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Price competition vs. quantity competition: the role of uncertainty

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We analyze the Nash equilibria of a one-stage game in which the nature of the strategic variables (prices or quantities) is determined endogenously. Duopolists producing differentiated products simultaneously choose either a quantity to produce or a price to charge. In the absence of exogenous uncertainty, there exist four types of equilibria with differing levels of output: (price, price), (quantity, quantity), (price, quantity), and (quantity, price). The multiplicity of equilibria stems from each firm's indifference between setting price and quantity, given its conjecture about its rival's strategy. But exogenous uncertainty about market demands, which makes firms uncertain about their residual demands, even in equilibrium, gives firms strict preferences between setting price and quantity. As a result, the number of equilibria is reduced. When uncertainty is exogenous, we analyze the effect of the slope of marginal costs, the nature of the demand disturbance, and the curvature of demand on firms' propensities to compete with price or quantity as the strategic variable. These three factors are likely to influence the nature and intensity of oligopolistic competition.

1. Introduction

■ Economists using games to represent oligopolistic competition debated the relative merits of models using prices or quantities as firms' strategic variables from as early as Bertrand's (1883) criticism of Cournot (1838). The extent to which firms can choose price or quantity must, of course, crucially affect the nature of competition.[1] If they have some choice, however, the extent to which firms want to choose price or quantity may be important.

When firms know both the market demand and, in equilibrium, other firms' choices of strategic variables, each firm is indifferent between setting price and quantity. Because

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[1] In an agricultural market farmers may have little choice but to pick a quantity to plant and then sell the output at the market-clearing price. A discount brokerage, on the other hand, must pick a price per share traded rather than a quantity, at least in the short run.

the residual demand is known with certainty, each strategic variable precisely determines the other, and either can be set to ensure the monopolistic price and quantity on the residual demand curve. But when firms are uncertain about their residual demands, because shocks to market demand are unobservable or because the level or type of strategic variables chosen by other firms is unknown, each firm's choice between setting a price and setting a quantity becomes important. Setting a price and setting a quantity generally give different expected losses relative to the maximum profits that could be achieved if the residual demand were perfectly known.

Weitzman (1974) emphasized the importance of the choice between setting price and setting quantity under uncertainty. The objective function in his model was expected total surplus. In our analysis firms maximize expected profits, and we incorporate their choices between price and quantity strategies into an oligopoly model to determine the strategic variables used in equilibrium.

We present a model of a duopoly in which firms endogenously select strategic variables. Firms simultaneously choose either a quantity to produce or a price to charge. Given the types and levels of the strategic variables selected, the remaining prices and quantities are determined to clear all markets. We are interested in the Nash equilibria of this one-stage game.

In the absence of uncertainty there are four types of equilibria: (price, price), (quantity, quantity), (price, quantity), and (quantity, price). Output levels, and hence the intensity of competition, differ among these equilibria. Although each firm is indifferent between choosing price and quantity, in each equilibrium it is crucial that each firm select the strategic variable that its competitor expects. Otherwise, its competitor would face a different residual demand curve and would want to choose a different action.

When exogenous uncertainty about market demands is introduced, the duopolists are no longer indifferent between setting price and quantity. As a consequence, the number of Nash equilibria is reduced. More precisely, consider four duopoly games under uncertainty, in each of which each firm is assigned one of its two possible strategic variables, but is allowed to choose its level. Call the equilibria of these games "candidate equilibria." When we make the choice of strategic variable endogenous by enlarging firms' strategy sets, we find that not all of the candidate equilibria are Nash equilibria. In the simple models we analyze only one equilibrium remains.

We consider the determinants of equilibria in games in which strategic variables are chosen endogenously in the presence of uncertainty. After offering an illustration in Section 2, we examine in Sections 3, 4, and 5 the role of the slope of the marginal cost curve, the nature of the demand uncertainty, and the curvature of demand and show how these factors affect the duopoly equilibria by influencing a firm's preference between price and quantity under uncertainty. In the models of these three sections the duopoly equilibrium is unique. In Section 6 we present an approximate general analysis for determining an individual firm's choice between setting price and setting quantity and show how the models of the three previous sections are special cases. Section 7 contains conclusions and suggestions for extensions.

Rather than provide a complete answer to the question, "What is the 'correct' oligopoly solution concept?"—an exercise that would require including a time dimension and a theory of when firms can credibly commit to which variables and at what cost—, our goal is simply to illustrate one component of the answer. We focus on how uncertainty will affect oligopolists' propensities to compete with price or with quantity as the strategic variable.

2. An illustration

■ Consider the case of consulting. If a consultant charges a fee per hour, he is setting a price. On the other hand, he might set a fixed fee per consultation. If he then spends less time on any one client when his office is busy than when demand is slack, so that his total work day is constant, he is, in effect, setting a quantity and adjusting price in accord with demand. In fact, many consulting firms seem to operate in the latter mode.[2]

In one international firm with both an auditing and a management consulting division, the management consulting division operates in a quantity-setting fashion, whereas the auditing division sets a price (i.e., a strictly defined fee per hour). We shall see that, because demand uncertainty in management consulting is high, the rapidly increasing marginal costs around full staff utilization (from close to zero to close to infinity) make fixing quantity the optimal choice for this division. In auditing, on the other hand, demand uncertainty is very low[3] so that our theory suggests that neither variable will be strongly preferred. Internal organizational factors dictate setting a price: charging a set price per hour for the hours actually worked both increases internal control and records exactly how long different tasks take, which is useful information for future audits.[4]

3. The general linear model

■ We analyze a differentiated products duopoly in which each firm simultaneously chooses either a quantity to produce or a price to charge. That is, each firm's strategy set is the union of all fixed quantities and all fixed prices. Given the types and levels of the two strategic variables selected, the remaining prices and quantities are determined to clear both markets. We do not model the process by which markets are cleared, just as, for example, it is not explicitly modelled in a Cournot game under uncertainty. Our interest is in characterizing the Nash equilibria of this one-stage game in which strategic variables are selected endogenously. We restrict our attention to pure-strategy equilibria.

Consider symmetric duopolists with linear marginal cost curves, $C'(q) = c_1 + c_2 q$, $c_1 > 0$, and the linear demand system,

$$p_i = \alpha - \beta q_i - \gamma q_j \tag{1a}$$

$$p_j = \alpha - \beta q_j - \gamma q_i, \tag{1b}$$

where $\alpha > c_1$, $\beta > 0$, and $\beta \geq \gamma \geq 0$. When $\gamma = \beta$, the products are perfect substitutes, whereas when $\gamma = 0$, demands are independent.

In the absence of uncertainty, if firm i conjectures that j is setting its quantity at $\bar{q}_j$, i's residual demand is $p_i = \alpha - \beta q_i - \gamma \bar{q}_j$, whereas if i conjectures that j is setting its price at $\bar{p}_j$, then i's residual demand is $p_i = \alpha(\beta - \gamma)/\beta + \gamma\bar{p}_j/\beta - q_i(\beta^2 - \gamma^2)/\beta$. This last equation is derived by solving (1b) for q_j in terms of q_i and $\bar{p}_j$ and substituting into (1a). Thus, given its conjecture about j's choice of strategic variable, i knows its residual demand exactly, and, regarding itself as a monopolist with respect to its residual demand, i identifies its profit-maximizing point (p_i^*, q_i^*). The absence of uncertainty guarantees that i can achieve this point by setting *either* its quantity $(q_i = q_i^*)$ *or* its price $(p_i = p_i^*)$ appropriately, and i is thus indifferent between these two strategies. As a consequence, without uncertainty there

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[2] Although they often quote a fixed rate per consultant per hour, when business is slack, more hours are worked on a project than are reported, and travel time is not charged. When the office is busy, however, travel, marginally related training, and even the time spent originally negotiating the project may all be charged to the client. In some consulting firms, project managers bid for staff resources, so that the firm internally resembles an auction market in which the real price of consulting time is bid up until demand is brought into balance with the (roughly) fixed supply.

[3] Market demand is roughly fixed owing to government regulation. In addition, the large costs to clients of switching between firms keep individual firms' demands stable and predictable.

[4] It is difficult to keep different sets of books for internal and for client use. The auditing division, in contrast to the management consulting division, has built up a strong ethic that staff should accurately report the hours they work.

exist four Nash equilibria that correspond to each firm's choosing either of its two possible strategic variables.[5] The equilibria are supported by each firm's choosing the strategic variable that its rival expects; although each firm sees its own choice between price and quantity as irrelevant, its choice is not irrelevant to its rival because the selection determines the residual demand curve that its rival faces.

Now suppose that both firms' demands are subject to a random shock that neither can observe at the time strategic variables are chosen.[6] In particular, let the shock be additive:

$$p_i = \alpha - \beta q_i - \gamma q_j + \epsilon \tag{2a}$$

$$p_j = \alpha - \beta q_j - \gamma q_i + \epsilon, \tag{2b}$$

where ϵ is a random variable assumed without loss of generality to have mean zero.[7] In each of the four games in which each firm is assigned one of its two possible strategic variables (price or quantity) and is allowed to choose its level, there is a unique equilibrium. Because of the linearity of the demand and marginal cost curves and the additivity of the shocks, these four "candidate equilibria" coincide with the four Nash equilibria in the game in which strategic variables are determined endogenously under certainty. Under uncertainty, however, the four candidate equilibria will not all be equilibria when firms can select which strategic variable to use, because the strategic variables assigned may not be the optimal ones to use.[8] Specifically, given its conjecture about j's choice of strategic variable ($\bar{q}_j$ or $\bar{p}_j$), i's position is that of a monopolist facing the uncertain (residual) demand given by

$$p_i = \alpha - \beta q_i - \gamma \bar{q}_j + \epsilon$$

or

$$p_i = \alpha\left(\frac{\beta-\gamma}{\beta}\right) + \frac{\gamma \bar{p}_j}{\beta} - q_i\left(\frac{\beta^2-\gamma^2}{\beta}\right) + \left(\frac{\beta-\gamma}{\beta}\right)\epsilon.$$

Under uncertainty a monopolist is not in general indifferent between (1) choosing a quantity and then selling at the resulting market price and (2) setting a price and then producing what the market demands. In both cases the resulting price-quantity pairs will diverge from the *ex post* optimal pairs, but the two strategies will lead to different expected losses relative to the profits that would be achievable if *ex post* adjustment were feasible.[9]

We use the following lemma.

