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Price Wars Caused by Switching Costs

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In many markets consumers have "switching costs", for example learning costs or transaction costs, of changing between functionally equivalent brands of a product, or of using any brand for the first time. We analyse a four-period complete-information model of a market with switching costs in which new entry occurs after the second period. The new entry results, in equilibrium, in a price war. That is, the new entrants' prices are higher in the post-entry period than in the entry period, and the incumbent's price falls in either the pre-entry period or the entry period and subsequently rises. We can interpret the incumbent's lowering its price in the pre-entry period as limit-pricing behaviour. We distinguish between two types of price war that can occur, and show how the type or mixture of types that arises depends on the size of switching costs.

1. INTRODUCTION

Recent explanations of price wars have included interpretations of them as low-price realisations of mixed-strategy equilibria with constant expected prices over time, as firms' pricing low to signal low costs and determination to stay in a market in order to encourage competitors to withdraw, and as a mechanism for enforcing collusion by punishing deviant firms.[1] The second explanation depends upon incomplete information about firms' costs, and the last explanation requires incomplete information about demand if a price war is ever actually to occur. We, however, will show that price wars occur quite naturally as pure-strategy equilibria in models with complete information, if consumers have costs of switching between competing brands of a product. (We will say that an industry has a price war if firms' prices first fall and then subsequently rise.)

In many markets consumers face substantial costs of switching between products that are ex ante undifferentiated. A number of computer manufacturers, for example, may make machines that are functionally identical but, if you have learned to use one firm's product line and invested in the appropriate software, you have a strong incentive to continue to buy from the same firm. Similarly, two banks may offer identical accounts,

1. For the first explanation see Shilony (1977) and Varian (1980). For the second explanation see Fudenberg and Tirole (1986a), Mailath (1984), and Zachau (1985). In Fudenberg and Tirole (1986b) and Riordan (1985) firms produce large outputs to strategically manipulate their rivals' information about the attractiveness of the market. For the last explanation see Stigler (1964) and Green and Porter (1984).

but there are high transaction costs in closing an account with one bank and opening another with a competitor.[2]

New entry triggers a price war in equilibrium in a market with switching costs. New entrants initially price low to capture market share, and the incumbent responds by cutting price to retain its own repeat purchasers. That prices fall after new entry is a common feature of many models; what is special here is that the price-cutting ends after a period of low prices. Entrants raise their prices after the entry period, when their repeat-customers' reservation prices for their products are higher by the value of the switching cost (or start-up cost). Moreover, an incumbent's price also rises after falling in the entry period. There are two reasons for this last result.

First, new entrants' prices may rise substantially in the post-entry period (relative to the size of switching costs), when these firms can exploit the fact that their customers are to some extent locked in by their switching costs. Since the incumbent cannot set its entry-period price too much greater than the entrants' price lest its customers defect to the entrants, the incumbent too raises its price in the post-entry period, even if its post-entry price is no higher than the entrants'. We will call a price war of this kind a "Type 1" price war, and will show that it arises when switching costs are small.

Second, the incumbent acquires a stock of high-reservation-price consumers before entry occurs, and so may be able to sustain a higher price than the entrants after the entry period, as well as during it. In this case, even if the entrants' prices do not rise substantially after the entry period, the incumbent's price will rise along with the entrants'. (The entrants' prices will always rise somewhat after their customers have built up switching costs.) We will call a price war of this kind a "Type 2" price war, and will show that it arises when switching costs are large.[3]

In addition to generating price wars, markets with switching costs may generate "limit pricing" behaviour in equilibrium with complete information. We will show that the incumbent may cut its price in the period immediately before entry in order to lock in customers to its product and so reduce the scale of new entry.

Section 2 presents a four-period model of a market with switching costs in which a dominant firm is a monopolist for two periods and entry takes place after the second period. Two periods after entry and one before entry are needed to discuss a price war. The additional pre-entry period separates the effects due to the dominant firm's own entry

2. Our model also applies when there are psychological costs of switching between brands, as when, for no obvious economically rational reason, consumers repeat purchase from simple habit or loyalty. The model could be applied to a market in which consumers are uncertain about products' qualities and so reluctant to switch between firms' products—see note 3. (Here the switching cost equals the insurance premium that a consumer would be willing to pay when buying from a new firm, to be guaranteed a product of the same quality as a previously purchased product.) In this case, however, there are additional complications which do not arise in our model, due to the possibility of prices being used as signals of quality and due to the existence of groups of consumers who tried a brand and did not like it. Our model also does not apply directly to cases in which "artificial" switching costs are created by contracts, repeat-purchase coupons, etc., in which there are no social costs of brand switching. For other examples and models of switching costs, see Klemperer (1983, 1986, 1987a, b), von Weizsäcker (1984), Banerjee and Summers (1987), and Farrell and Shapiro (1988).

3. Schmalensee (1982), Conrad (1983) and Doyle (1986) model entry when uncertainty about quality makes consumers reluctant to switch to new entrants' products (see note 2). Schmalensee assumes that the incumbent does not alter price in response to entry. Conrad, like us, considers an incumbent facing an entering competitive fringe. However, she assumes that consumers have no costs of switching between fringe firms. (In her model fringe firms' products are known to be of identical quality.) Therefore all fringe firms charge their production cost in every period and there is no price war. She also does not consider the incumbent's pre-entry behaviour, and does not explicitly compute the incumbent's price path after entry. In Doyle's model, entrants' prices are lower than incumbents' in the entry period and subsequently rise, but the incumbents' prices are constant across time because he assumes that all consumers have identical reservation prices.

in period 1 from those due to its strategic behaviour in period 2, and so allows us to analyse the possibility of limit pricing.

Section 3 demonstrates that the price paths are always those of a price war (Proposition 1) and argues that this result does not depend critically upon the formulation of our model. It also discusses limit pricing.

Section 4 further characterises the equilibrium price paths. It shows that the post-entry increase in the dominant firm's price can be divided into two parts corresponding to the two types of price war. With sufficiently small switching costs, we have a pure Type 1 price war (Proposition 2), whereas with sufficiently large switching costs, the price war is pure Type 2 (Proposition 3). With intermediate switching costs, the price war may be a mixture of both types.

Section 5 concludes.[4]

2. THE MODEL

We consider a four-period model of a dominant firm with a competitive fringe in a market with switching costs.

All consumers enter the market in period 1 and are in the market in every period. In each period the qth consumer has a reservation price $f(q)$, net of switching costs, for a single unit of the product. Any consumer who purchases from a different firm from the one from which he purchased in the preceding period, or who did not purchase in the preceding period, incurs a "switching cost" (or start-up cost), $s>0$.[5] We order the consumers so that $f(\cdot)$ is decreasing, and $f(\cdot)$ and $f^{-1}(\cdot)$ will be assumed continuous and differentiable as necessary.

