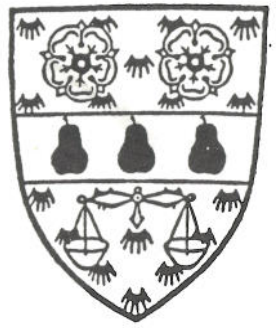
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MULTI-PERIOD COMPETITION WITH SWITCHING COSTS

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Multi–Period Competition with Switching Costs*

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Abstract

We analyse an infinite–period model of duopolistic competition in a market with consumer switching costs, in which in every period new consumers arrive and a fraction of old consumers leaves. We show that prices (and profits) are higher than in a market without switching costs, and that this result does not depend importantly on the specific assumptions of our model. We show that switching costs make the market more attractive to a new entrant, even though an entrant must overcome the disadvantage that a large fraction of the market is already committed to the incumbent's product. We examine the evolution of prices and of firms' market shares, and show how these are affected by differences between firms' costs, by interest rates, by the rate of turnover of consumers, and by growth in the size of the market. We also show how to use our model to examine some macroeconomic issues.

Keywords : Switching costs, Lock–in.

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1. Introduction

In many markets, *switching costs* give each consumer a strong incentive to continue buying from the firm from which he has previously purchased, even if other firms are selling functionally identical products. Examples of switching costs include the transactions cost of closing an account with one bank and opening another with a competitor, the learning cost incurred by switching to a new make of computer after having learnt to use another brand and the artificial switching costs created by frequent–flyer programs that reward customers for repeated travel on a single airline.[1]

Managers often seem more concerned with market shares than short–run profits. Switching costs explain why this may be rational behaviour : switching costs give firms a degree of monopoly power over their customers and so make current market shares an important determinant of future profits. Switching costs can thus help to explain phenomena such as the discounts firms offer to attract new purchasers, and the price wars that occur when there is new entry into a market.[2] Switching costs may, moreover, have important implications at the macroeconomic level. The value firms place on their market shares can, for example, explain both why price–cost margins appear to move counter–cyclically over the business cycle and why trade balances may respond so slowly to exchange rate changes.[3]

[1]Another kind of switching cost arises when uncertainty about product quality makes consumers reluctant to switch to untested products (see, for example, Schmalensee (1982)). Here however additional complications arise, due to the possibility of prices being used as signals of product quality and due to the existence of groups of customers who tried a brand and did not like it. We ignore these issues. For other examples of switching costs see Farrell and Shapiro (1989), Green and Scotchmer (1986), Klemperer (1987a) and the references they cite.

[2]For a discussion of new entry when there are switching costs see Klemperer (1989). See the discussion of two–period models below for an explanation of why firms offer discounts to new customers, and Klemperer (1987a) for further discussion and examples. The model we set out below applies when firms cannot offer discounts only to new customers (because old customers could successfully present themselves as 'new' or because the good could be resold).

[3]See Section 5.3 for discussion and references.

Several authors have used two–period models to analyse markets with switching costs.[4] In the second period of such models firms' power over their existing customers leads to high prices and in the first period, therefore, firms typically set lower prices than if there were no switching costs, in order to capture market share that will be valuable in the future. Such models do not, however, tell us what to expect from competition over many periods when 'old' locked–in customers and 'new' uncommitted customers are intermingled and firms cannot discriminate between these groups of customers. Will firms' temptation to exploit their current customer bases lead to higher prices than in the absence of switching costs, or will firms' desire to attract new customers lead to lower prices ? Similarly, two–period models cannot easily address whether consumer switching costs can reinforce a dominant firm's position, or protect an incumbent from new entry, when a flow of new uncommitted customers is arriving and replacing a fraction of old consumers in the market. Furthermore, two–period models may not be the most satisfactory ones for analysing the effects of business–cycle fluctuations, exchange–rate changes or other shocks, because of the special features of both the first and second periods of these models. This paper, therefore, analyses an infinite–horizon model of competition in a market with switching costs in which in every period new customers arrive and a fraction of old consumers leaves the market.

von Weizsäcker (1984) built a model of switching costs in continuous time but restricted firms to constant–price strategies and so abstracted from many of our interests. Farrell and Shapiro (1988) were the first to analyse a multi–period model in which firms compete in spot prices in each period, but their model incorporates a number of unusual features.

Section 2 presents our model and describes its advantages. Section 3 solves for the equilibrium.

[4]See Banerjee and Summers (1987), Beggs (1989), Caminal and Matutes (1989), Froot and Klemperer (1989) and Klemperer (1987a,b).

Section 4 examines the nature of the equilibrium. We show how market shares and prices evolve over time, both when firms have identical costs and when one firm has a cost advantage. We compare prices, profits and market shares to those in the absence of switching costs, and examine the effect of switching costs on the attractiveness of the market to a new entrant.

Section 5 pursues some extensions of our model. Section 5.1 distinguishes between the effects of switching costs that are real (social) costs (such as transaction costs) and the effects of artificial switching costs created by repeat–purchase discounts. Section 5.2 models a growing market, and Section 5.3 shows how our model can be applied to issues such as the variation of price–cost margins over the business cycle and the effects of anticipated changes (such as new taxes or exchange–rate changes) in markets with switching costs. We also sketch a continuous time version of our model (Section 5.4).

Although our model formally represents a market with switching costs, similar results may also apply to markets in which a firm's future profitability depends on its current market share for other reasons. Such a dependence could arise from consumers' search costs in discovering the existence or prices of competing products,[5] the fact that past sales advertise a firms' product, network externalities,[6] and even 'irrational' brand loyalty, in addition to switching costs.

Section 6 concludes.

2. The Model

Two firms A and B produce, at constant marginal costs c_A and c_B respectively, a good which cannot be stored. In each of infinitely many discrete periods every consumer has inelastic demand for a single unit of the product. In each period a cohort of ν 'new' customers enters the market. These consumers' tastes for different varieties

[5]See Phelps and Winter (1970).

[6]See Katz and Shapiro (1985).

of the product can be represented as being uniformly distributed along a line segment [0,1], with the firms A and B at 0 and 1 respectively. A new consumer at y has a 'transport' cost of ty of using A's product or t(1–y) of using B's product in the period he arrives, and values consuming the product in this period at r less his transport cost. In each period, every 'old' consumer (i.e. consumer who has previously purchased the product) values consuming the variety he has previously purchased at R.[7] It is assumed that once a consumer has bought a given brand it is too costly for him to switch to another. (It would be straightforward to include switching costs explicitly in the model and derive conditions for the size of switching costs such that it is never optimal for any consumer to switch or for any firm to induce any consumer to switch.[8]) For simplicity, we assume that an old consumer who fails to buy remains an old consumer next period, but a new consumer who fails to buy must leave the market.[9] After each period a fraction $1-\rho$ of both new and old consumers leaves the market ('dies'). In every period, every consumer who purchases has an equal likelihood of dying, irrespective of past history.

Firms and consumers have rational expectations and discount future revenues and costs using discount factors $\delta_F < 1$ and $\delta_C < 1$ per period respectively. We will be most interested in the case in which $\delta_F = \delta_C$, but note that our formulation allows us to show the effect of consumer expectations within the model. In particular, the case $\delta_C = 0$ corresponds to myopic consumer decision–making, which may be a more reasonable behavioural assumption than rational expectations.[10] Also, $\delta_C = 0$

[7]It would make very little difference to the model if old as well as new consumers had transport costs (see Section 4.2).

[8]In Farrell and Shapiro's (1988) multi–period model also, no consumer switches between firms in equilibrium, but in Klemperer's (1987b) two–period model some consumers do switch in the second period.

[9]Making different assumptions here would not affect the solution we find but would affect the range of parameters for which it is valid.

[10]It may be more reasonable, for example, to assume that when consumers see a firm charge a lower–than–expected price they do not change their expectations about future prices than to assume that they interpret the low price as foretelling a higher–than–originally–expected price (as would be the case in our model).

corresponds to a general model of competition for market share when current market share is a linear function of past market share and the difference in firms' prices (with rational expectations it evolves in the same way but the function of prices is endogenous to the model and depends on the consumers' discount factor).

Let the current stock of old consumers be S. We will be interested in the steady state where $\nu = (1-\rho)S/\rho$.

In each period each firm non–cooperatively and simultaneously chooses a price to maximise its total expected future discounted profit.[11] We write firm i's prices and its corresponding profits in a given period as p_i and π_i, and if its current stock of old consumers is x_i we write $V_i(x_i)$ for its expected discounted profits and $W_i(x_i)$ for the expected discounted utility of an 'old' consumer buying from it (i = A,B). Note that, since $x_A = S - x_B$, we could write all these expressions as functions of a single variable, but the current form keeps our equations symmetric.

Our model is similar to Farrell and Shapiro's (1988) model, but has the following important differences : (i) we do not restrict the consumers' discount factor to be zero, (ii) we incorporate some product differentiation between firms for new purchasers, (iii) we let firms choose prices simultaneously, (iv) we allow firms to have different marginal costs, but do not incorporate fixed costs, and (v) in our model, a randomly selected fraction of customers leaves the market each period, rather than each customer leaving the market after exactly two periods.

We believe that our assumptions are more natural and yield more plausible results : the first change from Farrell and Shapiro's model means we can consider rational consumer expectations, as well as showing within the model the effects of varying consumer expectations. Farrell and Shapiro by contrast assume that consumers

[11]All a firm's customers pay the same price. Thus this model does not apply directly to contractually created switching costs (repeat–purchase discounts, frequent–flyer programs etc.) in which firms receive lower net prices from repeat customers than from first–time buyers. We amend the model to analyse this case in Section 5.1 and show that when firms and consumers have a common discount factor this amendment makes no difference to firms' profits, the evolution of market shares or the effective prices that consumers pay.

are myopic, that is, that every consumer always buys from the firm offering the lowest price in the current period, without regard to the future. In the presence of switching costs, however, rational consumers consider expected future prices when making today's purchase decision, since today's purchase decision locks the customer in to repeat purchasing from the same firm in the future.[12]

The second change means that new consumers do not all buy from the same firm, so that firms' market shares can (and do) take any value in the interval (0,1). Farrell and Shapiro's assumption on the other hand (combined with their assumption that consumers live exactly two periods) generates the extreme result that in each period one firm sells to all the repeat purchasers and no new customers, while the other has no repeat business but sells to all the new customers. In their model, each firm always has a fifty per cent share of the market.[13]

Since the second change also smoothes firms' payoff functions, it also allows us to make the third change, which makes the model more plausible. Farrell and Shapiro's model has the unusual feature that firms set prices sequentially, with the firms taking turns to be the first mover (i.e. each firm sets price first in every second period).

The fourth change allows us to model asymmetric oligopoly. The fifth change allows us to vary the rate of consumer turnover. It also results in market shares evolving monotonically in equilibrium, rather than oscillating around their equilibrium values.