Lemma 1. Consider a monopolist facing the uncertain demand $p = A - Bq + \theta$, where $E\theta = 0$, $E\theta^2 = \sigma^2$, $A > 0$, and $B > 0$, and having the cost function $C(q) = c_1 q + c_2(q^2/2)$, where $0 < c_1 < A$ and $c_2 > -2B$ (the latter condition ensures that the second-order conditions are satisfied). Denote the optimal set quantity $\hat{q}$ and the optimal set price $\hat{p}$. The difference in expected profits between setting $\hat{q}$ and setting $\hat{p}$ is

[5] In the case of decreasing marginal costs we assume that $c_2 > -2(\beta^2 - \gamma^2)/\beta$, so that the second-order conditions are always satisfied, and that c_1 is large enough and $|c_2|$ small enough that prices are always positive in equilibrium.

[6] Whether firms are subject to the same or different shocks is not important. When firms' demands are subject to different shocks of arbitrary correlation, the proofs become more complicated, but the results are unaffected.

[7] As specified, this system allows the possibility that a firm setting a price may sell a negative quantity, or that a firm setting a quantity may receive a negative price, in each case depending on the values of the random variables and the other firm's strategic variable. Strictly, the demand system should incorporate constraints preventing these outcomes. Henceforth, we assume for every demand system and associated pair of cost curves that the support of the noise is small enough that these constraints never affect the equilibrium.

[8] When the assumptions of linearity and of additivity of the shocks are relaxed, the set of candidate equilibria under uncertainty will not match the set of equilibria under certainty.

[9] A monopolist would prefer to adjust both price and quantity in accord with the demand shock, but we are assuming that such *ex post* adjustment of both variables is infeasible and that one of them must be fixed *ex ante*.

$$E\Pi(\hat{q}) - E\Pi(\hat{p}) = \frac{\sigma^2 c_2}{2B^2}.$$

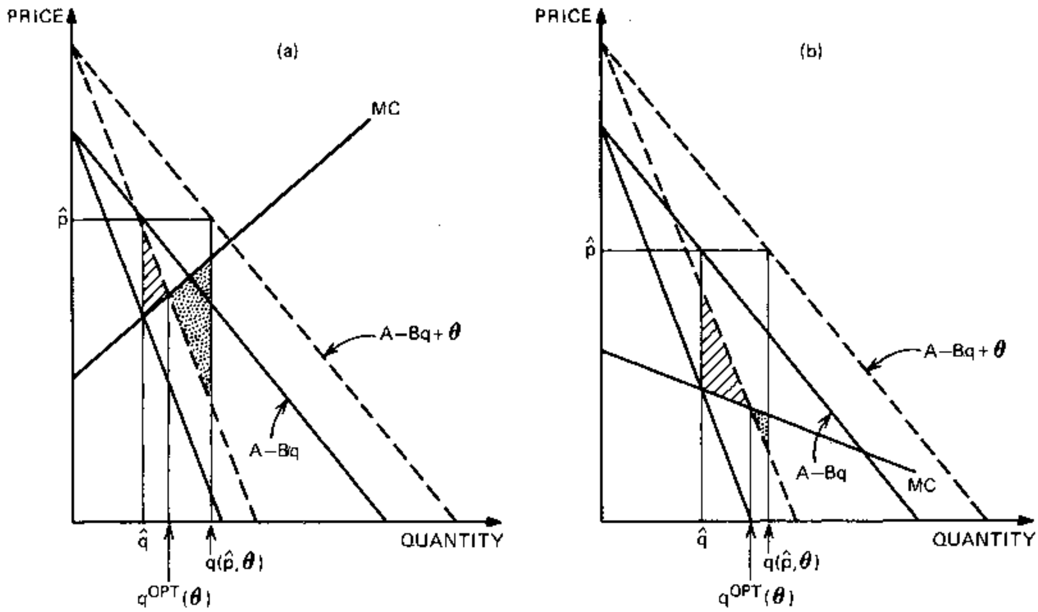
Hence, the monopolist strictly prefers to set quantity (price) when marginal costs slope upward (downward) and is indifferent only when marginal costs are flat.

The proof of Lemma 1 appears in the Appendix. For a graphical interpretation refer to Figure 1, which compares for a particular realization of θ the loss from setting $\hat{p}$ (dotted area) and the loss from setting $\hat{q}$ (hatched area) relative to the potential profit from setting price and quantity at their *ex post* optimal levels. Because of the linearity of demand and marginal costs, these losses are represented by similar triangles.[10] The loss from setting $\hat{p}$ exceeds the loss from setting $\hat{q}$ if the output resulting from $\hat{p}$, $q(\hat{p}, \theta)$, is farther from the monopolist's optimal output, $q^{opt}(\theta)$, than $\hat{q}$ is. Since for any θ the marginal revenue curve is twice as steep as demand, $q^{opt}(\theta)$ is halfway between $q(\hat{p}, \theta)$ and $\hat{q}$ when the marginal cost curve is flat, closer to $\hat{q}$ when marginal costs slope upward (Figure 1(a)), and closer to $q(\hat{p}, \theta)$ when marginal costs slope downward (Figure 1(b)). Thus, the steeper is the marginal cost curve, the less the *ex post* profit-maximizing quantity varies with θ, and the stronger is the preference for setting quantity.[11]

An alternative explanation for the critical role of the slope of marginal costs follows from the observation that for c_2 greater than (less than) zero, the cost function is strictly convex (strictly concave). With linear demand and an additive shock, expected output from setting $\hat{p}$ equals $\hat{q}$, and expected revenue from the $\hat{p}$ and $\hat{q}$ strategies is equal. The relative profitability of these strategies therefore depends only on the relative size of expected costs,

FIGURE 1

THE SLOPE OF MARGINAL COSTS



[10] Linearity and additivity also ensure that $\hat{p}$ and $\hat{q}$ are equal to the *ex post* optimal price and quantity when θ takes its mean value of zero.

[11] Weitzman's (1974) model shares the feature that the relative performance of a set price and a set quantity under uncertainty depends on the relative distances of the resulting outputs from the optimal output. But because the objective function in Weitzman's model is expected total surplus rather than expected profits, it is the sign of $c_2 - B$ that is critical.

and for convex (concave) costs, a fixed level of output is more (less) attractive than a random level with the same mean. With a linear cost function (constant marginal costs), $\hat{p}$ and $\hat{q}$ yield equal expected profit.

The slope of a linear demand curve affects only the magnitude and not the sign of the difference in expected profits between a price and a quantity strategy. A more elastic demand increases the fluctuations in both the *ex post* optimal quantity and the output from setting $\hat{p}$, and as a result increases the difference in expected losses from setting $\hat{p}$ and $\hat{q}$. An increase in the variance of the demand shock has the same effect.

We can now state the following proposition.

Proposition 1. Consider a differentiated products duopoly facing the demand system (2a) and (2b), where ϵ is a random variable, with mean zero, that is not observable by either firm at the time strategic variables are chosen. Let each firm's cost function be $C(q) = c_1 q + c_2(q^2/2)$, where $0 < c_1 < \alpha$. (a) If marginal costs slope upward, the unique Nash equilibrium involves both firms' choosing quantities. (b) If marginal costs slope downward (but $c_2 > -2(\beta^2 - \gamma^2)/\beta$ so that the second-order conditions are satisfied for each of the candidate equilibria), the unique Nash equilibrium involves both firms' choosing prices. (c) If marginal costs are constant, there exist four Nash equilibria, corresponding to the four possible pairs of strategic variable choices, except if the goods are perfect substitutes, in which case only the (quantity, quantity) and (price, price) equilibria exist.

Proof. When firm j sets quantity at $\bar{q}_j$, firm i is a monopolist facing the residual demand

$$p_i = (\alpha - \gamma \bar{q}_j) - \beta q_i + \epsilon,$$

which is linear with an additive shock. When j sets price at $\bar{p}_j$, i's residual demand is

$$p_i = \alpha\left(\frac{\beta - \gamma}{\beta}\right) + \frac{\gamma \bar{p}_j}{\beta} - q_i\left(\frac{\beta^2 - \gamma^2}{\beta}\right) + \left(\frac{\beta - \gamma}{\beta}\right)\epsilon,$$

which is also linear with an additive shock. Therefore, in both cases by Lemma 1 firm i's choice of strategic variable depends only on the slope of its marginal cost curve. For c_2 greater than (less than) zero, i prefers to set a quantity (price), whichever strategic variable j chooses. By symmetry, the only Nash equilibrium entails fixed quantities (prices) when marginal costs slope upward (downward).

When marginal costs are constant and $\gamma < \beta$, Lemma 1 implies that each firm is indifferent between setting price and setting quantity, whatever action its rival chooses. Hence, all four "candidate equilibria" are Nash equilibria of the game.