All firms have constant marginal costs, c per unit, and no fixed costs. Their products are identical (except that consumers must pay the cost s to switch between them), and the firms cannot price discriminate between new and old customers.[6] The products cannot be stored. Firms and consumers have rational expectations and use a discount factor $\lambda \in (0, 1)$ per period in calculating the present values of their production and consumption plans.

In period 1 a dominant firm—perhaps the first to develop a new technology—enters and is a monopolist, protected from entry. In period 2 the dominant firm is again a monopolist. In period 3 the dominant firm chooses a quantity and fringe firms then enter the market taking this quantity as given and acting competitively, that is, they enter until their expected discounted profits over the remaining periods (periods 3 and 4) are zero. We assume that the distribution of customers' reservation prices is the same for all entering fringe firms. In the final period, period 4, the dominant firm again chooses its quantity first and the period-3 fringe entrants then choose quantities taking the dominant firm's

4. Cassady (1963) gives numerous examples in which price wars follow entry into the retail trade. He also notes that in at least one region of the U.S. "there has been a strong tendency ... for established competitors to publicise deep-cut specials some weeks before the opening of a new competitive establishment" (page 32), that is, he observes "limit pricing". Consumers' switching costs in retailing include learning costs of familiarising themselves with new stores, but we do not claim that switching costs are more than one among a number of reasons for price wars.

5. An alternative assumption is that a consumer incurs a switching cost only if he purchases from a firm from which he has never previously purchased. In our context, the distinction between the two assumptions is of no importance.

6. Allowing firms to price discriminate between new and repeat purchasers would not change the nature of our results. See Klemperer (1986).

output as given. There are again no barriers to entry, so new fringe firms offer to sell at their marginal cost, c, in the final period. The dominant firm is unable to precommit to its output in any period in advance of that period.

In each period prices are such that the market clears, that is, all output is sold and each consumer weakly prefers the brand he has purchased (or no purchase if he made none) to any other brand and to no purchase, at the given prices and given rational expectations of future prices. Sometimes more than one price vector clears the market. (Consider, for example, a monopolist selling an output Q in each of two periods. In the second period any price between $f(Q)$ and $f(Q)-s$ would clear the market, but the natural assumption is that the price is $f(Q)$.) We therefore assume first that the dominant firm's price is the highest consistent with market-clearing, and then that the fringe firms' price is the highest that is consistent with market clearing.[8]

Which consumers buy from which firm is in large part determined by market prices, consumers' rational expectations of future prices, and the symmetry of the fringe firms, but remains to some extent arbitrary. In particular, if the dominant firm expands output in period 3, the distributions of reservation prices of the dominant firm's first-time buyers and of the fringe firms' customers may be indeterminate. In this case we assume, as is natural, that the proportion of the dominant firm's first-time buyers who have reservation prices above any specific price is not larger than the proportion of the fringe firms' customers whose reservation prices exceed the same price.[9] However, our main result (Proposition 1) is robust to any alternative assumption.[10]

We let q_t^D and q_t^F be the outputs in period t of the dominant firm and of the set of period-3 fringe entrants, respectively. (Thus we exclude any outputs of period-4 entrants from q_4^F, but we will show that in equilibrium such outputs are zero.) We let p_t^D and p_t^F be the corresponding market prices.

3. TWO TYPES OF PRICE WAR

Proposition 1 *In an equilibrium in which entry occurs in period* 3, $p_3^F < p_4^F$, $p_3^D < p_4^D$, *and either* $p_2^D > p_3^D$ *or* $p_1^D > p_2^D$.

Proof. See Appendix ‖

Thus the price paths resulting from new entry are always those of a price war—the period-3 entrants' price rises after entry and the dominant firm's price falls in either the entry period or the pre-entry period and subsequently rises.[11]

7. The order in which firms choose their outputs in the last period is in fact irrelevant, as is whether firms choose prices or quantities in this period. We think of the dominant firm as setting a quantity in period 3 because in this period the market prices alone would not necessarily specify the division of output between the dominant firm and fringe. However we could think of the fringe as entering in period 3 taking as given both price and the distribution of reservation prices of the consumers to which it will sell. (The latter is important since it determines the fringe's last-period profits.) The fringe could be thought of as setting either price or quantity in period 3.

8. At least in equilibrium, the order in which we apply these assumptions is irrelevant.

9. It seems natural that the distribution of each firm's first-time buyers' reservation prices should be the same. However, if the dominant firm expands output sufficiently, it must in equilibrium serve lower-reservation-price first-time buyers than those served by the fringe firms, since consumers will rationally expect the dominant firm to contract output in period 4. Our assumption allows for this possibility while also allowing the distributions to be the same when possible. In fact, our assumption is needed (in Lemma 3) for one specific price only: the price a firm would choose in period 4 if it had sold to all the consumers sold to (by all firms) in period 3.

10. The assumption is used to obtain the specific quantitative results of Propositions 2 and 3. Although it is never directly needed in the equilibria exhibited, it supports them by ruling out one kind of off-the-equilibrium-path behaviour.

11. We have not obtained results comparing p_2^D and p_3^D in the case in which $p_1^D > p_2^D$.

We will develop intuition for the Proposition and for the two types of price war that can arise, by considering in turn the cases of small and large switching costs. We will see that period-4 potential entrants sell no output in equilibrium, which justifies our focus on the period-3 entrants and the dominant firm.

3A. *Small Switching Costs: "Type 1" Price Wars*

If the switching cost, s, is sufficiently small, the binding constraint on firms' prices in period 4 (the last period) arises from the fact that period-4 potential entrants are willing to offer output at price c and so would attract all the customers of any firm whose price exceeded $c+s$. Thus both the dominant firm and the period-3 entrants set a period-4 price equal to $c+s$. Now the period-3 fringe entrants make zero profits over time so their period-3 price must be strictly below c. Therefore, since the dominant firm's period-3 price cannot exceed the fringe's price by more than the switching cost (if it did, all the dominant firm's customers would switch), the dominant firm's period-3 price is strictly less than $c+s$. Thus both the dominant firm's and the fringe entrants' prices are higher in period 4 than in period 3. These results, together with the standard result that an incumbent's price initially falls on new entry (industry output expands) show that Proposition 1 applies in this case.

