A Note on Ortega Reichert's "A Sequential Game with Information Flow"

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prepared for The Economic Theory of Auctions, Paul Klemperer (ed.)

May 1999

Although it was pathbreaking work, Armando Ortega Reichert's (1968) PhD thesis "Models for Competitive Bidding under Uncertainty" was never published. For reasons of space we publish just Chapter VIII, "A Sequential Game with Information Flow", which was a seminal analysis of a signalling game.[1] This note sets this chapter in context and explains the references it makes to earlier parts of the thesis.

The chapter examines a repeated first-price sealed-bid procurement auction in which all bids are reported at the end of each round. At the beginning of each period firms privately find out their own costs which are independently drawn from an exponential distribution with unknown parameter, so each firm's costs are correlated across the two rounds. Ortega Reichert shows each bidder has an incentive to bid less aggressively in the first auction than in a one-shot auction in order to alter its opponent's belief about the unknown parameter, and hence increase its opponent's estimate of the bidder's second-period costs.

[1] This chapter was enormously influential in, for example, guiding Milgrom and Roberts' (1982) analysis of limit pricing (personal communication from Paul Milgrom). It also significantly precedes Spence's (1972) PhD thesis on signalling games.

Ortega Reichert's thesis develops many other results including about how bidding is affected by bidders' attitudes to risk, asymmetries in their information, the number of bidders, their uncertainty about the number of other bidders, etc. It also contains an early model of the treasury bill auction market and derives revenue equivalence results similar to those in Vickrey (1962).

The model is an extension of the single-period model presented in Section 6.6.2. This is a one-shot first-price procurement auction in which each player's privately-known cost c_i, $i = 1, 2$, is independently distributed according to an exponential distribution with unknown state variable W,

$$f(c_i|W = w) = we^{-wc_i}, \quad c_i > 0.$$

The state parameter, W, which is interpreted as the "work content of the project", is assumed to have a gamma distribution with parameters (δ, r),

$$h(w) = \frac{\delta^r}{\Gamma(r)} w^{r-1} e^{-\delta w}, \quad w > 0$$

where it is assumed that $r \geq 2$, $\delta > 0$ and where $\Gamma(r) = \int_0^\infty x^{r-1} e^{-x} dx$.[2] The density of bidder i's estimate of j's cost is[3]

$$f_{ji}(c_j|c_i) = \int_0^\infty f(c_j|w)\, h(w|c_i)\, dw \qquad (6.4)$$

$$= (r+1)\frac{(\delta + c_i)^{r+1}}{(\delta + c_i + c_j)^{r+2}}. \qquad (6.53)$$

This has decumulative distribution (1 - cumulative distribution)

$$\bar{F}_{j|i}(c_j|c_i) = \frac{(\delta + c_i)^{r+1}}{(\delta + c_i + c_j)^{r+1}}. \qquad (6.54)$$

Assuming symmetric bidding functions $b(c)$ exist, bidder with type c_i would maximise expected profits by bidding as if it had the type $\hat{c}$ that solves[4]

$$\max_{\hat{c}} \left\{ [b_i(\hat{c}) - c_i]\, \bar{F}_{j|i}(\hat{c}|c_i) \right\}. \qquad (6.104)$$

To solve for the equilibrium we evaluate the first-order condition for $\hat{c}$ at $\hat{c} = c_i$; that is,

$$b_i'(c_i)\, \bar{F}_{j|i}(c_i|c_i) - [b_i(c_i) - c_i]\, f_{ji}(c_i|c_i) = 0. \qquad (6.106)$$

[2] That is, $\Gamma(r)$ is the conventional gamma function. If r is an integer, $\Gamma(r) = (r-1)!$.

[3] Equation numbering follows Ortega Reichert, but the equations have been slightly rewritten for clarity.

It is not hard to check that $h(w|c_i) = \frac{(\delta + c_i)^{r+1}}{\Gamma(r+1)} w^r e^{-(\delta + c_i)w}$, that is, is a Γ-density function with parameters $(\delta + c_i, r+1)$.

[4] The next two equations paraphrase Ortega Reichert's argument. We are following the method of solving for the bidding functions that is illustrated in Klemperer (1999, note 121).

Substituting in for the distribution functions and integrating up yields that the symmetric equilibrium bidding functions are[5]

$$b_i(c_i) = c_i + \frac{¢ + 2c_i}{r ¡ 1}: \qquad (6.107)$$

So bids are an a¢ne function of costs.

Example 6.6.2. and equation (6.107) are referred to in the text of Chapter VIII.

Chapter VIII can also be seen as an extension of Chapter II of the thesis. Chapter II has a similar structure, likewise assuming that there are two periods of bidding, but it uses a decision-theoretic approach whereas Chapter VIII is fully game-theoretic. Chapter II's model makes assumptions directly about the bidding behaviour which have a similar e¤ect to assuming costs are positively intertemporally correlated.[6] The analysis demonstrates that a bidder's second-period pro…ts are non-decreasing in the …rst-period bid. Thus bidding in a one-shot procurement game provides a lower bound for bids in the …rst period of the sequential procurement auction (that is, they bid less aggressively in the sequential auction). These conclusions are referred to in the last sentence of Chapter VIII.

References

Klemperer, P. D. (1999), 'Auction Theory: A Guide to the Literature', Journal of Economic Surveys, 13, 227-286, reprinted in this volume, P.D. Klemperer (ed.), The Economic Theory of Auctions. Cheltenham, UK: Edward Elgar.

Milgrom, P. R. and D. J. Roberts (1982), 'Limit Pricing and Entry under Incomplete Information: An Equilibrium Analysis', Econometrica, 50,

[5] This equation corrects a typographical error in Ortega Reichert's equation (6.107).

[6] It is assumed that a bidder who loses in period one will be more aggressive in period two than a bidder who wins in period one, and that the smaller the margin of defeat, the smaller the increase in aggressiveness of the losing bidder.

Another di¤erence with Chapter VIII's model is that only the winner's bid is observed in Chapter II, whereas both bids are seen in Chapter VIII.

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