For $\gamma = \beta$ (perfect substitutes), the demand system (2a) and (2b) as written is ill-specified. When we take proper account of the constraints that firms' quantities and prices can never be negative, the previous results still hold. For constant marginal costs, however, the asymmetric (price, quantity) and (quantity, price) equilibria do not exist. To show this, first observe that firm i's best response to any $\bar{p}_j > c_1$ is a price lower than $\bar{p}_j$ by an arbitrarily small amount, if $\bar{p}_j$ is less than the monopoly price. For higher $\bar{p}_j$ either a price is the unique best response or (for small uncertainty and large $\bar{p}_j$) the best price and the best quantity are equally profitable; in either case j obtains zero sales and hence zero profits.[12] Any $\bar{p}_j < c_1$ clearly involves losses. Hence, the only price that can be part of an equilibrium is c_1. If $\bar{p}_j = c_1 > 0$, i can earn expected profits of at most zero and is willing to choose either a price or a quantity. But i will not choose a quantity greater than $(\alpha + \underline{\epsilon} - c_1)/\beta$, where $\underline{\epsilon}$ is the lower bound of the support of ϵ, since to do so would risk earning negative profits. In

[12] The latter case arises when i can behave as a monopolist, unconstrained by j's price, and i's indifference between price and quantity then follows from Lemma 1. The former case arises if, with positive probability, the shock is so large that when i sets a quantity, j is left with positive demand.

response to $q_i \leq (\alpha + \underline{\epsilon} - c_1)/\beta$, j would prefer to choose a price above c_1 (or a quantity), and therefore no asymmetric equilibria exist.[13] *Q.E.D.*

Under uncertainty, the "cost" of choosing a particular strategic variable is the loss in expected profits relative to those earned when there is optimal *ex post* adjustment of both price and quantity. That is, it is the cost of ending up at the wrong point on the (residual) demand curve. The steeper is the marginal cost curve, the larger is this cost for setting price. This result is quite robust and will emerge again in the general formulation of Section 6. For the particular oligopoly model considered in this section, whenever marginal costs are upward sloping, setting quantity is always an optimal response for both firms, so the unique Nash equilibrium involves quantities as strategic variables.

□ **Setting price and maximum quantity.** It is often argued that "price competition" is better modelled by firms' setting both prices and maximum quantities that they are willing to sell.[14] We, instead, have deliberately focused on the simplest possible model (similarly, we have not allowed a firm setting quantity to refuse to sell if prices fall excessively low), and have preserved the symmetry between price competition and quantity competition. In fact, the difference between the two representations of price competition is typically small in our model. The optimal set price is determined approximately by the intersection of expected marginal revenue with marginal cost; the optimal maximum quantity (if allowed) is where the chosen price intersects marginal cost.

Without uncertainty, therefore, the maximum-quantity constraint can never bind in a (pure-strategy) equilibrium, because the firm for which it was binding would not be at a profit-maximizing point. Thus, if we give firms the choice between (a) setting a quantity and (b) setting a price and a maximum quantity, the only possible equilibrium outcomes are those corresponding to the four Nash equilibria of the simpler game. But one or more of the equilibria involving price as a strategic variable need not exist in this new game.[15]

It also follows that for a sufficiently small support of the uncertainty, the maximum-quantity constraint can never bind with positive probability.[16] Thus, the equilibria that match the equilibria of the simpler game with uncertainty are the only possible ones in the new game. Further, for the model of this section all of the equilibria in the simpler game with uncertainty survive in the new game: with increasing marginal costs the only equilibrium in the simpler game involves quantities and so remains an equilibrium, and with flat or decreasing marginal costs the quantity constraints are irrelevant, so that all the equilibria remain.

For sufficiently small uncertainty, therefore, Lemma 1 and Proposition 1 both hold when firms choose between either setting a quantity or setting a price and a maximum

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[13] The nonexistence of asymmetric (price, quantity) and (quantity, price) equilibria with perfect substitutes depends on the assumption that the constant level of marginal costs is strictly positive, an assumption that we make throughout the article. If marginal costs were zero, then in response to $\bar{p}_j = 0$, i would be willing to choose a quantity large enough ($q_i \geq (\alpha + \bar{\epsilon})/\beta$, where $\bar{\epsilon}$ is the upper bound of the support of ϵ) that $\bar{p}_j = 0$ is an optimal response. In all of the models we consider, asymmetric equilibria exist for the case of perfect substitutes and constant marginal costs if and only if marginal costs are zero.

[14] Kreps and Scheinkman (1983) for example, present a model in which capacity commitment followed by this kind of price competition yields the same outcome as single-stage quantity competition.

[15] The reason is that the residual demand of the competitor of the firm setting price is higher when the latter's constraint is binding than in the simpler game. The competitor may therefore prefer to reduce quantity (or raise price) from the equilibrium value of the simpler game to make its opponent's constraint binding. This behavior may upset an equilibrium of the simpler game, although it can never create a new one.

[16] If it did, one firm's marginal cost would be close to its price, and hence above its marginal revenue for all values of the shock. This would contradict the optimality of its response. With vertical marginal costs this argument does not apply, but choosing a quantity then achieves a firm's maximum possible profit for all realizations of the shock, so the equilibrium will not involve prices and maximum quantities.

quantity to produce. Similarly, the subsequent results in this article all hold for a game incorporating the alternative model of price competition for sufficiently small uncertainty with the exception that, where equilibria involving prices are claimed for cases with increasing marginal costs, these may not exist.[17]

4. The nature of the uncertainty

■ We demonstrate in this section that firms' preferences over strategic variables, and hence the nature of the Nash equilibria in our game, are sensitive to the manner in which random disturbances affect demands. Specifically, we show that if uncertainty affects the slopes of linear demand curves, rather than just the intercepts, then price competition will arise for flat or not excessively rapidly rising marginal cost curves.

As in Section 3, we begin by examining the behavior of a monopolist.

Lemma 2. Consider a monopolist facing a linear demand curve with fixed vertical intercept but random slope,

$$p = A - \frac{Bq}{\theta},$$

where θ is a strictly positive random variable with $E\theta = 1$ and $E\theta^2 = s^2 > 1$ and where $A > 0$ and $B > 0$. Let z denote $E(1/\theta)$, which, by Jensen's inequality, is strictly greater than one. Let the cost function be $C(q) = c_1 q + c_2 (q^2/2)$, where $0 < c_1 < A$ and $c_2 > -2B/s^2$. The latter condition ensures that the second-order conditions are satisfied. Denote the optimal set quantity $\hat{q}$ and the optimal set price $\hat{p}$. The difference in expected profits between setting $\hat{q}$ and setting $\hat{p}$ is

$$E\Pi(\hat{q}) - E\Pi(\hat{p}) = \frac{(A - c_1)^2}{2}\left(\frac{1}{2Bz + c_2} - \frac{1}{2B + c_2 s^2}\right).$$

Let $c_2^\dagger = (2(z-1)/(s^2-1))B > 0$. For all c_2 less than (greater than) $c_2^\dagger$, $E\Pi(\hat{q}) - E\Pi(\hat{p})$ is strictly negative (positive), and so setting a price (quantity) is strictly preferred.

The proof appears in the Appendix. The intuition is that rotation of the demand curve about the vertical intercept represents replication or shrinking of the market, with the distribution of consumers' reservation prices remaining unchanged. If marginal costs are constant, such a rotation does not alter the profit-maximizing price. Thus, setting a price is *ex post* optimal for flat marginal costs and, by continuity, is preferred to setting a quantity for slowly rising marginal costs. As in Section 3, however, as the slope of the marginal cost curve increases, the output fluctuations resulting from a set price become increasingly undesirable, because the *ex post* optimal quantity varies less and less. For a sufficiently steep marginal cost curve, setting quantity is preferred.

For another interpretation of this result, consider the dependence of price on θ for any fixed output. In Lemma 2 the price is strictly concave in θ and so has lower expected value than in Lemma 1, where the price is linear in θ. The output from setting price is linear in θ in both cases, with the same expected value. Therefore, since expected revenues from the optimal quantity and price strategies are equal in Lemma 1, the optimal quantity strategy yields strictly lower expected revenue than the optimal price strategy for the demand curve

[17] In some cases we can be sure that the claimed (price, price) equilibrium does exist, because the deviation that would be required of a firm to make its opponent's maximum-quantity constraint binding can be shown to be so large as to be unattractive. For example, in Proposition 2 for sufficiently small uncertainty and for marginal costs not rising excessively rapidly, driving the opponent to the point where its maximum-quantity constraint is binding requires a firm to reduce its own quantity below zero. Hence, under these conditions the claimed (price, price) equilibrium remains an equilibrium when the notion of price competition is generalized.

of Lemma 2. Against this must be set the difference in expected costs, which depends, as Section 3 showed, on the convexity or concavity of the cost curve.

For a given distribution of (multiplicative) uncertainty about the slope, the cutoff value c_2^* above which quantity is preferred is linear in the expected slope of demand. To see this, observe that multiplying the expected slope of demand and the slope of marginal costs by the same factor is equivalent to multiplying the size of the units in which we measure output and the value of the currency unit in which we measure price by that factor, and so does not alter the preference between setting price and setting quantity.

Proposition 2. Consider a differentiated products duopoly facing the demand system,

$$p_i = \alpha - \frac{\beta q_i}{\epsilon} - \frac{\gamma q_j}{\epsilon} \tag{3a}$$

$$p_j = \alpha - \frac{\beta q_j}{\epsilon} - \frac{\gamma q_i}{\epsilon}, \tag{3b}$$

where ϵ is a strictly positive random variable, with mean one, that is not observable by either firm at the time strategic variables are chosen. Let the cost curves be $C(q) = c_1 q + c_2(q^2/2)$. Assume that $c_1 > 0$ and that $c_2 > -2(\beta^2 - \gamma^2)/\beta s^2$, where $s^2 = E\epsilon^2$, so that the second-order conditions are always satisfied. There exists a $\bar{c}_2 \geq 0$ such that for $c_2 < \bar{c}_2$ in the unique Nash equilibrium firms set prices. For all $\gamma < \beta$, $\bar{c}_2$ is strictly positive. For the case of perfect substitutes ($\gamma = \beta$), $\bar{c}_2 = 0$, and for $c_2 = 0$, the unique Nash equilibrium entails price-setting.