We call a price war of this kind—characterised by the fact that the fringe's price rises by more than the switching cost—a "Type 1" price war. In a Type 1 price war the fringe firms' period-4 prices are exploiting their limited monopoly power over their higher-reservation-price customers at the cost of not selling to the less profitable lower-reservation-price consumers they sold to in period 3. (All consumers with reservation prices s above the fringe's period-3 price, which is below c, buy in period 3, but only consumers with reservation prices of at least $c+s$ will buy again in period 4.) Each entrant's period-3 output is high and price is low, not to attract the lower-reservation-price consumers (on whom firms lose money), but to win a large share of the higher-reservation-price consumers who are valuable in period 4. Examples include banks giving book-tokens and free banking services to students, and companies issuing discount coupons to attract valuable customers even though they also attract some consumers who will not repeat purchase.

We characterise the price paths for the case of small switching costs in more detail in Proposition 2 below.

3B. *Larger Switching Costs: "Type 2" Price Wars*

With larger switching costs, the threat of period-4 entry is not a binding constraint on the period-3 entrants' period-4 price. In this case, that price will equal the reservation price of the marginal customer the period-3 entrants sell to in period 4. This customer will be an "old" customer, that is, a repeat-purchaser, because a firm would have to set price c, yielding zero last-period profits, to attract any "new" consumers in the fact of competition from period-4 entrants. Since this customer obtains zero surplus in period 4, his switching cost must be paid for him in period 3 with a period-3 price that is at least s below his reservation price. Thus the fringe entrants' period-3 price is at least s below their period-4 price. Further, since the dominant firm sells to higher-reservation-price consumers than the fringe firms sell to (it entered the market first), and since the fringe firms' period-4 price is below that to which the threat of new entry constrains firms, the dominant firm's period-4 price will in general be higher than the fringe's period-4 price.[12] Therefore, since the dominant firm's period-3 price cannot exceed the fringe's period-3 price by more than s, the dominant firm's period-3 price must be less than its period-4 price. As before, the standard result than an incumbent's price initially falls on new entry is now sufficient to prove that we have a price war in this case.

We call a price war of this kind—characterised by the fact that the dominant firm can continue to charge a higher price than the entrants even after the entry period—a "Type 2" price war.

The price paths for the case of large switching costs are described in more detail in Proposition 3 below.

3C. Limit Pricing

We will see below that the dominant firm may choose a larger output in period 2 than in period 1. By increasing its pre-entry output, it not only increases its post-entry output but also denies high-reservation price consumers to the entrants. The entrants therefore sell to fewer low-reservation-price consumers (they are competing less vigorously for the fewer profitable consumers available) which prevents prices from falling so far in the entry period.

Our model thus provides an interpretation of limit pricing, since this behaviour implies that the incumbent's price immediately before the entry period is both lower than it was previously, and also lower than the short-run profit-maximising price. In contrast to the models of Bain (1949), Gaskins (1971), and others, this form of limit pricing reduces the amount of entry of *rational* potential entrants. In contrast to the models of Milgrom and Roberts (1982), Matthews and Mirman (1983), and Saloner (1984), among others, it arises in a model of complete and perfect information; we do not need to invoke the idea that the incumbent will dissipate first-period profits to signal information.

Note that this limit pricing does not completely deter entry, so Ordover and Willig's (1981) definition of predation, which relies on exit occurring, is missing the point here. These authors suggest predation occurs to drive out rivals so that price can later be raised; in our model predation occurs (in period 2) to corner a large share of the market and to reduce the entry of rivals so that later prices are higher (in period 3). Note also that since the incumbent's price may exceed its marginal cost in every period in our context, so its limit pricing may fail Areeda and Turner's (1975) criterion for predation. Of course, the incumbent's limit pricing does satisfy Williamson's (1977) test for predation, which considers an expansion of output by the incumbent when entry occurs as predatory.[13]

3D. Robustness

While the discussion above provides intuition, the formal proofs are intricate. The reason is that with switching costs, consumers consider expected future prices as well as today's

12. The dominant firm sells to the highest-reservation-price customers in periods 1 and 2 and sells only to repeat purchasers in period 3, because attracting new consumers in that period would require prices as attractive as those set by the period-3 entrants who earn zero profits over the last two periods. The Appendix establishes that the dominant firm's period-4 price equals the fringe's only in cases in which a Type 1 price war arises.

13. Strictly, our incumbent expands output just before entry. Note that it is difficult to define predation in our context: even when the incumbent maintains a constant pre-entry output, it will still in some cases produce a larger output at a lower price in both pre-entry periods than if it assumed the entrants' prices would be independent of its own behaviour, or than if it assumed the entrants' quantities would be independent of its own behaviour.

prices when making today's purchase decisions. Thus, for example, a low-reservation-price consumer who expects never to purchase again will buy at the lowest possible price (including any switching cost) today, while a higher-reservation-price consumer may prefer a higher-priced product if he (rationally) expects that product to be priced lower than other products in the future. Of course, the future prices of products depend on the choices made by different kinds of consumers today. Thus careful proofs are required even for statements that seem obvious, such as that in a pre-entry period the set of consumers buying from the incumbent monopolist consists of all those with reservation prices above some cutoff level (this is the statement of Lemma 1 in the Appendix).[14]

Nevertheless, we conjecture that our main result is quite robust to alternative model specifications. To understand why, observe that no rational customers will pay the discounted present cost of buying from the dominant firm in both periods 3 and 4, $p_3^D+\lambda p_4^D$, unless this is less than the discounted cost of buying from the entering fringe in those periods, $p_3^F+\lambda p_4^F$, plus the cost of switching to an entrant, s. That is,

$$p_3^D+\lambda p_4^D \leqq p_3^F+\lambda p_4^F+s. \tag{1}$$

Using this equation to substitute for $(p_3^F+s)-p_3^D$ into the identity

$$(p_4^D-p_3^D)=(p_4^D-p_4^F)+(p_4^F-(p_3^F+s))+((p_3^F+s)-p_3^D) \tag{2}$$

we obtain

$$(p_4^D-p_3^D)\geqq(p_4^F-(p_3^F+s))+(1+\lambda)(p_4^D-p_4^F). \tag{3}$$

Equation (3) thus depends on consumer rationality *alone*, provided only that some consumers buy from the incumbent in both the entry and the post-entry periods—it holds, for example, independent of whether firms choose prices or quantities and of whether the equilibrium is Nash or Stackelberg or collusive. Therefore, we have a price war, provided that (i) the incumbent's price at least equals the entrants' after entry and (ii) the entrants' price rises by at least the switching cost after it has locked customers in during the entry period, with at least one strict inequality in (i) and (ii). (This assumes that the incumbent's price initially falls during entry.) Furthermore, the same conditions guarantee a price war even if consumers are myopic. To see this, note that myopic consumers will purchase from the incumbent only if its period-3 price exceeds the entrants' price by no more than s, that is, $p_3^D\leqq p_3^F+s$, and substitute this condition into the identity (2).

