μ is the root of (A8) in the interval $(0.26\rho, 0.36\rho)$. Proposition 3 uses straightforward but tedious algebra to show that when $c_i = c_j$,

$$V_i(0) = k_i > t \left(\frac{S}{2\rho} \right) \left(\frac{1}{1 - \delta_F} \right) \text{ for } \rho > 0.$$

(In the absence of switching costs each firm's price-cost margin is t and per-period sales are $(1/2)(S/\rho)$.) Propositions 5 and 6 follow easily using the arguments in Section 4.5 and the results of Proposition 4.

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