The proof appears in the Appendix. Note that for a given firm k the shock affects the sensitivity of the firm's price to the quantity $\beta q_k + \gamma q_m$, $m \neq k$, but does not affect the relative sensitivity of the price to the firm's own and the other firm's quantity, i.e., the degree of substitutability of the products remains unchanged.

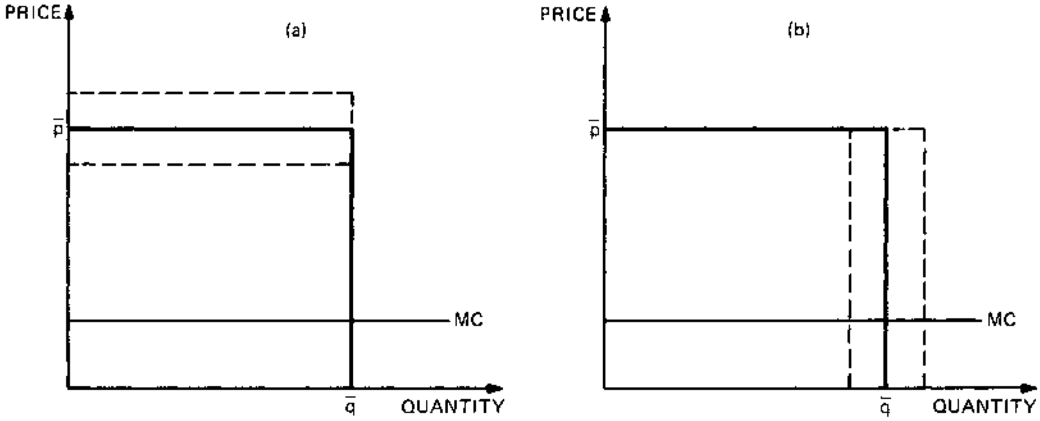
The uncertainty in the slopes of firms' residual demands makes the analysis and intuition similar to that of Lemma 2. For shallow marginal cost curves a firm's optimal response to either choice of strategic variable by its opponent is a set price, so for low values of c_2, there is a unique Nash equilibrium, with prices as strategic variables. Note that the cutoff value $\bar{c}_2$ below which we have a (price, price) equilibrium depends on the demand system. In particular, as the goods approach perfect substitutes, $\bar{c}_2$ approaches zero.[18] Even for perfect substitutes, however, the unique equilibrium for constant marginal costs and uncertainty about demand slopes is the (price, price) one, in contrast to Proposition 1, where both a (price, price) and a (quantity, quantity) equilibrium exist.

The models of Sections 3 and 4 have shown that the nature of competition depends in part on the nature of the uncertainty in market demands. For any demand curve, rotation about a fixed vertical intercept (i.e., $q = tg(p)$, where t varies) represents a change in the total size of a market in which the distribution of consumers' reservation prices remains unchanged. On the other hand, rotation about a fixed horizontal intercept (i.e., $p = tf(q)$) represents a particular type of change in the distribution of reservation prices, with the total size of the market remaining unchanged. A vertically additive shift of the demand function is an intermediate case. It involves a change in both the size of the market and the distribution of reservation prices. The contrast between the two extreme types of uncertainty is illustrated by the kinked demand curve of Figure 2, which has a horizontal segment at $\bar{p}$ and a vertical segment at $\bar{q}$. With marginal costs constant, uncertainty in the reservation price dictates setting the quantity $\bar{q}$ (Figure 2(a)), whereas uncertainty in the size of the market dictates

[18] As the goods become better substitutes, each firm's residual demand against a set price becomes flatter so that, as in Lemma 2, the cutoff slope of marginal costs above which setting quantity is preferred falls.

FIGURE 2

THE NATURE OF THE UNCERTAINTY



setting the price $\bar{p}$ (Figure 2(b)). Comparison of Proposition 2 with Proposition 1 shows that prices are more attractive as strategic variables when firms' uncertainty relates only to the size of the market than when the distribution of reservation prices is uncertain as well. The reason is simply that shocks that affect only the market size generate relatively little variation in the (*ex post*) optimal price. The variation is less, the flatter are marginal costs, and thus for not excessively rapidly rising marginal cost curves we expect to observe price competition in the presence of such shocks.[19]

5. The curvature of demand

■ This section explores how firms' choices between price and quantity as strategic variables are influenced by the curvature of their demands. The linear model of Section 3 showed that the slope of the demand curve affects the intensity of preference between price and quantity but not the ranking itself. The curvature of demand, on the other hand, does influence which strategic variable is preferred. It does so by introducing an asymmetry into the firm's response to positive and negative demand shocks.

As before, we first analyze the behavior of a monopolist.

Lemma 3. Consider a monopolist facing a demand curve that is subject to a vertically additive shock,

$$p = f(q) + \theta,$$

where $f(q)$ is decreasing and $E\theta = 0$. Denote the optimal set quantity $\hat{q}$ and the optimal set price $\hat{p}$. (Assume that the second-order conditions hold.) (a) If $f(\cdot)$ is strictly concave and marginal costs are increasing, constant, or not excessively rapidly decreasing, then the expected profits from setting $\hat{q}$ exceed those from setting $\hat{p}$. (b) If $f(\cdot)$ is strictly convex and marginal costs are decreasing, constant, or not excessively rapidly increasing, then the expected profits from setting $\hat{p}$ exceed those from setting $\hat{q}$.

[19] For constant marginal costs a monopolist prefers to set a quantity when the unobservable demand shock causes the demand curve to rotate about its intersection with the marginal cost curve. The reason is that for this type of shock, the *ex post* optimal quantity is constant. It follows that, for a sufficiently small level of constant marginal costs, quantity-setting is also preferred when the shock causes the demand curve to rotate about its horizontal intercept. When this type of uncertainty is incorporated into a duopoly model, the unique Nash equilibrium involves quantities as strategic variables.

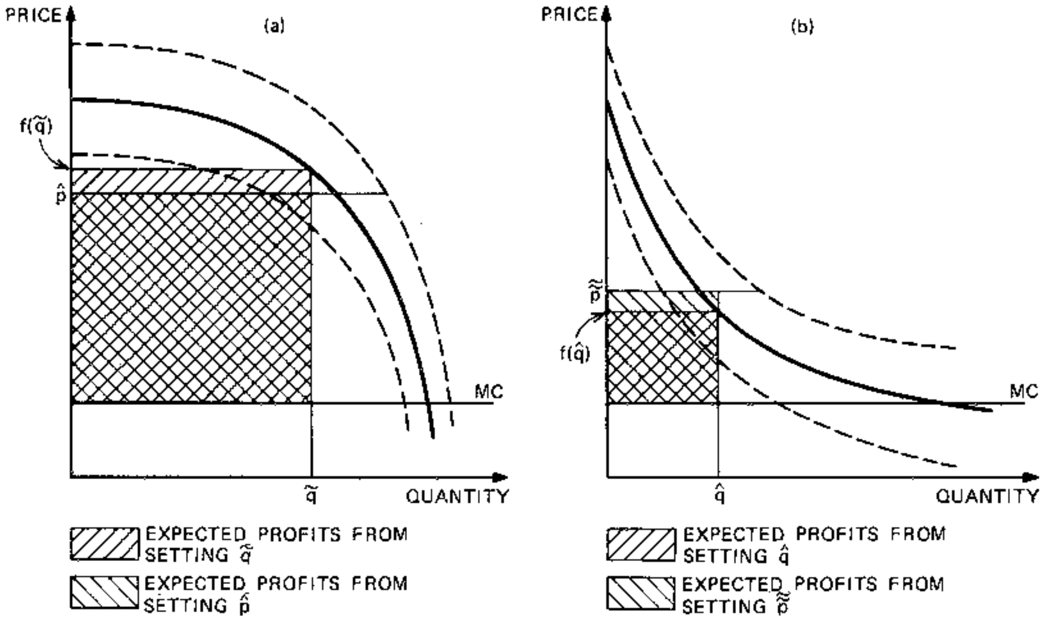
We prove Lemma 3 in the Appendix. For the intuition behind the result, observe that for a strictly concave or convex demand curve subject to a vertically additive shock, the output response under a set price is asymmetric with respect to positive and negative values of θ. In particular, when $f(\cdot)$ is strictly concave, the increase in output for a given $\theta > 0$ (relative to the output at $\theta = 0$) is less than the reduction in output for $-\theta$, and the reverse is true when $f(\cdot)$ is strictly convex. Therefore, for $f(\cdot)$ strictly concave (convex), the average output under a set price is less than (greater than) the output when the shock assumes its mean value. To apply this fact we compare, for $f(\cdot)$ strictly concave, expected revenue from setting the optimal price $\hat{p}$ with that from setting quantity at the level, $\tilde{q} < f^{-1}(\hat{p})$, of expected output under $\hat{p}$ (see Figure 3(a)). Expected revenue from the quantity strategy is higher because with a vertically additive shock the average price received equals $f(\tilde{q}) > \hat{p}$. Increasing marginal costs, implying convex total costs, simply reinforce the preference for setting a quantity. Since $\tilde{q}$ is preferred to $\hat{p}$, the best set quantity, $\hat{q}$, is preferred to $\hat{p}$.

For $f(\cdot)$ strictly convex we compare the best set quantity $\hat{q}$ with the price, $\tilde{\tilde{p}} > f(\hat{q})$, which yields average output equal to $\hat{q}$ (see Figure 3(b)). With a vertically additive shock average price from the quantity strategy equals $f(\hat{q})$, and hence expected revenue from setting $\hat{q}$ equals $\hat{q}f(\hat{q})$, which is less than $\hat{q}\tilde{\tilde{p}}$, the expected revenue from setting $\tilde{\tilde{p}}$. Decreasing marginal costs, implying concave total costs, reinforce the preference for a variable output level. Since $\tilde{\tilde{p}}$ is preferred to $\hat{q}$, the best set price, $\hat{p}$, is preferred to $\hat{q}$.