The advantage of our model of a dominant firm and entering fringe over, for example, an oligopoly model is simplicity.[15] Restricting strategic behaviour to a single player greatly reduces the number of kinds of possible future behaviour that have to be considered when computing equilibrium prices for any given period. This allows us to be more specific about the equilibrium price and quantity paths—see Section 4.

14. Why, then, do we work with consumers of a continuum of different reservation prices? The reason is that to obtain "Type 1" price wars we need a type of consumer that buys from an entrant in period 3 but not in period 4, and to obtain "Type 2" price wars we need two different types of consumer who do buy in period 4 (so that the dominant firm and entrants can sell at different prices in period 4). Since we therefore need consumers of at least three different reservation prices, it is easiest to work with a continuum.

15. Klemperer (1983) discusses some limited results for the case of an incumbent facing a single entrant with Cournot-Nash post-entry competition. Klemperer (1987a) describes a two-period symmetric duopoly in the equilibrium of which both firms sell to more consumers in the first period than subsequently and the price rises between the periods by more than the switching cost, as firms price low initially to gain the largest possible share of high-reservation-price consumers who can be milked subsequently—this is exactly our Type 1 effect. Klemperer (1986, pp. 108–110) discusses generalisations of the current analysis, including to additional periods.

4. FURTHER CHARACTERISATION OF THE PRICE PATHS

In our model the binding constraint on the dominant firm's period-3 price is equation (1) above (see Appendix A), so we can write

$$p_3^D + \lambda p_4^D = p_3^F + \lambda p_4^F + s \tag{1'}$$

and substitute this equation into (2) as before to obtain

$$p_4^D = p_3^D + T_1 + T_2, \tag{3'}$$

in which

$$T_1 \equiv p_4^F - (p_3^F + s) \text{ and } T_2 \equiv (1+\lambda)(p_4^D - p_4^F).$$

Thus the incumbent's price rise after entry can be divided into two components, T_1 and T_2, which reflect the Type 1 and Type 2 effects discussed above. We show below that with sufficiently small switching costs we have a pure Type 1 price war, whereas with sufficiently large switching costs we have a pure Type 2 price war. With intermediate values of the switching cost, the price war may be a mixture of the two types (that is, we can have $p_4^D > p_4^F > p_3^F + s$, so $T_1 > 0$ and $T_2 > 0$).[16]

Proposition 2 (*Small Switching Costs*). *There exists $s^L > 0$ such that if $s \leqq s^L$ then, in equilibrium, $q_1^D \lesseqgtr q_2^D = q_3^D = q_4^D$ and $T_1 > 0$, $T_2 = 0$: the price war is pure Type 1. The firms' price paths are as exhibited in Table 1. It is possible that $q_1^D < q_2^D$ in which case $p_1^D > p_2^D$, that is, the incumbent limit prices in the pre-entry period.*

Proof. Obtainable from the author on request.[17]

To understand these price paths, recall from Section 3A that with sufficiently small switching costs, the threat of new entry in period 4 is the binding constraint on all firms' period-4 prices, so $p_4^D = p_4^F = c + s$ (and $T_2 = 0$). Now the period-3 entrants compete their price down to the point at which they make zero profits over periods 3 and 4. We note that a period-3 fringe price of $c - x$ represents an effective price to new customers of $c + s - x$ and so attracts the $f^{-1}(c+s-x)$ consumers with reservation prices above this level, minus the q_3^D consumers who buy from the dominant firm. The repeat purchasers from the fringe in the final period are the $f^{-1}(c+s)$ consumers with reservation prices above $c+s$, minus those who buy from the dominant firm, so the zero discounted profits condition for the fringe is $-x(f^{-1}(c+s-x) - q_3^D) + \lambda s(f^{-1}(c+s) - q_3^D) = 0$, which uniquely determines x as a function of the dominant firm's period-3 quantity, q_3^D. The dominant firm's period-3 price is then the highest at which its old customers are willing to repeat-purchase from it rather than switch to the fringe: $p_3^D = p_3^F + s = c + s - x$.

With small switching costs the dominant firm sells to all its period-2 customers in periods 3 and 4: $q_2^D = q_3^D = q_4^D$. If the dominant firm sells to no new customers in period 2, $q_1^D \geqq q_2^D$, then its period-2 price will be the reservation price of its marginal customer, $p_2^D = f(q_2^D)$, since the firm is a protected monopolist in this period. In this case its period-1 price must be $p_1^D = f(q_1^D) - s$, so that the marginal period-1 customer is willing to purchase for the first time. (All customers are new in period 1.) If, on the other hand, the dominant firm sells to any new customers in period 2, $q_1^D < q_2^D$, it must set a period-2 price

16. For detailed discussion of the general case see Klemperer (1986).

17. The proofs of Propositions 2 and 3 can be found in Klemperer (1986) and are also available from the author upon request.

TABLE 1

"Type 1" Price wars: Price paths with small switching costs ($s \leqq s^L$)
Case (i): "q-constant" ($q_1^D = q_2^D$)

	t: 1	2	3	4
p_t^D	$f(\bar{q}) - s$	$f(\bar{q})$	$c + s - x(\bar{q})$	$c + s$
p_t^F	—	—	$c - x(\bar{q})$	$c + s$

Notes: x is defined by $x(q)(f^{-1}(c+s-x(q))-q) = \lambda s(f^{-1}(c+s)-q)$

$$q_1^D = q_2^D = q_3^D = q_4^D = \bar{q}$$
$$= \arg\max_q \{q[(f(q)-s-c)+\lambda(f(q)-c)+\lambda^2(s-x(q))+\lambda^3 s]\}.$$

Case (ii): "limit pricing" ($q_1^D < q_2^D$)

	t: 1	2	3	4
p_t^D	$f(q_1^D) - (1-\lambda)s$	$f(\bar{\bar{q}}) - s$	$c + s - x(\bar{\bar{q}})$	$c + s$
p_t^F	—	—	$c - x(\bar{\bar{q}})$	$c + s$

Notes: x is defined as above

$$q_2^D = q_3^D = q_4^D = \bar{\bar{q}} = \arg\max_q \{q[(f(q)-s-c)+\lambda(s-x(q))+\lambda^2 s]\}$$

$$q_1^D = (\arg\max_q \{q(f(q)-(1-\lambda)s-c)\} \text{ subject to } q < q_2^D$$

and to

$$q[(f(q)-c)+\lambda(s-x(q))+\lambda^2 s] < \bar{\bar{q}}[(f(\bar{\bar{q}}-s-c)+\lambda(s-x(\bar{\bar{q}}))+\lambda^2 s)])$$