[12]Given the prices that firms set in the belief that consumers are myopic, consumers are in fact optimising in Farrell and Shapiro's model by considering only this period's price. However if firms believed that consumers were rational they might set different prices. In our model, equilibrium prices do depend on whether consumers are rational or myopic.

[13]Farrell and Shapiro also consider a model with growing demand in which each period the number of new purchasers is larger by a factor γ than in the previous period, and in which the market shares are always $1/(1+\gamma)$ and $\gamma/(1+\gamma)$, and a model with fixed costs which sometimes yields complete monopoly. In an appendix they also set out a model in continuous time but their analysis of it makes non–primitive assumptions about value functions that are not satisfied by our model.

3. Solving the Model

We look for perfect (closed–loop) Markov equilibria, i.e. equilibria in which each firm's strategy depends only on the state (its stock of old consumers), but not otherwise on history.[14]

Our method of solution is constructive. We conjecture that firms' equilibrium prices and consumers' value functions are linear in the state, that firms' value functions are quadratic in the state and that consumers always buy. This allows us to derive in Section 3.1 equations which must be satisfied by equilibria of this form. We then show in Section 3.2 that these have a unique admissible solution. In our derivation we neglect certain constraints, such as non–negative sales, which must be satisfied. In Section 3.3 we therefore check that for a range of parameters our solution is indeed a solution to the constrained problem.

With myopic consumers, our model reduces to a linear–quadratic game, plus some constraints, so it is natural to look for linear solutions. With rational expectations, the situation is less clear but an easy inductive argument shows that the only possible pure strategy equilibrium of a finite horizon model is of this form, so it is natural to look for such solutions in the infinite horizon case. Indeed, it can be shown that the unique pure strategy equilibria of the finite horizon versions of our model converge to the equilibrium of the infinite horizon game.[15]

[14]We thus rule out the kind of punishment strategies first studied by Abreu (1988), which might allow more collusive equilibria to be supported. Our game is not a repeated game so his results are not strictly applicable, nor does it fit the assumptions of the folk theorem for time–dependent supergames proved by Friedman (1988), but these results are certainly suggestive.

[15]The proof of this result consists of deriving the recurrence relation between the coefficients of the value functions of the finite horizon problems with T and T+1 stages to go and applying the Contraction Mapping theorem. The algebraic detail is somewhat tedious and is omitted. For both the finite and infinite horizon games, we need to restrict ρ and $|c_A - c_B|$ to ensure that a pure strategy equilibrium exists (see Section 3.3).

3.1 Equations determining the solution

Assume that in equilibrium if firm i enters a period with x old consumers then

$$p_i(x) = d_i + e_i x \tag{1}$$

Assume also that firm i has value function

$$V_i(x) = k_i + l_i x + m_i x^2 \tag{2}$$

and that an old consumer buying from firm i has value function

$$W_i(x) = g_i + h_i x \tag{3}$$

To find the equilibrium we need to determine the three sets of coefficients associated with the functions above. Solving the new consumers' optimization problem (using (1) and (3)) allows us to find the equation of motion of the system that determines next period's state (x) as a function of this period's state. Since the value functions must be the sum of current payoffs plus the discounted value function starting next period, we can then solve for each of the coefficients of firms' and consumers' value functions (k_i,l_i,m_i,g_i and h_i) in terms of the parameters of the price function (d_i and e_i). Finally, solving the firms' optimization problem yields equations for each of the d_i and e_i in terms of the other parameters and so enables us to solve for all the unknowns in (1) to (3).

Let $f_i(x)$ be i's stock of old consumers next period if its current stock is x. Since the total stock of old consumers is always S, $f_j(S-x) = S - f_i(x)$, $j \neq i$. Let the new consumer who is just indifferent between purchasing from A and B be located at a distance $z_i(x)$ from i. Then

$$f_i(x) = (1-\rho)Sz_i(x) + \rho x \tag{4}$$

Equating the payoffs from buying from the two firms and substituting for W_i using (3)

$$\begin{aligned} &- d_i - e_i x + \rho\delta_C[g_i + h_i f_i(x)] - tz_i \\ &= - d_j - e_j(S-x) + \rho\delta_C[g_j + h_j(S-f_i(x))] - t(1-z_i) \end{aligned} \tag{5}$$

Substituting for $z_i(x)$ using (4) and re–arranging yields

$$f_i(x) = \eta_i + \mu x \tag{6}$$

where
$$\eta_i = \frac{(1-\rho)S\ (t - (d_i-d_j) + e_jS + \rho\delta_C(g_i-g_j-h_jS))}{2t - \rho(1-\rho)\delta_C S(h_A+h_B)} \quad (6a)^{16}$$

$$\mu = \frac{2\rho t - (1-\rho)S(e_A+e_B)}{2t - \rho(1-\rho)\delta_C S(h_A+h_B)} \quad (6b)$$

This is a first–order linear difference equation for the evolution of the state (that is of firm i's stock of old consumers) and is stable if $|\mu| < 1$.

Using (1)

$$W_i(x) = R - d_i - e_i x + \rho\delta_C W_i(f_i(x)) \quad (7)$$

and since (7) holds for all x we can equate the coefficients on x, using (3) and (6), to obtain

$$h_i = -e_i + \rho\delta_C h_i \mu \quad (8a)$$

So substituting for μ from (6a) we can thus solve for h_i in terms of e_A and e_B, as well as the parameters ρ, δ_C, S and t.[17]

Similarly, equating the constant terms in (7), using (3) and (6), yields

$$g_i = R - d_i + \rho\delta_C(g_i + h_i\eta_i) \quad (8b)$$

So, substituting for η_i from (6b) and h_i from above, we can solve for g_i as a function of d_A, d_B, e_A and e_B and the parameters R, ρ, δ_C, S and t.

Since consumers' survival rate is ρ, it follows from (6) that if in any period firm i enters with x old consumers its total sales are $(\eta_i + \mu x)/\rho$ in equilibrium. Therefore,

$$V_i(x) = (d_i + e_i x - c_i)(\eta_i + \mu x)/\rho + \delta_F V_i(\eta_i + \mu x) \quad (9)$$

Using (2) we can equate coefficients to obtain

$$m_i = e_i\mu/\rho + \delta_F m_i \mu^2 \quad (10a)$$

$$l_i = (d_i - c_i)\mu/\rho + e_i\eta_i/\rho + \delta_F l_i \mu + 2\delta_F m_i \mu\eta_i \quad (10b)$$

$$k_i = (d_i - c_i)\eta_i/\rho + \delta_F(k_i + l_i\eta_i + m_i\eta_i^2) \quad (10c)$$

[16] We will show that none of our expressions involves dividing by zero.

[17] Determining h_A and h_B in terms of e_A and e_B involves solving a quadratic for h_A+h_B in terms of e_A+e_B with two roots, one positive and one negative. We will show that only the negative one is relevant.

which we can solve for each of k_i, l_i and m_i in terms of d_A, d_B, e_A, e_B and the parameters of the problem.

Finally, we obtain conditions for firms' optimal pricing. If firms charge p_i and p_j then the the marginal new consumer, who obtains the same expected payoff buying from either firm, is located at the distance from i, $Z_i(p_i,p_j,x)$, that satisfies

$$-p_i + \rho\delta_C W_i((1-\rho)SZ_i(p_i,p_i,x) + \rho x)) - tZ_i(p_i,p_j,x)$$
$$= -p_j + \rho\delta_C W_j(S - ((1-\rho)SZ_i(p_i,p_j,x) + \rho x)) - t(1-Z_i(p_i,p_j,x)) \quad (11)$$

Substituting for W_i using (3) and re–arranging,

$$Z_i(p_i,p_j,x) = (p_j - p_i)/2t^* + \varphi_i(x) \quad (12)$$

in which $t^* = t - [\rho(1-\rho)\delta_C S(h_A+h_B)]/2$ and $\varphi_i(x)$ is a linear function of x, and depends on g_A, g_B, h_A, h_B and the parameters of the problem. Thus, with rational expectations one can think of consumers reacting to price changes as if they faced the effective transport cost t^* rather than t. With naive expectations, $\delta_C = 0$ so $t^* = t$ and $\varphi_i = 1/2$, independent of x.

Optimal behaviour by firm i therefore requires

$$p_i(x) \in \underset{p_i}{\operatorname{argmax}}\{(p_i - c_i)(x + ((1-\rho)/\rho)SZ_i(p_i,p_j,x)) + \delta_F V_i(\rho x + (1-\rho)SZ_i(p_i,p_j,x))\} \quad (13)$$

Differentiating (13) with respect to p_i, using (12), we obtain the first–order conditions for i's equilibrium strategy

$$x + S((1-\rho)/\rho)\left(\frac{p_j-p_i}{2t^*} + \varphi_i(x)\right) - \frac{(p_i-c_i)}{2t^*}((1-\rho)/\rho)S + (-(1-\rho)/2t^*)S\delta_F(l_i + 2m_i(\rho x + (1-\rho)S(\frac{p_j-p_i}{2t^*} + \varphi_i(x)))) = 0 \quad (14)$$

We remove the dependence on p_j by noting that in equilibrium (from (4) and (6))

$$(1-\rho)S(\frac{p_j-p_i}{2t^*} + \varphi_i(x_i)) + \rho x = \eta_i + \mu x$$

Therefore, since this equation holds for all x, we can equate the constant term and the coefficient of x with those in (1) to obtain

$$e_i = \frac{2t^*\mu}{S(1-\rho)} - 2\rho\delta_F m_i \mu \quad (15a)$$

$$d_i = c_i + \frac{2t^*\eta_i}{S(1-\rho)} - 2\rho\delta_F m_i\eta_i - \rho\delta_F l_i \quad (15b)$$

3.2 Solving the Equations

The right–hand sides of (15a) and (15b) are known functions of d_A, d_B, e_A and e_B and the parameters of the problem, from our earlier equations, so these four simultaneous equations can be solved to determine these four variables and hence all the other unknowns.

In fact, the problem can be reduced to solving a single equation : using (10a) to eliminate m_i from (15a) yields a linear equation for e_i in terms of t^* and μ, so, since exactly the same equation holds for A and B, we have $e_A = e_B = e$, say. Furthermore, we can use (8a) to eliminate h_A+h_B (and hence t^*), and using (8a) again to eliminate h_A+h_B from (6) gives an equation for μ in terms of e; so we can now eliminate μ to obtain an equation for e that is independent of c_A and c_B. It follows that e, hence also μ, hence (from (15a)) also $m_A = m_B = m$ and (from (8a)) $h_A = h_B = h$ are independent of the firms' costs.