For an alternative interpretation of these results, observe that the curvature of demand affects the ratio of the slope of marginal revenue, $2f'(q) + qf''(q)$, to the slope of demand, $f'(q)$. The more concave is demand, the steeper is marginal revenue relative to demand, and hence the smaller are the fluctuations in the *ex post* optimal quantity relative to the fluctuations in output under a set price. Concavity of demand therefore imparts a preference for stabilizing quantity.[20]

FIGURE 3

THE CURVATURE OF DEMAND



[20] On the cost side, the attractiveness of setting quantity depends on the slope of the marginal cost curve, whereas on the demand side, the critical factor is the slope of marginal revenue relative to the slope of demand. This asymmetry stems from the fact that, for a given set price and set quantity, changing the slope of marginal

The following proposition incorporates the effect of the curvature of demand into a differentiated products duopoly model.

Proposition 3. Consider the demand system,

$$p_i = f(q_i) + \lambda f(q_j) + \epsilon \tag{4a}$$

$$p_j = f(q_j) + \lambda f(q_i) + \epsilon, \tag{4b}$$

where $\lambda \in [0, 1)$ and where ϵ is a random variable, with mean zero, that is not observable by either firm at the time strategic variables are chosen. (Assume that the second-order conditions hold.) (a) If $f(\cdot)$ is strictly concave and marginal costs are increasing, constant, or not excessively rapidly decreasing, then the only Nash equilibria involve quantities as strategic variables. (b) If $f(\cdot)$ is strictly convex and marginal costs are decreasing, constant, or not excessively rapidly increasing, then the only Nash equilibria involve prices as strategic variables.

Proof. If firm j sets a quantity $\bar{q}_j$, then i's residual demand is

$$p_i = f(q_i) + \lambda f(\bar{q}_j) + \epsilon,$$

while if j sets a price $\bar{p}_j$, then i's residual demand is

$$p_i = (1 - \lambda^2) f(q_i) + \lambda \bar{p}_j + (1 - \lambda)\epsilon,$$

which is derived by solving (4b) for q_j and substituting into (4a). If $f(\cdot)$ is strictly concave and marginal costs are increasing or constant, then by Lemma 3, i strictly prefers to set a quantity regardless of the strategic variable j chooses. Moreover, for not excessively rapidly decreasing marginal costs, i prefers to set a quantity in both situations.[21] Hence, by symmetry, under these conditions, only (quantity, quantity) Nash equilibria exist. Similarly, if $f(\cdot)$ is strictly convex and marginal costs are decreasing, constant, or not excessively rapidly increasing, then by Lemma 3, i strictly prefers to set a price regardless of the strategic variable j chooses. Thus, by symmetry, under these conditions, only (price, price) Nash equilibria exist.[22] *Q.E.D.*

In the duopoly model considered in the proposition, each firm's residual demand has the same curvature regardless of the strategic variable the other selects, and hence each firm's preferred strategic variable is independent of the other's choice. Only one type of Nash equilibrium is therefore possible. In the presence of a vertically additive shock, concavity of the demand curve makes quantity the preferred strategic variable because of the asym-

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costs affects only the *ex post* optimal quantity, $q^{opt}(\theta)$, while changing the slope of marginal revenue alters the slope of demand and hence affects the output from a set price, $q(\hat{p}, \theta)$, as well. Changes in the curvature of demand, because they affect the slope of marginal revenue relative to the slope of demand, alter the relative sizes of $|q^{opt}(\theta) - \hat{q}|$ and $|q^{opt}(\theta) - q(\hat{p}, \theta)|$ and so affect the preference between $\hat{q}$ and $\hat{p}$. If all of the market power were on the other side of the market, as with a monopsonist facing an uncertain supply curve, the asymmetric roles of marginal revenue and marginal cost would be reversed. Setting quantity would be favored for steep marginal revenue curves and for convex supply curves; the latter condition implies that the marginal cost curve is steep relative to the supply curve.

[21] This follows from the continuity of $(E\Pi(\hat{q}_i) - E\Pi(\hat{p}_i))$ in the slope of marginal costs and in the other's strategic variable, and the fact that $(E\Pi(\hat{q}_i) - E\Pi(\hat{p}_i))$ is strictly positive for all values of the other's strategic variable when marginal costs are flat. By "continuous in the slope of marginal costs" we mean that, given any δ_1, there exists a δ_2 such that changing $C''(q)$ by not more than δ_2 at any q, but holding j's strategic variable constant, changes $(E\Pi(\hat{q}_i) - E\Pi(\hat{p}_i))$ by not more than δ_1.

[22] For $\lambda = 1$ the demand system is reasonable only if $f(\cdot)$ is linear, for which case see Proposition 1. (If $f(\cdot)$ were not linear, although $p_i = p_j$, the common price would respond to changes in q_i in a way different from that to changes in q_j.)

metrically large reductions in output that result from negative shocks under a set price; convexity of demand favors setting a price because of the asymmetrically large increases in output for positive shocks.

6. A general formulation

■ In this section we show how the effects illustrated in the three previous sections can be subsumed in an approximate general analysis of a monopolist's choice between setting price and setting quantity under demand uncertainty.

In the discussion of each of our lemmas, we saw that the ranking of the expected profits from the price and quantity strategies followed directly from the concavity or convexity of some element of the model. An alternative interpretation compared, for each value of the shock, the sizes of the deviations from the *ex post* optimal output levels under the two strategies. We now relate these two interpretations in a single formula. We exhibit a function whose local concavity or convexity provides an approximate formula for ranking price and quantity strategies. We then demonstrate that the same formula emerges from using the difference between output deviations under the two strategies to approximate the difference in their losses relative to *ex post* optimal adjustment.

Let the uncertain demand curve be $p = f(q, \theta)$, where $f(\cdot, \cdot)$ is twice continuously differentiable and $f_q < 0$ and $f_\theta > 0$ everywhere. Let $g(p, \theta)$ be defined by $f(g(p, \theta), \theta) = p$. The demand shock θ is assumed to be a continuous random variable. Let the cost curve be $C(q)$. We assume it is twice continuously differentiable.

We compare a set price p^* and a set quantity q^* that for some value of the demand shock, say θ^*, both yield the *ex post* optimal point on the corresponding demand curve. There is one such pair in which p^* equals the *ex ante* optimal price $\hat{p}$ and, in general, a different pair in which q^* equals the *ex ante* optimal quantity $\hat{q}$. If the ranking of the price and quantity strategies is the same for both pairs, this is sufficient to rank $\hat{p}$ and $\hat{q}$.[23]

Define the following three functions of θ:

$$\Pi^{opt}(\theta) = \text{unconstrained maximum profit;}$$

$$\Pi^{q}(q^*, \theta) = f(q^*, \theta)q^* - C(q^*)$$
$$= \text{profit from setting quantity at } q^*; \tag{5a}$$

$$\Pi^{p}(p^*, \theta) = p^*g(p^*, \theta) - C(g(p^*, \theta))$$
$$= \text{profit from setting price at } p^*. \tag{5b}$$

By construction $\Pi^q(q^*, \theta)$ and $\Pi^p(p^*, \theta)$ are both tangent to $\Pi^{opt}(\theta)$ at θ^*, and so also tangent to each other at this point. It follows that in a neighborhood of θ^*, either $\Pi^q(q^*, \theta)$ exceeds $\Pi^p(p^*, \theta)$ everywhere or $\Pi^p(p^*, \theta)$ exceeds $\Pi^q(q^*, \theta)$ everywhere, unless the two functions coincide.

To proceed formally we work directly with the function relating $\Pi^p(p^*, \theta)$ to $\Pi^q(q^*, \theta)$. Define for all θ the function T:

$$T(\Pi^q(q^*, \theta)) = \Pi^p(p^*, \theta).$$

Differentiating with respect to θ yields

$$T'(\Pi^q(q^*, \theta))\frac{\partial \Pi^q}{\partial \theta}(q^*, \theta) = \frac{\partial \Pi^p}{\partial \theta}(p^*, \theta). \tag{6}$$

[23] For example, if p^* is preferred to q^* for both pairs, then $\hat{q}$ is dominated by a nonoptimal price and hence by $\hat{p}$.

Differentiating again with respect to θ, substituting for T' by using (6), and solving for T'', we get

$$T''(\Pi^q(q^*,\theta)) = \left[\frac{\partial^2\Pi^p}{\partial\theta^2}(p^*,\theta)\frac{\partial\Pi^q}{\partial\theta}(q^*,\theta) - \frac{\partial^2\Pi^q}{\partial\theta^2}(q^*,\theta)\frac{\partial\Pi^p}{\partial\theta}(p^*,\theta)\right]\Big/\left(\frac{\partial\Pi^q}{\partial\theta}(q^*,\theta)\right)^3. \quad (7)$$

Since $\Pi^q(q^*, \theta)$ and $\Pi^p(p^*, \theta)$ are tangent at $\theta = \theta^*$, T is tangent to the 45°-line at the point $(\Pi^q(q^*, \theta^*), \Pi^p(p^*, \theta^*))$. It follows that if T is globally concave, profits from the quantity strategy are everywhere (weakly) larger than those from the price strategy, and therefore q^* is preferred to p^*. The reverse is true if T is globally convex.