$p_2^D = f(q_2^D) - s$ to attract the marginal new customer (who will be no better off in period 3 from having bought in period 2). However in this case, period-1 customers expect to save paying a switching cost in period 2 as a result of having bought in period 1 (since they would anyway want to buy from the dominant firm in period 2). Thus the marginal period-1 customer is willing to pay his reservation price, less his period-1 switching cost, plus the discounted value of his period-2 saving: $p_1^D = f(q_1^D) - s + \lambda s$. We can now compute the quantity choices, and the corresponding price paths, that maximise the dominant firm's discounted profits over time, $\sum_{i=1}^{4} \lambda^{i-1} q_i^D (p_i^D - c)$, subject to the constraint that its strategy must be time-consistent. Both the cases $q_1^D = q_2^D$ and $q_1^D < q_2^D$ are possible (see Klemperer (1986)), and in the case that $q_1^D < q_2^D$ the dominant firm is limit pricing. We discuss this case further below.

Proposition 3 (*Large Switching Costs*). *Let* $s^H = (1+\lambda)(p^m - c)$, *in which* p^m *is the price that would be chosen by a monopolist with marginal costs c operating on demand curve* $f(\cdot)$ *in a single period with no switching costs, and let* $f(\cdot)$ *be such that such a monopolist's profit function is strictly quasi-concave in the range of q such that* $p^m \leqq f(q) \leqq c + s$. *If* $s \geqq s^H$ *then, in an equilibrium in which entry occurs in period* 3, $q_1^D = q_2^D = q_3^D = q_4^D$ *and* $T_1 = 0$, $T_2 > 0$: *the price war is pure Type* 2. *The firms' price paths are as exhibited in Table* 2.

Proof. See footnote 17.

The intuition for these price paths is that with large switching costs, fringe firms sell only to consumers with relatively large reservation prices in period 3 and hence sell to

TABLE 2

"Type 2" Price wars: Price paths with large switching costs ($s \geqq s^H$)

	t			
	1	2	3	4
p_t^D	$f(\bar{q})-s$	$f(\bar{q})$	$c+\frac{s}{1+\lambda}-\lambda\left(p_4^D-\left(c+\frac{s}{1+\lambda}\right)\right)$	$\min\{f(\bar{q}), c+s\}$
p_t^F	—	—	$c-\frac{\lambda s}{1+\lambda}$	$c+\frac{s}{1+\lambda}$

Note: $q_1^D=q_2^D=q_3^D=q_4^D=\bar{q}=\arg\max_q\{q(f(q)-(1-\lambda)s-c)\}$.

all the consumers again in period 4. The threat of period-4 entry is not a binding constraint on the fringe's period-4 prices. Therefore, since the fringe firms' marginal consumers are the same in both periods, these consumers pay their reservation price in period 4 and pay their reservation price less the switching cost in period 3: $p_4^F=p_3^F+s$ (so $T_1=0$). Since the fringe firms' output is the same in both periods, the zero-profits condition implies $(p_3^F-c)+\lambda(p_4^F-c)=0$, hence

$$p_3^F=c-\frac{\lambda s}{1+\lambda} \quad \text{and} \quad p_4^F=c+\frac{s}{1+\lambda}.$$

The dominant firm's period-4 price is the minimum of the reservation price of its marginal consumer, $f(q_4^D)$, and the highest price it can charge without its customers defecting to period-4 entrants, $c+s$. From this price we can deduce its period-3 price, which will be the highest it can charge without its customers defecting to period-3 entrants, this is, $p_3^D=p_3^F+s-\lambda(p_4^D-p_4^F)$. (This is equation (1').) Finally, we can show that with large switching costs the dominant firm sells to the same customers in all four periods, so its period-2 price is the reservation price of its marginal customer, $f(q_2^D)$, and its period-1 price is this reservation price less the switching cost, $f(q_1^D)-s$. We can check, therefore, that the dominant firm's profits are $\sum_{i=1}^4 \lambda^{i-1}q_i^D(p_i^D-c)=(1+\lambda)(q(f(q)-(1-\lambda)s-c))$, in which $q=q_1^D=q_2^D=q_3^D=q_4^D$, and hence we can determine the dominant firm's optimal quantity choice and the corresponding price paths.[18]

Thus with large switching costs the incumbent never limit prices, that is, it never increases its output in the period immediately preceding entry (never sets $q_2^D>q_1^D$) or lowers its price in that period (never chooses $p_2^D<p_1^D$).[19] To see why limit pricing can arise when switching costs are small, but *cannot* arise with large switching costs, when

18. Note that although $q_2^D=q_3^D=q_4^D$ both for sufficiently small and for sufficiently large switching costs (Propositions 2 and 3), this is not true in general. For example, with intermediate switching costs, the dominant firm may strategically restrict q_3^F by choosing q_2^D and q_3^D larger than the monopoly output on $f(\cdot)$ if there were no switching costs, and then choose q_4^D equal to this monopoly level (so $q_4^D<q_3^D$). Choosing $q_3^D<q_2^D$ can be optimal when allowing the fringe to serve more high-reservation-price customers in period 3 raises p_4^F and hence also loosens the constraint, (1'), on p_3^D and p_4^D. With sufficiently small or sufficiently large switching costs, these complications are avoided because p_4^F is then pinned to $c+s$ or $c+s/(1+\lambda)$, respectively, which reduces the benefit the dominant firm can obtain from strategic manoeuvering. See Klemperer (1986) for detailed discussion.

19. The incumbent does choose a higher q_1^D and q_2^D and so a lower p_1^D and p_2^D than if no entry could occur in period 3, unless entry is prevented by the outputs that would be optimal if no entry were threatened. However, it chooses q_1^D and q_2^D taking the entrants' post-entry prices as given, and always $q_2^D=q_1^D$ and $p_2^D>p_1^D$. We show in Klemperer (1986) that this is true even if the switching cost is so large that it is optimal for the incumbent to completely block new entry, although the dominant firm's quantity choice and price path may then be different from those in Table 2.

post-entry profits are higher, observe from (1′) that the discounted sum of the dominant firm's prices over the last two periods is exactly s higher than the discounted sum of the entrants' prices. With large switching costs the entrants' prices are independent of the incumbent's behaviour (see Table 2) and their discounted sum equals the discounted sum of costs, $(1+\lambda)c$. With small switching costs, however, the discounted sum of entrants' prices depends on, and so can be manipulated by, the incumbent's behaviour (see Table 1) and also always exceeds the discounted sum of costs.