Having solved the equation for e, we can straightforwardly solve (15b) for d_A and d_B, and hence all the other variables. It remains, therefore, to show that the equation for e does indeed have a solution. It is easiest to examine an equivalent equation for μ (see Appendix A1 for the algebra)

$$\rho(\delta_F\mu^2+1) - \mu(\delta_F\mu^2-3)(\rho\delta_C\mu - (1+\rho^2\delta_C)) = 0 \qquad (16)$$

which is a quartic with four real roots in the intervals $(-\infty,-\sqrt{3/\delta_F})$, $(0,\rho)$, $(1,(1+\rho^2\delta_C)/\rho\delta_C)$, and $((1+\rho^2\delta_C)/\rho\delta_C,\infty)$, since it is easy to show that the left–hand side of (16) has opposite signs at the left and right end points of each interval.[18] We shall show in the next subsection, however, that only solutions with $\mu \in [-1,1]$ are admissible ($|\mu| > 1$ would imply $x \notin [0,S]$ within finite time – see (6)), so (16) always has a unique admissible solution for μ.

[18]At $\mu = \rho$ we use $\delta_F\rho < 1$ and at $\mu = 1$ we use $1/\rho - \delta_C(1-\rho) \geq 1/\rho - (1-\rho) > 1$. If any of δ_C, δ_F and ρ are zero then a similar argument shows that the conclusions of the paragraph are still correct.

In Appendix A2 we derive expressions for the other variables in terms of μ. These results are discussed in Section 4.

3.3 Range of Validity of the Solution

We now check the validity of the assumptions made in our derivation. Sufficient conditions under which the equations above are a solution to the model are that in every period (a) all new consumers prefer to purchase from either firm rather than not buy at all[19], (b) all old consumers wish to purchase, (c) equation (13) specifies a concave problem for each firm so that the first–order conditions are yielding maxima in (14), (d) no firm would prefer to raise its price above the level specified by (15) and serve only old consumers (the mathematics has (incorrectly) assumed that at sufficiently high prices a firm sells to a negative number of new customers and so it underestimated the profitability of very high prices – we must therefore consider such strategies separately), (e) the equations (15) do not require a firm to serve more than 100% of the new consumers (as noted above, the mathematics has not explicitly included this constraint).

A condition that ensures (a) holds is $r \geq \max_{i=A,B}\{d_i+eS+t-\rho\delta_C W_i(f_i(S))\}$, that is, every new consumer would prefer to pay the highest price ever charged in equilibrium rather than not buy at all. Condition (b) requires that $R \geq \max_{i=A,B}\{d_i+eS\}$. Simple algebra (see Appendix A3) shows that (c) is always satisfied provided $-1 < \mu < 1$, i.e. the system is stable, which is in any case required for (e). Condition (d) requires

[19]Note that it is not enough to guarantee that every consumer is always willing to buy from his preferred firm in equilibrium. The reason is that our analysis assumes that if a firm raised its price then any new customers it lost would buy from the other firm. Condition (a) ensures that this assumption is correct, so firms' costs of deviating from equilibrium are correctly computed.

$(R-c_i)x + \delta_F V_i(\rho x) \leq V_i(x)$ for all x. We now show that this is certainly satisfied if $R \leq \max_{i=A,B}\{d_i+eS\}$, so long as (e) holds.[20] Without loss of generality, let $c_i > c_j$, so $d_i > d_j$. Given (e), i sells to a non–negative number of new customers if it sets a price d_i+eS when it has S old customers and j charges d_j. From (12) it follows that it would certainly sell to a non–negative number of new customers at any smaller share of customers if it charged d_i+eS and its competitor followed the strategies specified by (15) (since $e \geq 0$, $t^* > 0$, $\varphi_i'(x) \leq 0$ – see Appendix A2). Since j has lower costs, customers prefer, ceteris paribus, to buy from j, so j would certainly sell to a non–negative number of new customers if it charged d_i+eS. (Note that j's rival's equilibrium prices are higher than i's rival's prices.) It follows that, since the previous analysis has evaluated the profitability of either firm charging any price up to d_i+eS correctly, neither firm will deviate.

The final condition to be checked is (e). An equivalent condition is that our solution specifies that each firm will always sell to a non–negative number of new customers, or that each firm has at least as many old customers next period as it would if it had sold to no new customers, i.e.

$$\eta_i + \mu x \geq \rho x \qquad \forall\, x$$

Now $\eta_i + \mu x + \eta_j + \mu(S-x) = S$, so $\eta_i + \eta_j = (1-\mu)S$. Therefore $\mu > 1$ would imply $\min\{\eta_i, \eta_j\} < 0$ so the condition would fail at $x = 0$, while $\mu < -1$ would imply $\min\{\eta_i,\eta_j\} < -\mu S$ so the condition would fail at $x = S$. Thus we require $|\mu| \leq 1$, hence our selection of the root for μ in the interval $(0,\rho)$ in Section 3.2. In the symmetric case $\eta_i = (1-\mu)S/2$, so the condition becomes

$$(1-\mu)S/2 + \mu S \geq \rho S \qquad (17)$$

[20]The simple arguments of this paragraph prove only that there is a single value of R consistent with (b) and (d). A very slightly harder argument shows that if $c_i = c_j$, (d) is certainly satisfied if $R \leq d_i + eS + t(4-7\rho)/(4-4\rho)$, so that an interval of values of R is consistent with (b) and (d). In fact, numerical computations show that the range of R for which our solution is valid is generally much larger.

This is clearly satisfied if $\rho \leq 0.5$. Further, we can use (16) to show that μ must be at least $2\rho/(6+5\rho^2\delta_C)$ – see Appendix A4, hence (17) is satisfied for all $\rho \leq 4/7$. It is hard to improve on this bound; for $\delta_C = \delta_F = 0$, (17) is violated for $\rho > 0.6$.

Since, from (A2.5), η_i is a linear function of costs, condition (e) will still hold so long as the firms are not too asymmetric. In fact with a little more work one can obtain an explicit bound on the permissible cost difference, $\bar{c}(\rho)$, for a given value of ρ. Since the algebraic details are not particularly informative, they are omitted. $\bar{c}(\rho)$ is continuous and decreasing in ρ. As ρ tends to zero, $\bar{c}(\rho)$ tends to 3t, the difference up to which both firms have positive sales in a standard model without switching costs. At $\rho = 1/3$, $\bar{c}(\rho)$ is greater than 1.5t, and at $\rho = 1/2$ it is greater than 0.5t.

We have therefore proved the following theorem :

Theorem 1 If $\rho \leq 4/7$ and $|c_A - c_B| \leq \bar{c}(\rho)$ then there is a range of R and r for which there is a unique Markov perfect equilibrium in which firms pursue linear strategies.

We have stated a theorem valid for all δ_C and δ_F. By restricting δ_C and δ_F one can improve on the bounds for ρ and $|c_A - c_B|$, but the improvement is slight.[21] The theorem does not claim that there is a unique Markov perfect equilibrium. It is possible that there are other solutions which do not have the linear form we have assumed. As we remarked at the beginning of the section, however, we know that, if ρ and $|c_A-c_B|$ are sufficiently small, the unique equilibria of the finite–period versions of this model are of this form and converge to the equilibrium of our model.

[21]We have not imposed r = R. It is easy to check that there is a range of values for which r = R and all the conditions (a) to (e) are satisfied, but we do not wish to restrict ourselves to r = R since first time users may obtain higher utility from the product but on the other hand may have to pay a start–up cost, and since we have in any case imposed a transport cost on new but not on old consumers.

When the proportion of new consumers in the market is very small (large ρ) or firms' costs are very different, our equations do not determine an equilibrium because (d) and (e) fail. At least one firm prefers to specialise in selling to its old customers alone and leave all the new consumers to be served by its rival, given the rival's conjectured behaviour. In fact, with asymmetric costs one firm may completely dominate the market. With symmetric firms but a very small flow of new consumers we conjecture that an equilibrium involves mixed strategies. This is because a firm whose rival was charging a relatively low price would set the (high) price R to fully exploit its own old consumers, but a firm whose rival was charging a high price at or close to R would undercut its rival slightly to attract the new consumers without giving up too much revenue on its old customers. Thus no pure strategy equilibrium may exist. For the remainder of this paper we assume $\rho \leq 4/7$ and the other parameters are such that an equilibrium in linear strategies exists.

4. The Nature of Competition

This section describes the nature of the equilibrium in our model. In Section 4.1 we explain why market shares evolve monotonically, in the direction of decreasing dominance, and converge rapidly. In Section 4.2 we examine the reasons why, when firms have equal costs, a market with switching costs always has higher prices than a market without switching costs. We perform comparative statics on the market price and we also analyse the relationship between our model and the corresponding two–period model. In Section 4.3 we discuss the dependence of firms' prices, when they have equal costs, on their market shares in the previous period and examine the ease of entry into a market with switching costs. Section 4.4 briefly considers the case of asymmetric costs.

Before proceeding, we collect together some expressions that will be useful for our analysis. We can write firm i's price as

$$p_i(x) = c_i + \omega + (\tfrac{x}{S} - \tfrac{1}{2})v + (c_j - c_i)\zeta \qquad (18)$$

or alternatively as

$$p_i(x) = c_i + 2\sigma_i\omega + (\tfrac{x}{S} - \sigma_i)v \qquad (18)'$$

in which i's steady–state market share is

$$\sigma_i = \frac{1}{2} + \frac{c_j - c_i}{4\omega + 2(1-\rho\delta_C)t} \qquad (19)$$

and

$$\omega = \tfrac{t}{1-\rho}(1+\rho^2\delta_C - \rho\mu\delta_C - (2\rho\delta_F/(3-\delta_F\mu^2)) \simeq \tfrac{t}{1-\rho}(1 + \tfrac{2}{3}\rho^2\delta_C - \tfrac{2}{3}\rho\delta_F)$$

$$v = \tfrac{t}{1-\rho}(2\rho(1-\delta_F\mu^2)/(3-\delta_F\mu^2)) \simeq \tfrac{t}{1-\rho}\left[\tfrac{2\rho}{3}\right]$$

$$\zeta = \frac{2\omega - v}{4\omega + 2(1-\rho\delta_C)t} \simeq \tfrac{1}{3}(1 + \tfrac{\rho\delta}{3}C - \tfrac{\rho\delta}{3}F)$$

(see Appendix A5). All the approximations are exact when $\delta_C = \delta_F = 0$ (and when $\rho = 0$, which is equivalent to the case when no consumers have switching costs). The relative error on ω never exceeds 1.5%, that on v never exceeds 2.5 % and that on ζ never exceeds 4%.The combined error from computing market shares using these approximations in (19) never exceeds 0.005, and the error from computing prices using these approximations in (18) and (18)' never exceeds 4.5% and 3%, respectively, of the average price–cost margin (ω). These bounds are derived numerically for the range of permissible parameters. Note that none of our results depend on these approximations.