Let us examine the sign of T'' at $\Pi^q(q^*, \theta^*)$. Since $\partial\Pi^q(q^*, \theta^*)/\partial\theta = \partial\Pi^p(p^*, \theta^*)/\partial\theta$ (because of tangency) and since $\partial\Pi^q(q^*, \theta^*)/\partial\theta > 0$ (because $f_\theta > 0$),

$$\operatorname{sgn}\left[T''(\Pi^q(q^*,\theta^*))\right] = \operatorname{sgn}\left[\frac{\partial^2\Pi^p}{\partial\theta^2}(p^*,\theta^*) - \frac{\partial^2\Pi^q}{\partial\theta^2}(q^*,\theta^*)\right].$$

Using the definitions (5a) and (5b) to evaluate the derivatives, we can write the expression in brackets on the right-hand side as

$$[p^* - C'(g(p^*,\theta^*)]g_{\theta\theta}(p^*,\theta^*) - C''(g(p^*,\theta^*))(g_\theta(p^*,\theta^*))^2 - f_{\theta\theta}(q^*,\theta^*)q^*. \quad (8)$$

Since (p^*, q^*) is the *ex post* optimal point on the demand curve $p = f(q, \theta^*)$, it satisfies the first-order condition for (*ex post*) profit maximization:

$$f(q^*,\theta^*) + q^* f_q(q^*,\theta^*) - C'(q^*) = 0,$$

or since $p^* = f(q^*, \theta^*)$ and $q^* = g(p^*, \theta^*)$,

$$p^* - C'(g(p^*,\theta^*)) = -q^* f_q(q^*,\theta^*). \quad (9)$$

Substituting (9) into (8) and replacing the derivatives of g with the appropriate derivatives of f (obtained by implicit differentiation of the identity $f(g(p, \theta), \theta) = p$), we conclude that

$$\operatorname{sgn}\left[T''(\Pi^q(q^*,\theta^*))\right]$$
$$= \operatorname{sgn}\left\{q^*[f_{qq}(q^*,\theta^*)f_\theta(q^*,\theta^*) - 2f_{q\theta}(q^*,\theta^*)f_q(q^*,\theta^*)] - C''(q^*)f_\theta(q^*,\theta^*)\right\}. \quad (10)$$

Equation (10) allows identification of the factors that influence the relative size of $\Pi^q(q^*, \theta)$ and $\Pi^p(p^*, \theta)$ in the neighborhood of θ^*. Reducing the slope of the marginal cost curve (lowering C'' as in Section 3), reducing the magnitude of the demand slope as the demand curve shifts out (raising $f_{q\theta}$ as in Section 4), and increasing the convexity of demand (raising f_{qq} as in Section 5) all increase the likelihood that profits from the price strategy exceed profits from the quantity strategy.

One way to interpret (10) is to recall that, as emphasized in the discussion of each of the lemmas, the costs of setting quantity and price depend on the distances of the resulting outputs, q^* and $g(p^*, \theta)$, from the *ex post* optimal output, $q^{opt}(\theta)$. We can therefore compare q^* and p^* by examining the relative sizes of $|q^{opt}(\theta) - q^*|$ and $|q^{opt}(\theta) - g(p^*, \theta)|$. The former is always larger (smaller) than the latter if $\partial[2q^{opt}(\theta) - g(p^*, \theta)]/\partial\theta$ is always positive (negative). Evaluated at (θ^*, p^*, q^*), the sign of this last expression is precisely the sign of the right-hand side of (10).

The flatter are marginal costs, the more the optimal quantity fluctuates with θ; the variation in the output resulting from a set price is, by contrast, unaffected by the slope of marginal costs. This result generalizes the conclusions of Lemmas 1 and 2 regarding the effect of c_2 in the linear model. A positive value of $f_{q\theta}$ means that demand becomes flatter as it shifts out with increases in θ and that, as a result, $q^{opt}(\theta)$ rises more rapidly with θ than if demand were simply translated vertically ($f_{q\theta} = 0$). The greater sensitivity of $q^{opt}(\theta)$ to θ makes setting a price relatively more attractive. This conclusion generalizes the contrast between the additive shock considered in Lemma 1 ($f_{q\theta} = 0$) and the rotational shock considered in Lemma 2 ($f_{q\theta} > 0$). Finally, as the demand curve becomes more convex, *ceteris paribus*, the ratio of the slope of marginal revenue to the slope of demand falls and causes the sensitivity of $q^{opt}(\theta)$ to θ to rise relative to the sensitivity of $q(\hat{p}, \theta)$ to θ. On average, $q^{opt}(\theta)$ and $q(\hat{p}, \theta)$ are relatively closer together, while $q^{opt}(\theta)$ and $\hat{q}$ are relatively farther apart, so the expected losses from setting price are reduced relative to the expected losses from setting quantity for greater convexity of demand. Conversely, for a very concave demand curve the marginal revenue curve is very steep relative to demand, so $q^{opt}(\theta)$ is very stable relative to $q(\hat{p}, \theta)$, and quantity-setting is preferred. Here we have a generalization of the result of Lemma 3.[24]

When any one of the three effects discussed in the previous paragraph is present in isolation, the local concavity or convexity of T, as determined by (10), holds globally as well. For more general environments, we can use (7) to examine the global behavior of T''.

Applying the approach of this section to identify Nash equilibria in a duopoly under uncertainty requires first calculating the candidate equilibria (for exogenously given strategic variables) and then determining for which candidate equilibria the firms' strategies remain optimal when strategic variables are endogenous. Suppose that demands are subject to a common shock and are given by

$$p_i = f(q_i, q_j, \epsilon) \tag{11a}$$

$$p_j = f(q_j, q_i, \epsilon), \tag{11b}$$

where f is twice continuously differentiable, $f_1 \leq f_2 \leq 0$ everywhere, and $f_3 > 0$ everywhere. Let each firm's cost function be $C(q)$, which we assume is twice continuously differentiable. When j sets a quantity $\bar{q}_j$, i's residual demand is

$$p_i = f(q_i, \bar{q}_j, \epsilon),$$

while when j sets a price $\bar{p}_j$, i's residual demand is

$$p_i = f(q_i, g(\bar{p}_j, q_i, \epsilon), \epsilon) \equiv F(q_i, \bar{p}_j, \epsilon),$$

where $g(\cdot, \cdot, \cdot)$ is the solution of (11b) for q_j in terms of p_j, q_i, and ϵ.

We must analyze the shape of the function T corresponding to i's residual demand curve in each candidate equilibrium. That is, using (10), we examine

$$\text{sgn}\,[q_i(f_{11}f_3 - 2f_{13}f_1) - C''f_3]$$

for the two candidate equilibria in which j sets quantity, and examine

$$\text{sgn}\,[q_i(F_{11}F_3 - 2F_{13}F_1) - C''F_3]$$

for the two candidate equilibria in which j sets price. If these expressions are all negative (positive), then by symmetry there is a unique Nash equilibrium in quantities (prices). This was the situation in the models of Sections 3, 4, and 5. In general, firm i's problem of selecting a strategic variable differs according to which candidate equilibrium we are examining. Different candidate equilibria involve different ranges of output, so the relevant value for the slope of the marginal cost curve may be different. Moreover, the shape of i's residual demand and the effect on it of the random disturbance depend on which variable j is keeping set. Thus, determining which candidate equilibria are equilibria when firms can

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[24] Only for a vertically additive shock ($p = f(q, \theta) = \tilde{f}(q) + \theta$) is $f_{q\theta}$ identically zero. For a horizontally additive shock, for example, we have $f(q, \theta) = \tilde{f}(q - \theta)$, so $f_{q\theta} = -\tilde{f}''(q - \theta)$, and thus an increase in f_{qq} reduces $f_{q\theta}$. For this case the right-hand side of (10) reduces to $\tilde{f}'(q\tilde{f}'' + C'')$, which falls as $\tilde{f}''$ increases (since $\tilde{f}' < 0$). Thus, for a horizontally additive shock, greater convexity of demand increases the likelihood that setting quantity will be preferred, and greater concavity has the reverse effect. (See Figure 2(b) for an illustration of the latter point.) In general, therefore, comparative statics on demand curvature requires recognizing that changes in f_{qq} and in $f_{q\theta}$ are interdependent.

choose which strategic variable to use often requires more specific knowledge of demand and cost conditions.

We can, however, make some generalizations. First, for sufficiently steep marginal costs the Nash equilibrium is always unique and involves quantities as strategic variables. As marginal costs become steeper, setting a quantity becomes relatively more attractive in response to both a price and a quantity strategy and eventually is preferred in both situations. Second, changes in curvature and in the effect of the uncertainty alter both of a firm's residual demand curves ($f(\cdot, \cdot, \cdot)$ and $F(\cdot, \cdot, \cdot)$) in the same direction. That is, as f_{11} increases (decreases), *ceteris paribus*, F_{11} increases (decreases); similarly, as f_{13} rises or falls, *ceteris paribus*, F_{13} moves in the same direction. Thus, very large or very small values of f_{11} or f_{13} increase the likelihood that a firm's preferred strategic variable will be independent of the variable its rival selects and hence the likelihood that there will be a unique Nash equilibrium. Finally, as demand for each good becomes more elastic and reduces both $|f_1|$ and $|F_1|$, the fluctuations in the *ex post* optimal output and in the output under a set price increase.[25] This in turn increases the difference in expected profits from setting quantity and setting price (see Lemma 1) and so intensifies the firm's preferences over strategic variables. Greater uncertainty also strengthens these preferences. Thus, we would expect the factors we have identified—the slope of marginal costs, the nature of the demand uncertainty, and the curvature of demand—to play a larger role in determining the nature and intensity of competition in industries in which demands are relatively elastic and are affected by significant unobservable shocks.

7. Conclusion

■ We have shown that in an environment of uncertainty about market demands, oligopolists generally have strict preferences between setting quantity and setting price and that, as a consequence, the Bertrand and Cournot outcomes are typically not both equilibria in a game in which firms can choose which strategic variables to use. Recognizing firms' responses to uncertainty thus provides some insight into the nature of oligopolistic competition.[26]

In contrast with our approach, imposing a requirement in the spirit of Selten's (1975) concept of (trembling-hand) perfection does not eliminate any of the four types of equilibria under certainty. For each of the firms in any equilibrium it is possible to find a tremble for the opponent that makes the firm strictly prefer the strategic variable it uses in the equilibrium. There is, however, no economic rationale for the conjectured trembles. We, on the other hand, explicitly model the random effect that gives firms strict preferences over strategic variables.