Finally, we confirm our claim that period-4 fringe entrants sell no output in equilibrium (though we have seen that their potential entry may constrain the behaviour of the other firms). To see this, note that the fringe firms' period-3 price is always below c, since they make zero profits over periods 3 and 4 together. Therefore all consumers with reservation prices above $c+s$ buy from some firm in period 3, and will repeat-purchase from the same firm in period 4 (since no firm charges more than $c+s$ in period 4). Thus the period-4 potential entrants can attract no customers at the price, c, at which they are willing to sell.

5. CONCLUSION

In our model of a market in which consumers have switching costs, the entry of new firms leads to a price war, and we have argued (in Section 3D) that this conclusion is robust. A price war could also be caused by the entry of new consumers who are not yet committed to any particular firm, as firms cut prices to attract them. More generally, if each period a small fraction of consumers leaves the market and is replaced by new consumers, price wars arise if, as seems likely, firms use the same period to cut prices to replenish their customer bases.[20] Finally, if consumers have incomplete information about firms' costs, each firm has an incentive to price low in early periods to signal that it has low costs and so will charge low future prices, because consumers who have switching costs consider expected future prices when making their current choices between competing brands.[21] This provides an additional reason for a price war.

APPENDIX

We write D for the dominant firm and F for the set of period-3 fringe entrants, and let r_t^D and r_t^F be the reservation prices of the lowest-reservation-price consumers sold to in period t by D and F, respectively.

(A) *Proof of Proposition* 1 *and Equation* (1′)

In period 4, the free entry of new fringe firms implies that a firm's price could not exceed c if it were to sell to any consumers it had not previously sold to, so its period-4 profits would be non-positive. However each firm that was in the market in period 3 sold to some consumers with reservation prices above c in that period (otherwise its profits over the last two periods would be negative) and so can make positive profits at at least some prices above c. Thus in period 4, neither D nor F sells to any consumer that it did not supply in period 3. That is,

$$q_4^D \leqq q_3^D \tag{A1}$$

20. When any one firm cuts price, other firms are unable to sustain high prices without their customers switching to the discounter, so we conjecture that firms will hold sales synchronously. (Of course, if there are sufficiently many new consumers in each period, firms may "hold sales" in every period so that we will not observe "price wars".) Note that if there are more new consumers in "booms" than in "busts", price wars will be more frequent in booms than in busts, so real wages will be procyclical. Bils (1985) develops a related intuition in a model of monopolist selling a product about the quality of which first-time customers are uncertain.

21. Bagwell (1987) develops a related intuition in a two-period model of a monopolist when consumers have search costs rather than switching costs.

and

$$q_4^F \leqq q_3^F. \quad (A2)$$

Hence

$$p_4^D = \min\{r_4^D, c+s\} \quad (A3)$$

and

$$p_4^F = \min\{r_4^F, c+s\}. \quad (A4)$$

(Each firm's price is constrained by its customers' total cost, $c+s$, of paying the switching cost and buying from a "new" fringe firm at price c. The constraint that a consumer must not want to pay the switching cost to buy from another firm that was present in period 3 does not bind.)

We note that a firm's lowest-reservation-price period-3 customer expects no gain in period-4 utility from having bought in period 3 (as compared with not having bought in period 3)—in period 4 the consumer will not buy or will pay his reservation price or will pay an amount equal to the total cost of buying from a new fringe entrant. Thus F's marginal consumers must at least break even in period 3. Therefore,

$$p_3^F \leqq r_3^F - s \quad (A5)$$

and, since F sells only to repeat purchasers in period 4,

$$p_3^F \leqq r_4^F - s. \quad (A6)$$

The zero-profits condition for the fringe firms and $p_4^F > c$ imply

$$p_3^F < c. \quad (A7)$$

Collecting (A4), (A6) and (A7) together establishes

$$p_3^F \leqq p_4^F - s. \quad (I)$$

Further progress requires three lemmas. Lemmas 1 and 2 use consumer rationality to determine how consumers distribute themselves across firms. Lemma 3 tells us that (if $q_3^F > 0$)

$$q_3^D \leq q_2^D \quad (A8)$$

and

$$q_3^D + q_3^F > q_2^D. \quad (A9)$$

The lengthy proofs are in Part B of the Appendix. (The intuition for (A8) is that to attract new customers in period 3, D's price would have to equal F's (below c), and its profitability would fall towards F's (zero-profit) level. The intuition for (A9) is that if entry did not increase industry output, D could completely prevent entry by maintaining its period-2 output in period 3, and (if its period-2 output were optimal) this would be a more profitable strategy than allowing entry.)

From Lemma 1, $r_2^D = f(q_2^D)$. Applying Lemma 2 first to (A1) and then to (A8), we have $r_4^D \geqq r_3^D \geqq f(q_2^D)$. Substituting in (A3),

$$p_4^D \geqq \min\{f(q_2^D), c+s\}. \quad (A10)$$

Also, noting (A9), the lowest-reservation-price consumer that purchases in period 3 must buy from F, so substituting in (A5), $p_3^F \leqq f(q_3^D + q_3^F) - s$. If the inequality were strict, however, more than $(q_3^D + q_3^F)$ consumers would wish to purchase in period 3, so

$$p_3^F = f(q_3^D + q_3^F) - s. \quad (A11)$$

If we had $p_3^D > p_3^F + s$, then D's lowest-reservation-price period-3 consumer would prefer to switch to F in that period since this consumer gains no utility in period 4 from buying from D in period 3. Therefore

$$p_3^D \leq p_3^F + s. \quad (A12)$$

Equations (A7) and (A12) together imply $p_3^D < c+s$, and (A9), (A11) and (A12) together imply $p_3^D < f(q_2^D)$, so using (A10) we have:

$$p_3^D < p_4^D. \quad (II)$$

There are now two cases. If $q_1^D \geqq q_2^D$, then by Lemma 1 D sells only to consumers with reservation prices greater than or equal to $f(q_2^D)$ in period 2, and all these consumers bought from D in period 1. Therefore $p_2^D \geqq f(q_2^D)$, since no consumer can be made worse off in future periods by buying from D in period 2. Recalling from the previous paragraph that $p_3^D < f(q_2^D)$, we have