In equilibrium, firms' market shares evolve according to a linear difference equation. Writing $\sigma_{i,T}$ for firm i's market share in period T, $\sigma_{i,T} = x_{i,T+1}/S$ (firms' current shares of old customers are their previous period market shares), so $\sigma_{i,T} = \eta_i/S + \mu\sigma_{i,T-1}$, from (6), so firm i's steady–state market share is $\sigma_i = \eta_i/((1-\mu)S)$ and

$$\sigma_{i,T} = \sigma_i + \mu^T(\sigma_{i,0} - \sigma_i) \qquad (20)$$

4.1 The Evolution of Market Share

Market shares evolve monotonically ($\mu > 0$), in the direction of decreasing dominance ($\mu < 1$), and converge rapidly (μ is small, approximately $\rho/3$).

To understand why the evolution of market share is monotonic, it is easiest to begin by noting that with equal costs firms with larger current sales sell at higher prices. The same result arises in the corresponding single–period model without switching costs for an analogous reason : the firm that is selling to more customers in total this period gives up more revenue from those customers by making a small price cut but, since the total market size is fixed, each firm would gain the same number of customers (from the other) by such a price cut. Therefore, for both firms to be in equilibrium, the currently larger firm must gain more per new customer affected by the price cut. That is, the larger firm must have a larger price–cost margin in equilibrium. Of course, in our model firms must consider the effect of a price cut on their value functions as well as on their current profits. Note, however, that if the static result were expected to be overturned in future periods, that is if larger firms charged lower prices after this period, then firms' value functions would tend to be *concave* – indeed would have to be in a linear model like ours – which would increase the gains to the smaller firm from a price cut relative to the gains to the larger firm. Thus the currently smaller firm must charge the lower price in the current period and so of course in all future periods as well.

Since the larger firm sets the higher price it sells to fewer new customers and so must be selling to more old customers. Therefore, since all customers die at the same rate in our model, it must have sold to more customers in total in the previous period. Thus the larger firm always remains larger : market shares converge monotonically.

A similar intuition applies even when firms have unequal costs : a firm currently selling to more than its steady–state share of consumers must, to be in equilibrium, be selling at a higher price–cost margin than it does in steady state. Since its opponent is therefore selling less than in steady–state, the opponent's price–cost

margin is below its steady–state margin. Hence the first firm sells to fewer new customers than it does in steady state. Hence it must be selling to more old customers, and so must have had more than its steady–state output in the previous period. Therefore convergence is monotonic.

Note that the last step of these arguments would not apply in a model like that of Farrell and Shapiro in which all customers live exactly two periods. In such a model the firm currently selling to more old customers must in the previous period have sold to more new customers, that is, have been the smaller and lower–priced firm in the previous period. Thus market shares oscillate about their equilibrium values in such a model.[22] We believe our result is the more natural one.

If firms have equal costs, their market shares converge to equality from any starting point, so the dominance of the initially larger firm decreases. This is in spite of the fact that both joint profits and the sum of the firms' value functions are maximised at the boundary (market shares of 0 and 1).[23]

These results contrast with those of Harris (1988) and Vickers (1986), who find in a general model of competition for market share that if joint flow profits are maximised on the boundary then one firm becomes increasingly dominant. The reasons for the difference are fairly straightforward. First, in our model, unlike theirs, the effort rates (prices) affect current flow profits (in their model the profits are additively separable from the costs of attracting customers) and this makes it cheaper for the follower (smaller firm) to work harder (offer lower prices). Thus in our model the follower works harder than the leader, whereas in theirs the leader works harder. Second, we explicitly model consumer behaviour so, since in our model consumers

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[22]Of course in their model total market shares are always exactly one half per firm, but firms' shares of new customers and of old customers oscillate between zero and one.

[23]As noted above, the larger firm charges the higher price so profits and value functions are convex in previous–period market shares. To confirm this, use equations (A1.1), (A1.2), (A1.5) and $0 < \mu < \rho$ to check that $m_i > 0$.

prefer, ceteris paribus, to be attached to the smaller firm (it is expected to offer lower prices in the future), the smaller firm would attract a larger proportion of new customers even if the two firms worked equally hard (offered the same price).[24] Third, in our model dominance is further eroded by the turnover of customers. Since old consumers leave the market and are replaced by new consumers, the larger firm would have to attract a larger share of new consumers than the follower simply to retain its current level of dominance.

These effects mean that market shares converge to their steady state values very rapidly. We can show that $\mu \in (0.26\rho, 0.36\rho)$ (see Appendix A4), so for $\rho = 0.5$, for example (equal numbers of new and old consumers in each period), a new entrant's market share exceeds 41% in the first period and 48% in the second when firms' costs are equal.

Since one reason for the erosion of dominance is that rational consumers prefer to be attached to smaller firms, convergence is slightly faster when consumers have rational expectations (or high discount factors) than when they are myopic (or have low discount factors). On the other hand, the convexity of firms' value functions means that new customers are worth more to a larger firm than to a smaller one, which narrows the amount by which the larger firm's price exceeds the smaller firm's and slows the erosion of dominance. The higher are firms' discount factors, the more important this effect is and the slower is convergence. When firms and consumers' discount rates are held equal the consumer effect dominates : convergence is very slightly more rapid when discount factors are higher.[25]

[24]In terms of our analysis, $\partial\varphi_i/\partial x_i < 0$ in (12) – see Appendix A2.

[25]We omit detailed derivations of the comparative statics here and elsewhere in Section 4 as they are straightforward exercises in manipulating the relevant equations (here (16)).

4.2 Symmetric Steady–State Prices and Profits

In steady–state equilibrium with symmetric costs $c_A = c_B = c$, the coefficients of v and ζ are zero, so $p_i = p_j = p = c + \omega$. For comparison, in a market without switching costs in which all customers have transport costs t in every period, $p = c + t$. It is clear that the market with switching costs always has the higher prices. It is instructive to examine the reasons for this a little more closely.

The naive argument why prices will be higher in the presence of switching costs is that firms will seek to exploit their monopoly power over their locked–in customers. If customers and firms were myopic ($\delta_C = \delta_F = 0$), this would be the only force at work and prices would be $c + (t/(1-\rho))$. If however firms care about the future then they will compete more fiercely for new customers, since these customers will become valuable repeat purchasers in the future. In the two–period models of Klemperer (1987a,b) this is sufficient to imply that prices are generally lower in the first period than in the absence of switching costs. Suppose for a moment that firms had to commit themselves once and for all to a single price (and customers are myopic). It is easy to check that, if they are symmetrically placed, the equilibrium prices will be $c + (t/(1-\rho))(1-\rho\delta_F)$. This exceeds c+t, the price without switching costs, since $\delta_F < 1$. Thus the fact that firms discount the future is sufficient to imply that the desire to exploit old customers outweighs the desire to attract new ones.

This conclusion is reinforced when one takes account of the fact that firms cannot commit themselves to a single price. In the first place, each firm will be less tempted to cut price since it will realise that a smaller market share for its rival today means that the rival will be more aggressive tomorrow. If customers were myopic, prices would be $c + (t/(1-\rho))(1-(2\mu\delta_F/(1+\delta_F\mu^2)) \simeq c + (t/(1-\rho))(1-(2\rho\delta_F/3))$. Secondly, if customers are rational they will recognise that a firm that cuts prices today to attract new customers will charge higher prices in the future. This reduces their responsiveness to price cuts and lowers still further the incentive to cut price. From the firms' point of view it is as if consumers' effective transport costs were

increased to $t^* = (1+(\rho-\mu)\rho\delta_C)t > t$. Thus prices rise to $c + (t^*/(1-\rho))(1-(2\mu\delta_F/(1+\delta_F\mu^2)))$, which, with a little algebra (see Appendix A5), we can show equals $c + \omega$.

It is arguable that even this underestimates the extent to which prices exceed those in a market without switching costs. For we have compared a model where old consumers have no transport costs to one in which all consumers have transport costs.[26] If, in our model of switching costs, all consumers have transport costs in every period and each consumer remains at the same position on the line (unchanged tastes) in every period, then a new consumer who takes into account the entire expected discounted sum of the transport costs he will pay acts as if he had a one–time transport cost $t(1+\rho\delta_C+(\rho\delta_C)^2+..)=t/(1-\rho\delta_C)$. Equilibrium prices are therefore $c + \omega/(1-\rho\delta_C) > c + \omega$.[27] On the other hand, our model in which only new consumers have transport costs has the same equilibrium prices as one in which all consumers have transport costs, but consumers' future positions on the line are independent of their current positions – when each new consumer's future position is, in expectation, equidistant between the firms, the incorporation of future transport costs has no effect on the consumers' current preference between the two firms if they are risk neutral. Thus when old as well as new customers have transport costs, equilibrium prices depend on the assumptions made about the constancy of consumer tastes but may be even higher than in our model, and so yet further above their level in the absence of switching costs.[28]

These results contrast with those of von Weizsäcker (1984), who finds that switching costs are likely to lower prices. To understand why, note that in his model firms commit themselves to a single price, which cannot be altered in future periods. If

[26]In the absence of switching costs there is no pure strategy equilibrium if old consumers have no transport costs.

[27]When old consumers have transport costs, the values of r and R for which the equilibrium is of the form we have examined are slightly altered.

[28]In the absence of switching costs periods are independent of one another so prices do not depend on whether or not consumers' tastes change over time.

this were the case in our model we would, as we have shown, obtain $p = c+(t/(1-\rho))(1-\rho\delta_F)$ for the case of myopic consumers. If however customers are rational they will realise that a price cut will bring benefits in all future periods and so, if they pay a transport cost only when they are new, their sensitivity to price cuts will be the same as if they were myopic but had an effective transport cost $t/(1+\rho\delta_C+..) = t(1-\rho\delta_C)$. Thus equilibrium prices would be $c + (t/(1-\rho))(1-\rho\delta_F)(1-\rho\delta_C)$, which may be greater or smaller than c+t. This would also be the case if we assumed, as von Weizsäcker does, that consumers pay transport costs in every period but that consumers' tastes change over time.[29] Thus under assumptions similar to his, we too obtain ambiguous results as to whether switching costs make markets more or less competitive.[30] This is, however, an extreme case. Making any one or more of the changes (a) giving consumers constant tastes, *or* (b) looking for a closed loop equilibrium (in which firms cannot precommit themselves to future prices), *or* (c) or making consumers myopic (or giving them expectations that firms' future prices will be equal to each other) would give the unambiguous result that the market with switching costs would have higher prices.[31] Thus our analysis suggests a strong presumption that prices are higher in the presence of switching costs than in their absence.

As is clear from the above discussion, prices increase in ρ, the fraction of customers who have switching costs in any period.