In our model uncertainty arises from exogenous demand shocks. It could alternatively arise from each firm's uncertainty about its rival's behavior, owing to uncertainty about some characteristic of the rival's cost or demand function. There is thus no need for actual variation in market conditions; a lack of perfect information about the rival's behavior is sufficient to give firms strict preferences between setting prices and setting quantities.[27] Whichever the kind of uncertainty, an arbitrarily small amount can move us away from the case with four equilibria.

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[25] Suppose, for example, that f_1 and f_2 are both reduced by a factor of λ ($\lambda > 1$). Then $F_1 = (f_1^2 - f_2^2)/f_1$ is reduced by a factor of λ as well.

[26] In a similar spirit Daughety (1985) introduces uncertainty to choose between Cournot-Nash and Cournot-von Stackelberg equilibria by giving firms the choice between playing early or waiting to infer information about market demand and playing late.

[27] In this case we look for a Bayesian Nash equilibrium in which each "type" of firm has correct conjectures about the mapping from its rival's "type" to the rival's choice of strategic variable, and chooses its own strategic variable to maximize expected profits, given these conjectures.

Three factors affect firms' preferences between quantity and price as strategic variables in the presence of uncertainty: the slope of marginal costs, the effect of the random disturbance on demand, and the curvature of demand. Each affects the relative "costs" of setting quantity and price by altering the sensitivity to the shock of the *ex post* optimal price-quantity pairs relative to the sensitivity of the price-quantity pairs resulting from the allowable strategic variables. The most compelling result concerns the slope of the marginal cost curve. When marginal costs are steep, setting quantity is preferred because the *ex post* optimal quantity is relatively stable compared with the output that would result from setting a price and producing to meet market demand. This formalizes the commonly expressed intuition that when marginal costs rise rapidly, firms will compete as Cournot players, but that when marginal costs become flatter, firms will be forced to compete on price.[28] If we enriched the model to include other costs of choosing strategic variables besides those arising from uncertainty, we would find that competition in industries with rapidly increasing marginal costs was most often through quantities, whereas competition in industries with flatter or decreasing marginal costs was most often through prices. Firms' preferences between strategic variables are stronger in industries with more elastic demands and greater uncertainty.

These results explain why the firm described in Section 2 effectively sets quantity in its management consulting division but sets price in its auditing division. They may also help explain the contrast between Eastern Airlines' competitive strategy on its shuttle routes involving New York, Boston, and Washington and the form of competition in much of the rest of the airline industry. Whereas for most routes schedules are first set and prices subsequently adjusted in response to demand fluctuations, on the shuttle routes Eastern sets a price and then provides as many planes as are necessary to meet demand. Eastern's willingness to set price and risk quantity fluctuations may reflect the comparatively low marginal cost of adding a shuttle flight.[29]

□ **Future extensions.** Recent work by Bresnahan (1982) on the identification of market power may be useful for testing our model empirically. He considers a model in which there are two sources of exogenous uncertainty that affect the slope and the intercept of market demand, and he shows that under these conditions the degree of market power, the marginal cost curve, and the demand equation are all separately identified. Suppose that we used our model to predict firms' choices of strategic variables for various industries on the basis of the estimated demand and marginal cost equations. We could then compare the implications of these choices for the degree of market power with the econometric estimates of market power.

Extension of the model to more than two firms is straightforward. In the absence of uncertainty, there are 2^n equilibria, corresponding to each firm's choosing one of its two

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[28] Roughly, the intuition is that with vertical marginal cost curves the only "reasonable" supply curves or reaction curves to have, or to conjecture, are vertical ones, i.e., Cournot. Incorporating uncertainty makes this intuition rigorous by incorporating the costs that firms bear when their demand is not as expected. Another formalization of the intuition is provided by Bresnahan's (1981) consistent conjectures equilibrium, which looks more like a Cournot equilibrium as the slope of marginal cost rises.

[29] The planes are smaller, the routes are very short so that staff and equipment can return to their correct location at the end of a day at relatively low cost, and the high frequency of flights means that the expected incremental number of return flights incurred by a given extra flight is close to zero, since excess resources may build up at either end of the route. With many flights per day, a given flight reduces return costs with roughly the same probability that it increases them. With only infrequent flights, on the other hand, an extra flight almost certainly incurs a return cost.

Another reason for setting a price on the shuttle may be that the other costs of setting quantity rather than price (operating a reservation system and adjusting discounts to meet demand) are greater relative to revenues and thus greater relative to the costs arising from uncertainty. Our model has not considered these other costs. Eastern's strategy on the shuttle also differentiates its product from the products of airlines that require reservations.

possible strategic variables. With uncertainty the results of Propositions 1, 2, and 3 continue to hold, since under the specified conditions each firm's preferred strategic variable is independent of the strategic variables chosen by the other $n - 1$ firms. The propositions also hold for firms having different levels of costs and facing different demand shocks, and can easily be generalized to incorporate other asymmetries in demand.

Our model takes firms' cost curves as exogenous and independent of their choices of strategic variables. In fact, a firm adopting a quantity strategy might experience a cost advantage relative to adopting a price strategy and having to retain a flexible production technology until the realization of demand was known. This cost advantage would, however, be reduced in a model that allowed inventories and backlogging of orders to limit the variability in production associated with a price-setting strategy. More generally, inventories help to separate the production and sales decisions. They therefore reduce the effect of cost conditions on the equilibrium choice of strategic variables, and so make the curvature of demand and the nature of the uncertainty relatively more important.

A more ambitious model would recognize that factors in a firm's internal structure affect the relative costs of setting a price and setting a quantity. Examples of such factors are the relationships among the firm's divisions, its reporting structures, the contracts it has with its employees, its organizational values, and its procedures and rules. In a multiperiod model the choice of both internal structure and production technology should be made before the choice of strategic variable.[30] Such a model would include fuller game-theoretic interactions, but the insights from our single-period model should still apply.[31]

A common suggestion is that price competition is best represented by each firm's choosing a price in conjunction with a maximum quantity that the firm is willing to produce (see Section 3). In Klemperer and Meyer (1985) we pursue this line in a more general framework by enlarging firms' strategy sets. In practice we think that, even in a static framework, competition is best modelled as some mixture of price and quantity competition. Our management consulting firm, for example, would typically find itself adjusting quantity as well as price in response to a demand shock. We can capture this idea by allowing firms to choose any supply schedule relating quantity to price. Setting a quantity is equivalent to choosing a vertical supply schedule, and setting a price is equivalent to choosing a horizontal supply schedule. Again there are a large number of equilibria under certainty, but they are dramatically reduced by incorporating uncertainty.

Appendix

■ Proofs of Lemmas 1 and 2, Proposition 2, and Lemma 3 follow.

Proof of Lemma 1. The optimal set quantity solves

$$\max_q E\left(q(A - Bq + \theta) - c_1 q - c_2 \frac{q^2}{2}\right),$$

for which the first-order condition is

$$A - 2Bq - c_1 - c_2 q = 0,$$

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[30] *Ceteris paribus*, a shallower short-run marginal cost of production curve will be associated with higher total costs of production than a less flexible technology, at the less flexible technology's optimal scale (Stigler, 1939). If a firm receives no new information about demand between the time it chooses its technology and the time it selects its price or quantity, this is a factor in favor of choosing an inflexible technology and subsequently setting a quantity. If, however, some information about demand is revealed between these choices (as, for example, if a single choice of technology is followed by several periods of production under different market conditions), the firm may choose a more flexible technology, which then makes price relatively more attractive in the subsequent market game.

[31] Singh and Vives (1984) and Cheng (1985) make an important start on such a model by showing that under fairly general conditions committing to set a quantity dominates committing to set a price in an environment with costless commitment and no uncertainty.

since $E\theta = 0$. This yields

$$\hat{q} = \frac{A - c_1}{2B + c_2}.$$

Substituting for $\hat{q}$ in the objective function gives

$$E\Pi(\hat{q}) = \frac{1}{2}\frac{(A - c_1)^2}{(2B + c_2)}.$$

To find the optimal set price, invert the demand equation to obtain

$$q = \frac{A + \theta - p}{B},$$

and solve

$$\max_p E\left(p\left(\frac{A + \theta - p}{B}\right) - c_1\left(\frac{A + \theta - p}{B}\right) - c_2\frac{(A + \theta - p)^2}{2B^2}\right).$$

The first-order condition is

$$\frac{A}{B} - \frac{2p}{B} + \frac{c_1}{B} + \frac{c_2(A - p)}{B^2} = 0,$$

which yields

$$\hat{p} = \frac{(A + c_1)B + c_2 A}{2B + c_2}.$$

For a given θ, the quantity resulting from setting price at $\hat{p}$ is

$$q(\hat{p}, \theta) = \frac{A - c_1}{2B + c_2} + \frac{\theta}{B},$$

and

$$E\Pi(\hat{p}) = \frac{1}{2}\frac{(A - c_1)^2}{(2B + c_2)} - \frac{\sigma^2 c_2}{2B^2}.$$