$$\text{if } q_1^D \geqq q_2^D \text{ then } p_2^D > p_3^D. \tag{IIIa}$$

If instead $q_1^D < q_2^D$ (this is in fact the case for some $f(\cdot)$, c, s, and λ—see Section 4), D sells in period 1 to the consumers who have reservation prices greater than or equal to $f(q_1^D)$, by Lemma 1. All these consumers gain λs (in period-1 terms) in the future from buying in period 1, since any consumer with reservation price greater than or equal to $f(q_2^D)$ is willing to pay the switching cost, s, to buy in period 2 even if he did not buy in period 1. Therefore $p_1^D = f(q_1^D) - s + \lambda s$. In period 2, D sells to consumers with reservation prices as low as $f(q_2^D)$, these consumers pay the switching cost to buy from the firm in that period (since $q_2^D > q_1^D$), and the maximum utility that could be gained in future periods from having bought from D in period 2 is the cost of switching to D in the next period, or λs in period-2 terms. Therefore $p_2^D \leqq f(q_2^D) - s + \lambda s$. Hence

$$\text{if } q_1^D < q_2^D \quad \text{then } p_1^D > p_2^D. \tag{IIIb}$$

Results (I), (II), (IIIa), and (IIIb) together establish Proposition 1. Note that the proof of Lemma 3 shows that even if (A8) did not hold, $p_3^D = p_3^F \leqq f(q_3^D + q_3^F) - s < c$ but $p_4^D > c$, $p_4^F > c$, and $p_2^D \geqq f(q_2^D) - s$, so Proposition 1 holds. It follows that Proposition 1 does *not* depend on the assumption (that is required for (A8)) about the distribution of D's consumers' reservation prices if D expands output in period 3.

We now show $p_4^D \geqq p_4^F$. If, instead, $p_4^D < p_4^F$, then, since $p_4^F \leqq c + s$ by (A4), (A10) implies $p_4^D \geqq f(q_2^D)$. Therefore $p_4^F > f(q_2^D)$, so in period 4, and hence also in period 3, F serves some consumers with reservation prices above $f(q_2^D)$. Using Lemma 1, these consumers must switch from D to F in period 3, but if in equilibrium $p_4^F > p_4^D$ no consumer who expects to buy in period 4 will be willing to make this switch unless $p_3^D > p_3^F + s$ which contradicts (A12). Therefore

$$p_4^D \geqq p_4^F. \tag{A13}$$

Finally, p_3^D is the maximum price that satisfies (i) $p_3^D \leqq r_3^D$, (ii) $p_3^D \leqq p_3^F + s$, and (iii) $p_3^D + \lambda p_4^D \leq p_3^F + \lambda p_4^F + s$. Condition (i) ensures that all D's period-3 consumers prefer buying from D to not buying from any firm in that period. (We use the facts that by (A8) and Lemmas 1 and 2 D's period-3 consumers all bought from it in period 2, and that the lowest-reservation-price consumer served gains no utility in period 4 from buying from D in period 3.) Condition (ii) ensures that no period-3 customer of D who expects not to buy or to be indifferent about buying from D in period 4 (we know that at least D's lowest-reservation-price customer is in this position) prefers to switch to F—this is equation (A12). Condition (iii) ensures that no consumer expecting to buy from D in both periods 3 and 4 would do better to switch to F. Since we know $r_3^D \geqq f(q_2^D)$, (A9) and (A11) ensure that (i) is weaker than (ii). By (A13), (ii) is weaker than (iii), so constraint (iii) always binds. That is, $p_3^D + \lambda p_4^D = p_3^F + \lambda p_4^F + s$, which can be rearranged to obtain (1′). ‖

(B) Proofs of Lemmas used in Part A

We begin with a preliminary lemma.

Lemma 0. *A firm can attract new customers and lose old customers in the same period only if (i) the old customers buy from no firm in that period, and (ii) the new customers all have reservation prices (net of switching costs) at least $(1-\lambda)s$ higher than the highest reservation price among the old customers.*

Proof. Consider a 'new' customer c_n who switches to firm X in the same period, t, that an 'old' customer c_0 (who bought from X in period $t-1$) does not buy from X.

To show (i), let c_0 switch to another firm Y in period t and let p_{t+j}^X and p_{t+j}^Y be the prices that consumers rationally expect the firms X and Y to charge in period $t+j$. Since an individual consumer has zero mass the prices are independent of the movements of c_0 and c_n.

Let k_0 = number of periods c_0 stays with Y before switching again or leaving the market. Then

$$\textstyle\sum_{i=1}^{k_0} p_{t+i-1}^X \lambda^{i-1} \geqq \sum_{i=1}^{k_0} p_{t+i-1}^Y \lambda^{i-1} + s. \tag{*}$$

Let k_n = number of periods c_n stays with X before switching again or leaving the market, so

$$\textstyle\sum_{i=1}^{k_n} p_{t+i-1}^X \lambda^{i-1} \leqq \sum_{i=1}^{k_n} p_{t+i-1}^Y \lambda^{i-1}. \tag{**}$$

So if $k_0 \leqq k_n$, c_n would be better off buying from Y for k_0 periods and then switching to X, by (*). But if $k_0 \geqq k_n$, c_0 would be better off staying with X for k_n periods and then switching to Y, by (**), so we have a contradiction.

To show (ii) let c_0, c_n, have reservation prices r_0, r_n, respectively. By (i) c_0 must leave the market so $p_t^X \geqq r_0$, else c_0 would do better to stay in the market one more period. But $p_t^X \leqq r_n-(1-\lambda)s$, else c_n would do better not to buy in period t and defer paying the switching cost at least to period $t+1$. So $r_0 \leqq r_n-(1-\lambda)s \Rightarrow$ (ii). ‖

Lemma 1. *The dominant firm sells to the q_t^D consumers with the highest reservation prices in each pre-entry period $t=1,2$.*

Proof. Assume instead that there is a first period $t \leqq 2$ in which a consumer c_l with reservation price $r_l \leqq f(q_t^D)$ buys from D in the same period that a consumer c_h with higher reservation price r_h does not buy. Then either neither consumer bought from D in period $t-1$ or both consumers bought from D in period $t-1$. (For c_h but not c_l to have bought in period $t-1$ contradicts Lemma 0 (ii).) Let k_l (k_h) be the first period after t in which c_l (c_h) does not (does) buy from D, respectively, or equal 5 is there is no such period.

First let $k_l \leqq 3$. Now c_l must weakly prefer his path to c_h's path for periods $t \dots, k_l-1$. Also, c_l loses less than c_h by not buying in a period. So since c_l buys in every period to k_l-1, and c_h does not buy in period t, c_h must prefer c_l's path for periods $t, \dots, k_l-1$, (by at least the value of r_h-r_l in period t), which is a contradiction.

Next let $k_h < k_l$. Now c_l weakly prefers his path for periods $t, \dots k_h$. So, since c_h's path involves not buying in period t, c_h strictly prefers c_l's path, which is a contradiction.