[29]In his numerical computations it is assumed that the probability a consumer's tastes change in any period is quite large relative to the interest rate, so that only a small fraction of the present value of a consumer's lifetime is spent before his tastes change.

[30]von Weizsäcker's model is, of course, different from ours in that consumers live forever (ρ=1) but switching costs are small enough so that some switching does occur. Our point is that his assumptions (commitment to never change prices together with changing consumer tastes) gives a strong bias to more competitive results.

[31]We saw that in our model, in which firms do not precommit themselves, making consumers myopic lowers prices. Making change (c) in conjunction with (b) (and/or (a)) nevertheless results in prices that are higher than in the absence of switching costs.

Also, prices decrease in δ_F, because market share becomes more valuable to firms as they care more about the future and so they compete more fiercely for it. This effect seems fairly robust. It is only partly mitigated by the effect of firms knowing that fiercer behaviour today results in a fiercer rival tomorrow.

We have also seen that prices increase in δ_C since customers who value the future more are less sensitive to price cuts. This result is less robust to the specification of the model. It is nullified if consumers believe expected prices are equal across firms and is reversed if firms precommit themselves to constant prices across time.[32]

In our original formulation the coefficient on δ_C is smaller in magnitude than that on δ_F so with either rational or myopic expectations an increase in the interest rate raises prices.[33]

If our model had only two periods then prices in the first period would be $p_1 = c + t + (t/(1-\rho))((2/3)(\rho^2\delta_C - \rho\delta_F))$, and in the second $p_2 = c + t + (\rho t/(1-\rho))$. Recall that in a market without switching costs prices are $c + t$. In our model $p - (c+t) \simeq (t/(1-\rho))((2/3)(\rho^2\delta_C - \rho\delta_F)+\rho)$. Thus the effect of switching costs on prices in each period of our model is closely approximated by the sum of the two effects that arise in a two period model. Since the 'second–period effect' in the two–period model is always positive, prices are always higher in our model than in the first period of the two–period model. The 'first–period effect' may, however, be of either sign. Thus it is possible that prices in every period of our model are higher than in the second period of the corresponding two–period model. It follows that in a (symmetric) finite–period

[32]This result might also be reversed, even with rational expectations and without precommitment, if consumers were imperfectly informed about costs. Then, we conjecture, consumers might anticipate that firms with lower prices had lower costs and so would charge lower prices in the future. Consumers would then become more sensitive to price cuts as δ_C increased.

[33]The result is not, however, reversed if consumers have transport costs (and constant tastes with rational expectations) in every period. In this case prices increase rapidly in δ_C because consumers care more about their future transport costs.

model, prices may be higher in the middle stages than either at the beginning or the end.[34]

4.3 Asymmetric Market Shares

Each firm's price increases linearly in its market share – firms with more old customers have a relatively greater incentive to exploit existing customers than attract new ones – and the rate of increase, v, is both independent of firms' costs and also roughly independent of both firms' and consumers' discount rates.[35] Thus this coefficient is roughly what one obtains in the last period of a finite–period model.

Since firms' prices are linear in their market shares, and market shares converge monotonically, an incumbent's price falls monotonically towards that of a new entrant. This contrasts with Klemperer (1989), in which the incumbent and entrants both charge lower prices in the entry period and both subsequently raise price. In the latter model the new entrants serve a pool of consumers with low reservation prices who had not been served by the incumbent and the entrants subsequently raises prices to exploit them. In the present model there is no expansion of industry output (which keeps prices from falling too far in the entry period) and there is customer turnover in subsequent periods (which keeps the average price from rising after entry). On the

[34]In our model the condition for prices to be higher in the infinite period model (or in the middle period of a very–many–period model) than in the last period of a finite period model is, approximately, $\rho\delta_C > \delta_F$, and the result cannot hold if $\delta_C \leq \delta_F$. If however consumers have transport costs in every period (and constant tastes) then the price in the infinite–horizon model (and in the middle periods of a very–many–period model) is multiplied by $1/(1-\rho\delta_C)$ and the price in the last period of a finite horizon model is unaffected, and the former must be greater than the latter if $\delta_C = \delta_F$. The reason is that the expected discounted value of a new consumer's transport costs is falling towards the end of the market and so prices fall.

[35]In fact v decreases with δ_F and increases with δ_C. The former effect is due to the fact the convexity of firms' value functions means that new consumers are more valuable to a larger firm than a smaller one, which narrows the amount by which the larger firm's price exceeds the smaller firms' price, and that this is more important the more firms value the future. The latter effect is tiny.

other hand, our model would also generate a price war if both firms entered simultaneously without any customer base.

With symmetric costs we can rewrite (18) as $p = c + t + (t/(1-\rho))(\rho\delta_C(\rho-\mu)+(2\rho(1-\delta_F)/(3-\delta_F\mu^2))) + (x/S)v$, so $p > c + t$ for all x. Thus even a new entrant with no customer base to exploit (x=0) charges a higher price than in the absence of switching costs (the intuition for this is that a new entrant's rival is so much less aggressive than if there were no switching costs). Since a new entrant sells to less than half the market, its profits in the entry period may be either larger or smaller than in the absence of switching costs. Once market shares become more equal, however, its profits will be higher than if there were no switching costs and it it can be shown that, as a result, the value function (discounted profits) of a new entrant is always higher with switching costs than without them. Thus, when firms have equal costs, entry is more attractive into a market with switching costs (that is would take place in the face of higher fixed entry costs). For consumer survival rates such that our equilibrium is valid, the disadvantage that a fraction of the market is locked into the incumbent's product is outweighed by the higher markups in the presence of switching costs.[36]

4.4 Asymmetric Costs

Simple manipulation of (18)' and (19) shows that the difference between firms' steady–state prices equals $(1-\rho\delta_C)t/(2\omega+(1-\rho\delta_C)t)$ times the difference in their costs, which is less than the price difference that would apply without switching costs (1/3 times the cost difference).

The less efficient (higher–cost) firm's steady–state market share is almost always larger than in the absence of switching costs. The reason is simply that the dominant firm is less aggressive as it wishes to exploit its old consumers.

[36]The results of this paragraph remain true if, in the market with switching costs, consumers have transport costs (and constant tastes) in every period. In this case $p > c + t/(1-\rho\delta_C)$ for all x.

With very few exceptions, both firms' steady–state prices and profits are higher than in the absence of switching costs. In fact, each firm's value function (discounted profits) is greater than in the absence of switching costs even when it has no customer base, for all except a small range of parameters.[37] Thus, as with symmetric costs, the presence of switching costs generally facilitates new entry of either lower– or higher–cost firms.[38]

5. Extensions

5.1 Competition with Artificial (or 'Contractual') Switching Costs

When switching costs are contractually created by, for example, repeat–purchase discounts, the problem differs from that modelled thus far, because firms receive lower net prices from repeat customers than from new customers (or from any customers who switch from the rival firm). We show in this section that firms' profits and the evolution of market shares are unaffected by this difference.

Assume that each firm is committed to giving a repeat–purchase discount of s to each old customer in each period that he repeat purchases (so old customers face a cost s of switching between firms' products in addition to any difference in gross

[37]The exceptions are restricted to large cost differences together with small ρ and either very low or very high δ_F and δ_C. The lower–cost firm's steady–state price and profits and industry steady–state profits are always increased by switching costs.

[38]With minor exceptions the comparative statics of firms' steady–state prices and profits with respect to ρ,δ_F,δ_C, and a common discount factor $\delta_C=\delta_F$ are as for the case when they have equal costs. The exceptions are that a high–cost firm's steady–state price and profits may decrease in δ_C, its price may decrease in ρ and a very–low cost firm's profits may increase in $\delta_C=\delta_F$. The lower–cost firm's steady–state market share is increasing in δ_F, in $\delta_C=\delta_F$, and, for ρ greater than about 3/7, in δ_C. If old customers have transport costs (and unchanged tastes), this increases the effective differentiation between firms. It is then unambiguous that the higher–cost firm's steady–state market share, price, and profits are all larger with switching costs than without, and that its price increases in ρ. Further, the effective differentiation is increasing in δ_C, so both firms' prices and profits and the higher–cost firm's market share increase in δ_C and increase in $\delta_C = \delta_F$. The other results of this section are unaffected.

prices).[39] We assume that s is sufficiently large that no customer will ever wish to switch in equilibrium and no firm will find it profitable to induce customers to switch. Let a firm with x old consumers quote a price $\tilde{p}_i(x)$ to new consumers in equilibrium and so also charge its old consumers a net price $\tilde{p}_i(x) - s$. It is straightforward to perform an analysis paralleling that of Section 3 to obtain the following proposition :[40]

Proposition 1 If $\rho \leq 4/7$ and $|c_A - c_B| \leq \bar{c}(\rho)$ then there is a range of r and R for which there is a unique Markov perfect equilibrium in which firms pursue linear strategies. Firms' prices to new consumers are given by $p_i(x) + \rho\delta_F s$, where $p_i(x)$ are the strategies that would apply in Theorem 1 (and are described in (18)–(19)) if there were real switching costs and firms gave no discounts. Firms' profits and the evolution of market share are the same as if there were real switching costs.

One way of seeing this result is to note that in any period the current problem is equivalent to one in which each firm will receive a 'bonus' of s for every new consumer that it attracts. Since switching costs are large enough that no switching actually occurs, one could equally well assume that the firm will receive a sum of $s(1-\rho\delta_F)$ in every period that the new consumer remains in the market (which has the same expected discounted value, s, to the firm). This is, however, exactly as if the firms' costs were lower by $s(1-\rho\delta_F)$ each period and it received no bonus – the fact that the firm does not actually receive a bonus on old consumers is irrelevant to its marginal profitability and so does not affect any action it takes. Thus the (unique

[39] Analysing the circumstances under which firms would wish to make such commitments is beyond the scope of this paper. It will however be seen that all firms' profits are higher when such commitments are the industry practice than when no switching costs are created in this way. Further, Caminal and Matutes (1989) show that such commitments arise endogenously in a model very much like the two–period version of this one.

[40] If we perform the analysis in terms of the price $p = \tilde{p} - s$ that their old customers pay, many of the equations (1–8,10a,11,12,15a,16,17,19,20) are unchanged, while the others are only slightly modified to account for the bonus that firms are paid by new customers.

linear) equilibrium in this model simply corresponds to that of the original problem with firms' costs lower by $s(1-\rho\delta_F)$: prices are $p_i(x) - s(1-\rho\delta_F)$ to old customers, or $p_i(x) + s\rho\delta_F$ to new ones.