Therefore,

$$E\Pi(\hat{q}) - E\Pi(\hat{p}) = \frac{\sigma^2 c_2}{2B^2}. \quad Q.E.D.$$

Proof of Lemma 2. The optimal set quantity solves

$$\max_q E\left(q\left(A - \frac{Bq}{\theta}\right) - c_1 q - c_2\frac{q^2}{2}\right),$$

for which the first-order condition is

$$A - 2Bzq - c_1 - c_2 q = 0,$$

since $E(1/\theta) = z$. This yields

$$\hat{q} = \frac{A - c_1}{2Bz + c_2}.$$

Substituting for $\hat{q}$ in the objective function gives

$$E\Pi(\hat{q}) = \frac{1}{2}\frac{(A - c_1)^2}{(2Bz + c_2)}.$$

To derive the optimal set price, invert the demand equation to obtain

$$q = \frac{\theta(A - p)}{B}$$

and solve

$$\max_p E\left(\frac{p\theta(A - p)}{B} - \frac{c_1\theta(A - p)}{B} - \frac{c_2\theta^2(A - p)^2}{2B^2}\right).$$

The first-order condition is

$$\frac{A - 2p}{B} + \frac{c_1}{B} + \frac{c_2 s^2(A - p)}{B^2} = 0,$$

since $E\theta = 1$ and $E\theta^2 = s^2$. This yields

$$\hat{p} = \frac{(A + c_1)B + c_2 s^2 A}{2B + c_2 s^2}.$$

For a given θ, output is

$$q(\hat{p}, \theta) = \frac{(A - c_1)\theta}{(2B + c_2 s^2)},$$

and expected profits are

$$E\Pi(\hat{p}) = \frac{1}{2}\frac{(A-c_1)^2}{(2B+c_2s^2)}.$$

Thus, we have

$$E\Pi(\hat{q}) - E\Pi(\hat{p}) = \frac{1}{2}(A-c_1)^2\left(\frac{1}{2Bz+c_2} - \frac{1}{2B+c_2s^2}\right)$$

$$= \frac{1}{2}(A-c_1)^2\left(\frac{c_2(s^2-1)+2B(1-z)}{(2Bz+c_2)(2B+c_2s^2)}\right).$$

The denominator is positive by the assumption that the second-order conditions are satisfied, i.e., $c_2 > -2B/s^2$. The numerator is linearly increasing in c_2, since $s^2 - 1 > 0$. Therefore, there exists a c_2^* such that for $c_2 < c_2^*$ the expression is negative, and for $c_2 > c_2^*$ the expression is positive, where c_2^* satisfies

$$c_2^*(s^2-1) + 2B(1-z) = 0,$$

so

$$c_2^* = \frac{2B(z-1)}{s^2-1} > 0.$$

Note that the cutoff level c_2^* depends on the distribution of the shock and on the expected slope of demand, but not on the vertical intercepts of the demand or marginal cost curves. For a given distribution of the shock, c_2^* is linear in B. *Q.E.D.*

Proof of Proposition 2. When j sets quantity at $\bar{q}_j$, i's residual demand curve is

$$p_i = \left(\alpha - \frac{\gamma\bar{q}_j}{\epsilon}\right) - \frac{\beta q_i}{\epsilon}.$$

Both the slope and the intercept vary with the shock ϵ. For constant marginal costs ($c_2 = 0$), firm i's optimal set quantity in response to $\bar{q}_j$ solves

$$\max_{q_i} E\left(q_i\left(\alpha - \frac{\gamma\bar{q}_j}{\epsilon} - \frac{\beta q_i}{\epsilon}\right) - c_1 q_i\right).$$

Differentiating and rearranging yield

$$\hat{q}_i = \frac{\alpha - \gamma\bar{q}_j z - c_1}{2\beta z},$$

where $z = E(1/\epsilon)$, and so

$$E\Pi(\hat{q}_i) = \frac{(\alpha - \gamma\bar{q}_j z - c_1)^2}{4\beta z}.$$

With $c_2 = 0$ firm i's optimal set price in response to $\bar{q}_j$ solves

$$\max_{p_i} E\left((p_i - c_1)\left(\frac{\alpha\epsilon - \gamma\bar{q}_j - \epsilon p_i}{\beta}\right)\right),$$

and the solution is

$$\hat{p}_i = \frac{\alpha - \gamma\bar{q}_j + c_1}{2}.$$

$$E\Pi(\hat{p}_i) = \frac{(\alpha - \gamma\bar{q}_j - c_1)^2}{4\beta}.$$

Since $z > 1$ by Jensen's inequality and since $\gamma \geq 0$ by the assumption that the products are substitutes,

$$E\Pi(\hat{p}_i) > E\Pi(\hat{q}_i),$$

so for flat marginal costs firm i strictly prefers to set a price, whatever set quantity j has chosen. By the continuity of $(E\Pi(\hat{p}_i) - E\Pi(\hat{q}_i))$ in both c_2 and $\bar{q}_j$ and the continuity in c_2 of j's quantity, $\bar{q}_j(c_2)$, in the candidate (quantity, quantity) equilibrium, there exists a $c_2^Q > 0$ such that for $c_2 < c_2^Q$, firm i strictly prefers to set a price when j sets $\bar{q}_j(c_2)$. It follows that for $c_2 < c_2^Q$ the candidate (quantity, quantity) equilibrium is not a Nash equilibrium when strategic variables are chosen endogenously.

Now suppose that j sets price at $\bar{p}_j$. Solving equation (3b) for q_j and substituting into (3a) give i's residual demand,

$$p_i = \alpha - k_1 - \frac{q_i k_2}{\epsilon},$$

where $k_1 = \gamma(\alpha - \bar{p}_j)/\beta$ and $k_2 = (\beta^2 - \gamma^2)/\beta$. The shock affects only the slope and not the intercept of i's residual demand curve, while j's price affects only the intercept. Hence, Lemma 2 implies that for all c_2 less than

$c_2^p = (2k_2(z-1)/(s^2-1)) = (2(z-1)/(s^2-1))((\beta^2-\gamma^2)/\beta)$, i strictly prefers to set a price against any set price of firm j. Similarly, j prefers to set a price against any set price of firm i. It follows that for $c_2 < c_2^p$, there exists a (price, price) Nash equilibrium and there do not exist (price, quantity) or (quantity, price) Nash equilibria. Note that $c_2^p > 0$ for imperfect substitutes ($\gamma < \beta$) and $c_2^p = 0$ for perfect substitutes ($\gamma = \beta$).

Define $\bar{c}_2 = \min\{c_2^q, c_2^p\}$. For imperfect substitutes $\bar{c}_2 > 0$. Using the results above, we find that for $c_2 < \bar{c}_2$ the unique Nash equilibrium involves prices as strategic variables. For perfect substitutes $\bar{c}_2 = 0$. For $c_2 = 0$ the unique Nash equilibrium involves prices ($p_i = p_j = c_1$). This follows because: (i) with $\bar{p}_j = c_1$ setting $p_i = c_1$ is a best response; (ii) asymmetric (price, quantity) and (quantity, price) equilibria are ruled out by the same argument used for the corresponding case ($\gamma = \beta$, $c_2 = 0$) in Proposition 1; and (iii) setting a price is a strict best response against any quantity, as shown above.

Proof of Lemma 3. Let the cost function be $C(q)$. Consider first the case in which $f(\cdot)$ is strictly concave and $C''(q) \geq 0$. Define $g(\cdot) = f^{-1}(\cdot)$. Since $f(\cdot)$ is decreasing and strictly concave, $g(\cdot)$ is strictly concave. When the firm sets the price $\hat{p}$, output satisfies $f(q) + \theta = \hat{p}$, so $q = g(\hat{p} - \theta)$, which for a sufficiently small support of θ is nonnegative for all θ. Define $\hat{q}$ to be the expected output from setting a price $\hat{p}$: $\hat{q} = E[g(\hat{p} - \theta)] > 0$. Define $\tilde{p} = f(\hat{q})$. Since $g(\cdot)$ is strictly concave, Jensen's inequality implies that $\hat{q} < g(\hat{p} - E\theta) = g(\hat{p})$, and, therefore, since $f(\cdot)$ is decreasing, $\tilde{p} > \hat{p}$. Now

$$
\begin{aligned}
E\Pi(\hat{p}) &= \hat{p}E[g(\hat{p}-\theta)] - E\left[\int_0^{g(\hat{p}-\theta)} C'(q)dq\right] \\
&\leq \hat{p}E[g(\hat{p}-\theta)] - \int_0^{E[g(\hat{p}-\theta)]} C'(q)dq \quad (\text{since } C''(q) \geq 0) \\
&< \tilde{p}\hat{q} - \int_0^{\hat{q}} C'(q)dq \quad (\text{from } \tilde{p} > \hat{p} \text{ and the definition of } \hat{q}) \\
&= E[(f(\hat{q}) + \theta)\hat{q}] - \int_0^{\hat{q}} C'(q)dq \quad (\text{from } E\theta = 0 \text{ and the definition of } \tilde{p}) \\
&= E\Pi(\hat{q}) \\
&\leq E\Pi(\hat{q}) \quad (\text{by the definition of } \hat{q}).
\end{aligned}
$$

The difference $(E\Pi(\hat{q}) - E\Pi(\hat{p}))$ is strictly positive for $f(\cdot)$ strictly concave and $C''(q) \geq 0$, and is continuous in the slope of marginal costs. Thus, setting quantity is preferred for $f(\cdot)$ strictly concave and marginal costs not excessively rapidly decreasing ($C''(q)$ not excessively negative).

The proof for $f(\cdot)$ strictly convex and $C''(q) \leq 0$ proceeds similarly by defining $\tilde{\tilde{p}}$ such that $E[g(\tilde{\tilde{p}} - \theta)] = \hat{q}$ and by using the strict convexity of $g(\cdot)$ to show that $E\Pi(\hat{p}) \geq E\Pi(\tilde{\tilde{p}}) > E\Pi(\hat{q})$. Again, since the inequality $E\Pi(\hat{p}) > E\Pi(\hat{q})$ is strict, continuity implies that setting price is preferred for $f(\cdot)$ strictly convex and marginal costs not excessively rapidly increasing ($C''(q)$ not excessively positive). *Q.E.D.*

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