These prices have the same expected value over a consumer's lifetime, discounting at rate δ_F, as the prices $p_i(x)$, so firms' profits are unaffected by whether there are (real) social costs in switching or whether firms artificially create switching costs and (apparently) pay for them with repeat–purchase discounts. New customers' expected discounted lifetime payments are unaffected if $\delta_C = \delta_F$.[41]

This line of reasoning also shows that firms' profits and the evolution of market shares would be unaffected if consumers of different ages were offered different discounts : all that matters to firms is the expected discounted cost to them of a consumer's lifetime discounts.[42]

When all repeat purchasers receive the same discount s, the net price is s higher for new than for old consumers. Also, when there are real switching costs, and consumers must pay the switching cost (or start–up cost) to make their first purchase from either firm, the effective total price including switching costs is s higher for new than for old consumers. Thus the pattern of effective prices consumers pay is the same whether switching costs are artificial or real. In the latter case the effective price is $\rho\delta_F s$ – the annualised value of the switching cost – higher than in the former case, so the entire burden of the real switching costs falls upon the consumers.

[41] Consumers' expected payments are different if $\delta_C \neq \delta_F$, but the relative values of the two firms' price paths is unaffected so consumers' behaviour is unaffected (for appropriate values of r and R). Note that firms' per–period profits are affected even in steady state (a fraction ρ of consumers pays $(1-\rho\delta_F)s$ less and a fraction $(1-\rho)$ pay $\rho\delta_F s$ more than when switching costs are real) because in any given period a firm has benefited from its old consumers having paid higher prices in past periods.

[42] Nor need discounts be positive every period, provided that consumers expect future gains from repeat purchase. Frequent–flyer and trading–stamp programs, for example, give awards only after a number of repeat purchases.

5.2 Pricing in a Growing Market

In this section we ask how competition in the presence of switching costs is affected by steady growth in the market size. We find that although the form of the equilibrium is similar, prices and possibly even discounted profits are lower than in a static market.

We assume that in every period the number of consumers is larger than in the previous period by a factor γ. We again look for Markov perfect equilibria. Now however the natural choice of state variable is a firm's current share of old customers, rather than its absolute number of old customers. Provided $\gamma\delta_F$ is less than unity, so value functions are bounded, we obtain the following proposition :

Proposition 2 If $\rho/\gamma \leq 4/7$ and $|c_A - c_B| \leq \bar{c}(\rho/\gamma)$ then there is a range of R and r for which there is a unique Markov perfect equilibrium in which firms pursue linear strategies. These are the same functions of shares of old customers as would apply in Theorem 1 (and are described in (18)–(19), reading x/S as current share of old customers) if there were no growth but the survival rate were ρ/γ and discount factors were $\gamma\delta_C$ and $\gamma\delta_F$.

To see this, first consider the problem in which the survival rate is also multiplied by the factor γ. In this case the ratio of new to old consumers is unchanged, so for any given current sales, the numbers of new consumers and of old consumers attached to each firm next period would be γ times the number in the original static model. For given consumer demand, therefore, we have simply multiplied the value of each successive period by γ, so the effect on firms is exactly as if we had multiplied their discount factors by γ and left the survival rate unchanged. For consumers, given this behaviour, the fact that their probability of reaching each successive period is multiplied by γ is equivalent to multiplying their discount factor by γ and leaving the survival rate unchanged. Thus the solution to the original static problem when both discount factors are multiplied by γ is the solution to the problem in which both the survival rate and the growth rate is multiplied by γ. The solution to the problem in which only the growth rate is multiplied by γ is of course the solution to this problem with the survival rate divided by γ. That is, when the number of new consumers is multiplied by a factor of γ every period, the solution is that of the original problem with δ_F, δ_C and ρ replaced by $\gamma\delta_F$, $\gamma\delta_C$ and ρ/γ, respectively.

Analysing a growing market using this two–step approach shows that prices are lower in a more rapidly growing market for two reasons. First, more rapid growth reduces the proportion of locked–in old consumers and so lowers prices. Second, the increased relative importance of the future means that firms would compete more vigorously for future profits, which also lowers prices. The second effect would arise even if the proportion of old consumers were held constant as the growth rate increased.[43] (The firm effect dominates the effect of consumers valuing the future more provided $\delta_C \leq \delta_F$: see Section 4.) In fact, more rapid growth may reduce prices so much that firms' current–period profits and expected total discounted profits may be lower in a more rapidly growing market (holding constant the number of repeat–purchasers in the current period). Note however, that no matter how rapidly growing the market, prices and profits still exceed those in a (static or growing) market without switching costs.

5.3 Temporary Changes in Parameters

Section 4 was confined to an examination of the effects of permanent changes in the parameters. For some applications, however, for example the investigation of pricing across the business cycle or the effects of future tax changes, we need to explore what happens when some parameters are temporarily different from their long–run values.

Suppose that costs are currently $\tilde{c}_A$ and $\tilde{c}_B$ and discount factors are $\tilde{\delta}_C$ and $\tilde{\delta}_F$, but these parameters will be c_A and c_B and δ_C and δ_F in all future periods (changes in

[43] Farrell and Shapiro obtain the opposite result, that prices are higher in a growing market. Their result is due to the unusual timing of moves in their game; they do not consider it intuitive.

ρ are harder to handle as they affect the ratio of new to old consumers in all future periods). The equation, (13), determining firm i's optimal behaviour becomes :

$$p_i(x) \in \underset{p_i}{\text{argmax}} \{(p_i-\tilde{c}_i)\,(x + ((1-\rho)/\rho)S\hat{Z}_i(p_i,p_j,x,\tilde{\delta}_C,\delta_C,\delta_F,\rho,c_A,c_B)) + \tilde{\delta}_F\, V_i(\rho x + (1-\rho)S\hat{Z}_i(....))\} \qquad (13)'$$

in which $V_i(x)$ is the function depending on $c_A,c_B,\delta_C,\delta_F$ and ρ determined by our previous analysis, and $\hat{Z}_i(.)$ specifies how consumers divide themselves between the two firms as a function of both current and future parameter values. It is now straightforward to determine $\hat{Z}_i(.)$ using the analogous equation to (11) (in which $W_i(x)$ is the function depending on $c_A,c_B,\delta_C,\delta_F$ and ρ determined previously) and hence to obtain expressions for p_A and p_B. For example, in symmetric steady–state equilibrium (with $\tilde{c}_A=\tilde{c}_B=\tilde{c},c_A=c_B,x=S/2$),

$$p_i = \tilde{c} + \frac{t}{1-\rho}\left[1 + (\rho-\mu)\rho\tilde{\delta}_C - \frac{2\tilde{\delta}_F\rho}{3-\delta_F\mu^2}\right]. \qquad (21)$$

(see Appendix A6).

We can now address a number of questions in addition to the straightforward comparative statics on costs and discount factors. For example, an anticipated future increase in the corporate tax rate from $\tilde{\tau}$ to $\tau > \tilde{\tau}$ multiplies the value of all revenues beyond this period by $(1-\tau)/(1-\tilde{\tau})$ and so is equivalent to multiplying firms' current period discount factor, $\tilde{\delta}_F$, by $(1-\tau)/(1-\tilde{\tau}) < 1$. We can see from (21) that such a change reduces the value of market share and so increases current profits, but since (in symmetric equilibrium) firms' current market shares will remain unchanged, before–tax future profits are all unchanged.[44]

As another example, a one–period 'boom' in which all consumers' demands are multiplied by $Y > 1$ scales up the value of this period to all market participants by the

[44]It is easy to use (21) to show that a proportional tax increase reduces symmetric firms' future *after*–tax profits by more, in present value, than it increases their current profits. An increase in the marginal tax rate that had a smaller effect on the average rate would, however, reduce future after–tax profits by less while having the same effect on current profits and so could benefit firms in net.

factor Y and so is equivalent in its effect on prices to scaling both $\tilde{\delta}_F$ and $\tilde{\delta}_C$ by $1/Y$ (on net raising prices if $\tilde{\delta}_C \leq \tilde{\delta}_F$ – see (21)). On the other hand, to model the effect of only new consumers' demands being multiplied (for this period) by Y we would re–solve (13)' with the term $((1-\rho)/\rho)S\hat{Z}_i(.)$ multiplied by Y and $\tilde{\delta}_C$ multiplied by $1/Y$. Both the latter changes lower price, and this result – lower–price cost margins in a boom – seems to be the empirically correct one (see Bils (1987)).[45]

As a final example, we can model an international oligopoly in which a domestic firm A competes with a foreign firm B in the domestic market. A permanent depreciation of the domestic currency corresponds to a permanent increase in B's costs, $\tilde{c}_B$ and c_B (measuring all variables in domestic currency), whereas a temporary depreciation corresponds to only a temporary increase in B's current costs, $\tilde{c}_B$, in conjunction with a temporary increase in firms' and consumers' discount factors, $\tilde{\delta}_C$ and $\tilde{\delta}_F$.[46] Froot and Klemperer (1989) showed in the context of a two–period model that import prices may therefore respond much less to exchange–rate changes that are expected to be temporary than to those that are expected to be permanent. In our model, also, the effects of a temporary increase in B's costs are both less than those of a permanent increase in B's costs and also opposed by the effects of the changes in discount factors (assuming $\tilde{\delta}_C \leq \tilde{\delta}_F$). Thus we can confirm that Froot and Klemperer's

[45]We suspect that this result arises even when all consumers' demands are multiplied by Y, if firms' (managers') utilities are strictly concave in current–period profits and additive across periods (perhaps because of managers' compensation schemes, or capital–market imperfections, risks of bankruptcy etc.) – Gottfries (1988) obtains a result similar to this in a two–period model of search costs which has a similar reduced form to ours, but confirming this conjecture in our infinite–horizon model with switching costs is beyond the scope of the present paper. We also suspect the same result would arise if we modelled a boom as a one–period increase in the number of new customers (and indeed Bils (1985) obtains a similar result in a model of a monopolist selling an experience good), but we cannot check this in the current model since such a 'boom' would lead to a subsequent increase and then a slow decrease in the number of old consumers.

[46]If world interest rates remain constant and there are no barriers to capital mobility, then domestic interest rates must fall if the domestic currency is expected to regain value.

main theoretical results are not special to a two–period model but also apply in an infinite horizon analysis.[47]

To analyse cases in which the model parameters are expected to change more than once, we would solve (13)' repeatedly – solving first for the period before the last anticipated parameter change, hence obtaining new functions V_i and $\hat{Z}_i$ which we would substitute into (13)' to solve for the preceding period, etc. It might also be possible, though analytically hard, to look for a steady–state solution of a case of infinitely repeated cycles, each cycle of which has a finite number of periods. Analysis of this kind might allow us to perform richer investigations of, for example, pricing across the business cycle.

Note that although we have derived our equations from optimising behaviour in the presence of switching costs, the reduced form of our model is a model of competition for market share where this is a linear function of lagged market share and of firms' prices. Thus analyses of the kind suggested in this subsection would apply in environments in which market share matters for reasons other than switching costs, provided that market shares evolve in this way.

5.4 A Continuous Time Model

By developing the multi–period model in discrete time, we have facilitated comparisons with existing two–period models; our model also makes analyses of the kind described in Section 5.3 quite straightforward. In this section, however, we sketch how a continuous time version of our model can be developed.

As before, we assume that every new consumer derives instantaneous utility r less a transport cost from purchasing the good at the time at which he arrives. In the interval (t,t+dt), every surviving old consumer has probability dt $[+o(dt^2)]$ of wishing to make a purchase, in which case he derives instantaneous utility R from a single unit of the variety he previously purchased, and has probability θdt $[+o(dt^2)]$ of leaving the market. New consumers arrive at rate θS, so the mass of consumers is constant at S.

[47]See Froot and Klemperer (1989) for further discussion and empirical evidence.

Let the firms' and consumers' discount rates be $\lambda_F > 0$ and $\lambda_C > 0$ respectively.[48] As before, the state of the system is firm i's stock of old consumers, x.

By performing an analysis that closely parallels that of Section 3, we can show that for large enough θ (that is, when a large enough fraction of sales are to new consumers) and not–too–dissimilar costs, there is a range of r and R for which there is a unique Markov perfect equilibrium in which firms' prices are linear functions of their shares of old consumers. As before, firms' value functions are quadratic and consumers' value functions are linear functions of the state. Firms' shares of sales converge to their steady–state values at an exponential rate.

The main results of our discrete time model apply. When firms have equal costs, both firms' prices exceed their prices in the absence of switching costs, whatever their market shares.[49] In symmetric steady–state equilibrium, prices are increasing in firms' discount rate, decreasing in consumers' discount rate, and increasing in the fraction of sales that are made to old consumers. The value function of a new entrant is always higher with switching costs than without them. Details are given in Appendix A7.

6. Conclusion

This paper has presented a model of dynamic competition with switching costs.

We have shown that prices and profits are higher than in a market without switching costs. Our analysis in Section 4.2 showed that while it is possible to construct models in which the opposite is true, there are several reasons to believe that our result is the more general one. We also showed prices rise as firms discount the

[48]The assumption that old consumers buy at rate one is without loss of generality since we can always choose units of time for which this is the case. The other details of the model are as for the discrete time case. In particular, we assume that the good is not storable.

[49]As in discrete time, the price at $x = 0$ tends to the price without switching costs in the limit as $\lambda_C \to \infty$ and $\lambda_F \to 0$.

future more, fall as consumers do so, fall as the turnover of consumers increases and fall as the rate of growth of the market increases.

The higher profits in a market with switching costs mean that new entry may be more attractive than in their absence, even though an entrant must overcome the disadvantage that a fraction of the market is locked into the incumbent's product. After new entry or any shock, market shares converge monotonically and fairly rapidly to stable steady–state. With symmetric costs the convergence is to equal shares, so the flow of new consumers into a market with switching costs results in the decreasing dominance of the initially larger firm.

Section 5.3 suggested some applications of our model which we hope to explore in future work.

Appendix

(A1) Derivation of equation (16)

From (10a)

$$m_i = \frac{e_i \mu/\rho}{1 - \delta_F \mu^2} \tag{A1.1}$$

Hence using (15a) and rearranging

$$e_A = e_B = e = \left[\frac{1-\delta_F \mu^2}{1+\delta_F \mu^2}\right] \frac{2t^* \mu}{S(1-\rho)} \tag{A1.2}$$

Substituting for e into (6b) and using the definition of t* we have

$$\mu \left[\frac{3-\delta_F \mu^2}{1+\delta_F \mu^2}\right] = \frac{\rho t}{t^*} \tag{A1.3}$$

Now from (8a) $h_A = h_B = h = -e/(1-\rho\delta_C\mu)$, so using this expression to substitute for e in (6b)

$$\mu = \frac{\rho t + (1-\rho)S(1-\rho\delta_C\mu)h}{t - \rho(1-\rho)\delta_C S h}$$

Rearranging yields

$$h = -\frac{t(\rho-\mu)}{S(1-\rho)} \tag{A1.4}$$

Substituting for h into the definition of t* we obtain

$$t^* = t(1 + (\rho-\mu)\rho\delta_C) \tag{A1.5}$$

Finally, substituting into (A1) yields

$$\mu \left[\frac{3-\delta_F \mu^2}{1+\delta_F \mu^2}\right] = \frac{\rho}{1 + (\rho-\mu)\rho\delta_C}$$

This is equivalent to (16) in the main text.

(A2) Expressions for other variables

We can eliminate $d_i - c_i$ from (10b) using (15b) and eliminate e_i using (A1.2) to obtain

$$l_i = \frac{\eta_i}{\rho}\left[\frac{2t^*\mu}{S(1-\rho)} + e_i\right] = \frac{\eta_i}{\rho}\left[\frac{4t^*\mu}{S(1-\rho)(1+\delta_F\mu^2)}\right] \qquad \text{(A2.1)}$$

Substituting for l_i into (15b) and eliminating m_i using (A1.1) and (A1.2) we obtain

$$d_i - c_i = \psi\,\eta_i$$

in which

$$\psi = \frac{2t^*(1-2\delta_F\mu-\delta_F\mu^2)}{S(1-\rho)\ (1+\delta_F\mu^2)} \qquad \text{(A2.2)}$$

Now using the fact that $e_A = e_B$ and $h_A = h_B$ we can use (6b) to rewrite (6a) to read

$$\eta_i = \frac{(1-\mu)}{2}S - \frac{(1-\rho)}{2t^*}S((d_i-d_j) - \rho\delta_C(g_i-g_j)) \qquad \text{(A2.3)}$$

After eliminating $g_i - g_j$ using (8b) and (8a) we can subtract the expression for η_j from that for η_i to obtain

$$\eta_i - \eta_j = -\frac{(1-\rho)}{2t^*}\left[(d_i-d_j)\left[1 + \frac{\rho\delta_C}{1-\rho\delta_C}\right] + \left[\frac{\rho\delta_C^2}{1-\rho\delta_C}\right]\frac{t(\rho-\mu)}{S(1-\rho)}(\eta_i-\eta_j)\right]$$

which simplifies to

$$\xi(\eta_i - \eta_j) = d_j - d_i \qquad \text{(A2.4a)}$$

in which

$$\xi = \frac{t^* - \rho\delta_C t}{(1-\rho)S} \qquad \text{(A2.4b)}$$

Adding the expressions for η_i and η_j yields $\eta_j = (1-\mu)S - \eta_i$. Finally, solving using (A2.2) we obtain

$$\eta_i = \frac{c_j - c_i}{2(\xi+\psi)} + \frac{(1-\mu)}{2}S \qquad \text{(A2.5)}$$

and

$$d_i = c_i + \psi\left[\frac{c_j - c_i}{2(\xi+\psi)} + \frac{(1-\mu)}{2}S\right] \qquad \text{(A2.6)}$$

Equations (A1.1),(A1.2),(A1.4),(A1.5),(A2.1),(A2.5) and (A2.6) now directly provide expressions for m,e,h,t^*,l_i and l_j, η_i and η_j,d_i and d_j respectively as functions of μ and the parameters of the problem (ρ,δ_F,δ_C,c_A,c_B). It is useful to note from (A1.4)

and (A1.2) that for the root of interest $\mu \in (0,\rho)$, we have $h < 0$ and $t^* > t$. Finally, $\varphi_i(x) = (t+\rho\delta_C(g_i-g_j-hS)+2\rho^2\delta_C hx)/2t^*$, so $\partial\varphi_i/\partial x < 0$.

(A3) Proof that condition (c) always holds

Differentiating (14) with respect to p_i and simplifying yields

$$\rho(1-\rho)Sm_i\delta_F \le 2t^*$$

as the condition for the second–order condition for i to hold. Using (10a) this is equivalent to

$$\frac{(1-\rho)Se_i\mu\delta_F}{1 - \delta_F\mu^2} \le 2t^*$$

From (6b) and the fact that $e_A = e_B = e$ it follows that

$$(1-\rho)Se = \rho t - \mu t^* \qquad \text{(A3.1)}$$

$$\le t^*(\rho - \mu), \text{ using (A1.5) and } \mu \in (0,\rho).$$

Hence, since $\mu \ge 0$

$$\frac{(1-\rho)Se\mu\delta_F}{1-\delta_F\mu^2} \le \frac{t^*\delta_F(\rho\mu - \mu^2)}{1-\delta_F\mu^2} \le \frac{t^*(1-\delta_F\mu^2)}{1-\delta_F\mu^2} = t^* < 2t^*$$

(A4) Bounds on μ

From (16) $\mu = ((\mu^2+1/\delta_F)/((3/\delta_F)-\mu^2))(\rho/(1+\rho^2\delta_C-\rho\delta_C\mu))$ so, for the root of interest ($\mu \in (0,\rho)$), we have $\mu > \rho/(3(1+\rho^2\delta_C)) > \rho/6$. Substituting $\mu > \rho/6$ in the equality yields $\mu > 2\rho/(6+5\rho^2\delta_C)$.

Furthermore, for $\rho \le 4/7$ we have $\mu > 0.26\rho$.

Also, the equality yields $\mu < ((\mu^2+1)/(3-\mu^2))\rho$, so $\rho \le 4/7 \Rightarrow \mu < 0.36\rho$.

(A5) Expressions for prices

We have

$$\begin{aligned} p_i(x) &= d_i + e_i x \\ &= c_i + \psi\eta_i + e_i x \qquad \text{(using (A2.2))} \end{aligned}$$

$$= c_i + \psi\eta_i + \sigma_i e_i S + (\tfrac{x}{S} - \sigma_i)e_i S$$

Now (6) implies that $x_i = \eta_i + \mu x_i$, in which $x_i = \sigma_i S$ is i's steady–state share of old customers, so $\sigma_i = \eta_i/((1-\mu)S)$, so we have

$$p_i = c_i + 2\sigma_i\omega + (\tfrac{x}{S} - \sigma_i)v \qquad (18)'$$

in which $\omega = \frac{1}{2}(e_i S + (1-\mu)\psi S)$ and $v = e_i S$. (A5.1)

Using (A1.2) and (A1.3), $v = \left[\frac{2\rho t}{1-\rho}\right] \frac{1-\delta_F\mu^2}{3-\delta_F\mu^2}$

Using (A1.2) and (A2.2) $\omega = \frac{t^*}{1-\rho}\left[1 - \frac{2\mu\delta_F}{1+\delta_F\mu^2}\right]$, and now using (A1.3)

$\omega = \frac{t^*}{1-\rho}\left[1 - \frac{2\rho\delta_F}{(3-\delta_F\mu^2)}\frac{t}{t^*}\right]$, so using (A1.5) $\omega = \frac{t}{1-\rho}(1 + \rho^2\delta_C - \rho\mu\delta_C - (2\rho\delta_F/(3-\delta_F\mu^2))$.

We can rewrite (18)' as

$$p_i(x) = c_i + \omega + (\tfrac{x}{S} - \tfrac{1}{2})v + (\sigma_i - \tfrac{1}{2})(2\omega - v) \qquad (A5.2)$$

so noting again that $\sigma_i = \eta_i/((1-\mu)S)$ and using (A2.5),

$$p_i(x) = c_i + \omega + (\tfrac{x}{S} - \tfrac{1}{2})v + (c_j - c_i)\zeta \qquad (18)$$

in which $\zeta = (2\omega - v)/(2(1-\mu)S(\xi+\psi))$. Now $(1-\mu)\psi S = 2\omega - eS$ from (A5.1), so substituting for ξ using (A2.4b) and for e using (A3.1), we obtain $\zeta = (2\omega - v)/(4\omega + 2(1-\rho\delta_C)t)$. Comparing (A5.2) and (18),

$$\sigma_i = \tfrac{1}{2} + (c_j - c_i)/(4\omega + 2(1-\rho\delta_C)t) \qquad (19).$$

The approximations for ω, v and ζ are obtained by $\mu \simeq \rho/3$ and neglecting terms of high order.

(A6) Temporary changes in parameters

Assume the model parameters are $\tilde{c}_A, \tilde{c}_B, \tilde{\delta}_C$, $\tilde{\delta}_F$ and ρ in the current period and $c_A, c_B, \delta_C, \delta_F$ and ρ in all future periods. Then the position, $\hat{Z}_i(p_i, p_j, x)$, of the marginal new consumer is determined by

$$-p_i + \rho\tilde{\delta}_C W_i((1-\rho)S\hat{Z}_i(p_i,p_j,x) + \rho x)) - t\hat{Z}_i(p_i,p_j,x)$$
$$= -p_j + \rho\tilde{\delta}_C W_j(S - ((1-\rho)S\hat{Z}_i(p_i,p_j,x) + \rho x)) - t(1 - \hat{Z}_i(p_i,p_j,x)) \qquad (11')$$

where $W_i(x)$ is the function of $c_A, c_B, \delta_C, \delta_F$ and ρ determined in the original analysis, so

$$\hat{Z}_i(p_i,p_j,x) = \frac{(p_j - p_i)}{2\hat{t}^*} + \hat{\varphi}_i(x) \qquad (12')$$

in which $\hat{t}^* = t(1 + (\rho-\mu)\rho\tilde{\delta}_C)$ and $\hat{\varphi}_i(x) = (t + \rho\tilde{\delta}_C(g_i - g_j - hS) + 2\rho^2\tilde{\delta}_C hx)/\hat{t}^*$

Differentiating equation (13') of the text with respect to p_i we obtain the first–order condition

$$x + ((1-\rho)/\rho)S(\frac{p_j - p_i}{2\hat{t}^*} + \hat{\varphi}_i(x)) - \frac{(p_i - \tilde{c}_i)}{2\hat{t}^*}((1-\rho)/\rho)S$$
$$+ (-(1-\rho)/2\hat{t}^*)S\tilde{\delta}_F(l_i + 2m(\rho x + (1-\rho)S(\frac{p_j - p_i}{2\hat{t}^*} + \hat{\varphi}_i(x))) = 0 \qquad (14')$$

Adding (14') to the corresponding first–order condition for p_j, substituting for l_i and m using (A1.1),(A2.1) (and manipulating using (A1.2) and (A1.3) to yield $l_i + l_j + 2mS = 4t/((1-\rho)(3-\delta_F\mu^2))$ and noting $\hat{\varphi}_i(x) + \hat{\varphi}_j(S-x) = 1$, yields

$$p_i + p_j = \tilde{c}_i + \tilde{c}_j + 2\left[\frac{t}{1-\rho}\right]\left[1 + (\rho-\mu)\rho\tilde{\delta}_C - \frac{2\rho\tilde{\delta}_F}{3-\delta_F\mu^2}\right]$$

Setting $c_i = c_j$, $\tilde{c}_i = \tilde{c}_j = \tilde{c}$, and $x = S/2$ yields (21) in the text, and to solve for the general asymmetric case we subtract the first–order condition for p_j from that from p_i, (14'), to yield an additional equation for $p_i - p_j$.

(A7) Continuous time

We again look for solutions in which firms' prices and consumers' value functions are linear in the state, and firms' value functions are quadratic. We use the same symbols as before since there is no risk of confusion :

$$p_i(x) = d_i + e_i x \qquad (A7.1)$$
$$V_i(x) = k_i + l_i x + m_i x^2 \qquad (A7.2)$$
$$W_i(x) = g_i + h_i x \qquad (A7.3)$$

If the marginal new consumer is located a distance $z_i(x)$ from firm i, the evolution of x (i's stock of old consumers) is governed by

$$\dot{x} = -\theta x + \theta S z_i(x) \tag{A7.4}$$

and

$$z_i(x) = \frac{1}{2} + \frac{p_j(S-x)-p_i(x)}{2t} + \frac{W_i(x)-W_j(S-x)}{2t} \tag{A7.5}$$

Substituting (A7.1) and (A7.3) into (A7.5) and applying this to (A7.4) yields

$$\dot{x} = \eta_i + \mu x \tag{A7.6}$$

Note that the system is stable if $\mu < 0$, unstable if $\mu > 0$.

The expected discounted life–time utility of an old consumer attached to firm i is

$$W_i(x) = \int_0^\infty e^{-n(\theta+\lambda_C)} (R - p_i(x(n))) \, dn \tag{A7.7}$$

Using (A7.6) and (A7.1) to substitute for $p_i(x(n))$ this can be integrated up to yield

$$W_i(x) = \frac{(R+(\eta_i/\mu)-d_i)}{\theta+\lambda_C} - \frac{e_i(x-(\eta_i/\mu))}{\theta+\lambda_C-\mu} \tag{A7.8}$$

Equating coefficients with (A7.3) allows us to solve for g_i and h_i in terms of d_A, d_B, e_A, e_B, and the parameters of the system.

Similarly the value functions of firms are given by

$$V_i(x) = \int_0^\infty e^{-n\lambda_F} (p_i(x(n))-c_i)(\theta S z_i(x) + x) dn \tag{A7.9}$$

and so we can again substitute for $p_i(x(n))$ using (A7.6) and (A7.1), and then integrate up and equate coefficients with (A7.2) to obtain k_i, l_i, and m_i as functions of d_A, d_B, e_A, e_B and the parameters of the system.

If firms charge p_i and p_j the location of the marginal new consumer is given by

$$Z_i(p_i,p_j,x) = \frac{p_j-p_i}{2t} + \varphi_i(x) \tag{A7.10}$$

where $\varphi_i(x) = 1/2 + (W_i(x)+W_j(S-x))/2t$ is a linear function of x. Hence x will evolve according to

$$\dot{x} = -\theta x + \frac{\theta S(p_j-p_i)}{2t} + \theta S \varphi_i(x) \tag{A7.11}$$

$$= \alpha_i(p_i,p_j,x) \tag{A7.11}'$$

The current profits firm i makes by charging p_i are

$$\pi_i(p_i,p_j,x) = (p_i - c_i)(\theta S Z_i(p_i,p_j,x) + x)$$

Hence we can find optimal prices for the firms by solving the continuous time Bellman equation

$$0 = \max_{p_i} \{\pi_i + V_i' \alpha_i - \lambda_F V_i\} \tag{A7.12}$$

Looking for solutions of the form (A7.1), (A7.12) gives us enough equations to determine d_i and e_i and hence all the unknowns of the system. As before, e_i and m_i are independent of i.

As in the case of discrete time, it is simplest to solve for μ first. Using (A7.8), (A7.9) and (A7.12) we obtain

$$\mu + \theta = \frac{2(\mu-\lambda_F/2)(\mu+\theta+1)}{\theta+1+\lambda_F/2} \left[\frac{1}{\theta+\lambda_C-\mu} + 1\right] \tag{7.13}$$

It can be checked that this has a unique negative root, lying between $-(1+\theta)$ and $-\theta$. Only this root is admissible, since $\mu > 0$ would imply that the system would violate its constraints.

As in the case of discrete time, we must check that it was safe to ignore the bounds on the system. The obvious analogues to conditions (a) and (b) are satisfied if $r \geq \max_{i=A,B} \{d_i+eS-W_i(S)\}$ and $R \geq \max_{i=A,B} \{d_i+eS\}$. (c) is always satisfied. An easily sufficient condition for (d) is $R \leq \max_{i=A,B} \{d_i+eS\}$. (e) will certainly hold if $\theta \geq 1$ in the symmetric case (the argument is the analogue of that used to show $\rho \leq 0.5$ in the discrete time case, and this bound – like that one – can be improved) and (e) will also be valid if the model is not too asymmetric.

If one imposes the constraint that no firm can take more than 100% of the new customers, (A7.11) becomes

$$\dot{x} = -\theta x + \min\{\max\{\theta S Z_i(p_i,p_j,x),0\},\theta S\} \tag{A7.11}''$$

Note that the right–hand side is still a Lipschitz function of p_i,p_j and x. Hence, the equation for $\dot{x}$ will be Lipschitz in x if i and j use strategies which are Lipschitz functions of x. Moreover, it is non–negative at $x = 0$ and non–positive at $x = S$. Therefore, using Hirsch and Smale (1974) p.172, there exists a unique solution to (A7.11)'' from any initial position in [0,S], which is defined for all t. The usual sufficiency argument then shows that the solution we have found for i is indeed an optimal response in the class of Lipschitz strategies to j. We have therefore found an equilibrium when players use Lipschitz strategies. ((A7.11)'' really ensures that the presence of the finite boundary is irrelevant.)

It is now not hard to develop formulae for d_i,e and the other variables of interest. In particular,

$$e = -\frac{(\mu-\lambda_F/2)(\mu+\theta+1)}{(\theta+1+\lambda_F/2)}\frac{2t}{\theta S} \qquad \text{(A7.14)}$$

and, in the symmetric case, with $c_i = c_j = c$

$$d - c = \frac{\mu}{\theta}\frac{(2\mu+\theta+1-\lambda_F)t}{(\theta+1+\lambda_F)} \qquad \text{(A7.15)}$$

The results are a little different from the discrete time case because choices at any particular instant do not affect the evolution of the state and a new consumer instantly becomes an old consumer. Thus, for example, in (A7.10) consumers' immediate responsiveness to a price change is a function of t rather than of t* as in (12). Nevertheless it is not hard to check that the major results of the discrete time model carry through to the continuous time formulation.

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