So $k_h \geqq k_l \geqq 4$, hence c_h does not buy the product in periods $t, \dots, 2$.

Now let U_{jk} be c_j's net discounted utility of following c_k's price path from period t on, j, $k=h$, l. With rational expectations, $U_{ll} \geqq U_{lh}$ and $U_{hh} \geqq U_{hl}$, so

$$U_{hh}-U_{lh} \geqq U_{hl}-U_{ll}. \tag{†}$$

Observe

$$U_{hh}-U_{lh} = \textstyle\sum_{i=t}^4 \lambda^{i-t}(r_h-r_l)I(c_h^i)$$

and

$$U_{hl}-U_{ll} = \textstyle\sum_{i=t}^4 \lambda^{i-t}(r_h-r_l)I(c_l^i)$$

where

$$I(c_j^i)=\begin{cases}1 & \text{if consumer } c_j \text{ buys from some firm in period } i\\ 0 & \text{if not.}\end{cases}$$

But, from above, c_h does not buy in periods $t, \dots, 2$ and c_l buys at least in periods $t, \dots, 3$ so

$$U_{hh}-U_{lh} \leqq \textstyle\sum_{i=3}^4 \lambda^{i-t}(r_h-r_l) < \sum_{i=t}^3 \lambda^{i-t}(r_h-r_l) \leqq U_{hl}-U_{ll},$$

which contradicts (†). ‖

Lemma 2. *If, for any firm X, $q_{t-1}^X \geqq q_t^X$, then $r_{t-1}^X \leqq r_t^X$.*

Proof. The result follows from Lemma 0 (ii), since X cannot attract new customers in period t without also losing old ones. ‖

Lemma 3. *In an equilibrium in which entry occurs in period* 3 ($q_3^F>0$),

$$q_3^D \leqq q_2^D \tag{A8}$$

and

$$q_3^D+q_3^F > q_2^D \tag{A9}$$

Proof. We use the fact that (A1)-(A7) and (I) hold independent of q_1^D, q_2^D, and q_3^D. The zero-profits condition for the fringe is

$$(p_3^F-c)q_3^F+\lambda(p_4^F-c)q_4^F=0. \tag{B1}$$

Substituting for q_3^F in (B1) using (A2) and noting (A7), we have $(p_3^F-c)q_4^F+\lambda(p_4^F-c)q_4^F \geqq 0$. Substituting now for p_3^F using (I) implies $p_4^F \geqq c+s/(1+\lambda)$. Substituting instead for p_4^F using $p_4^F \leqq c+s$ (from (A4)) implies $p_3^F \geqq c-\lambda s$. Finally, in period 4 one option available to period-3 fringe entrants is to leave their output unchanged: $q_4^F = q_3^F$. In this case Lemma 2 and (A5) imply $r_4^F \geqq p_3^F + s$. Equations (A4) and (A7) would then imply $p_4^F \geqq p_3^F + s$. Since, therefore, $q_4^F = q_3^F$ and $p_4^F = p_3^F + s$ cannot yield profits over the last two periods for the fringe given q_3^F and p_3^F, we have $(p_3^F - c)q_3^F + \lambda(p_3^F + s - c)q_3^F \leqq 0$ and so $p_3^F \leqq c - \lambda s/(1+\lambda)$. Collecting these results together,

$$c - \lambda s \leqq p_3^F \leqq c - \frac{\lambda s}{1+\lambda} \tag{B2}$$

and

$$c + \frac{s}{1+\lambda} \leqq p_4^F \leqq c + s. \tag{B3}$$

Now $q_3^F \geqq q_4^F$ from (A2) and $(p_3^F - c) < 0$ from (B2) so using (B1) we have $(p_3^F - c) + \lambda(p_4^F - c) \geqq 0$. Part A of this Appendix uses arguments that are independent of any in this proof to show that *if* (A8) and (A9) hold then $p_3^D + \lambda p_4^D = p_3^F + \lambda p_4^F + s$ (this is (1′)) so also $(p_3^D - c) + \lambda(p_4^D - c) - s \geqq 0$. Since $p_4^D \leqq c + s$ from (A3), therefore, we have

$$\text{if (A8) and (A9) hold,} \quad (p_3^D - c) \geqq (1-\lambda)s. \tag{B4}$$

We repeat that (B1), (B2) and (B3) are independent of (A8) and (A9).

We prove (A8) by showing that, given q_2^D, choosing $q_3^D > q_2^D$ is less profitable than choosing $q_3^D = q_2^D$. As usual, no firm's lowest-reservation-price period-3 consumer gains any utility in period 4 from having purchased in period 3. Therefore, noting Lemma 1, if $q_3^D > q_2^D$, we need $p_3^D \leqq p_3^F$ for D's lowest-reservation-price period-3 consumer to prefer buying from it to buying from F in period 3. Similarly we need $p_3^F \leqq p_3^D$, so $p_3^D = p_3^F \leqq f(q_3^D + q_3^F) - s$ for the lowest-reservation-price consumer who buys from any firm to be willing to buy. By (B2), therefore, $p_3^D < c$. If instead $q_3^D = q_2^D$, then (A8) and (A9) and hence (B4) apply, so $p_3^D \geqq c$, so D's period-3 profits are higher. Now if after choosing $q_3^D > q_2^D$, D chooses $q_4^D \leqq q_2^D$, D's period-4 profits are no higher than if it had chosen $q_3^D = q_2^D$, since it could then choose any $q_4^D \leqq q_2^D$ and receive a price as high as if it had expanded output in period 3. So for D to choose $q_3^D > q_2^D$ in period 3, it must be the case that $q_4^D > q_2^D$ in period-4 equilibrium. Then also $p_4^D = p_4^F$ in equilibrium, since $p_3^D = p_3^F$ so otherwise D could not attract any new customers in period 3 who expect to buy from it in period 4. Therefore, by our assumption about the distributions of reservation prices of consumers served by the different firms in period 3 (see Section 2), D sells in period 4 to a no greater fraction than F of those customers it served for the first time in period 3. Therefore D, like F, makes at most zero profits on its "new" period-3 customers over periods 3 and 4 together. As before, D makes less profits on the q_2^D "old" customers in period 3 than if it had chosen $q_3^D = q_2^D$, since $(p_3^D - c)$ is negative, and makes no more profits on these consumers in period 4 than if it had chosen $q_3^D = q_2^D$, since it could then have chosen $q_4^D = q_2^D$ and received at least as high a price. So D chooses $q_3^D \leqq q_2^D$.

We prove (A9) by assuming for contradiction that $q_3^D + q_3^F = \tilde{q} \leqq q_2^D$. Then in period 3 F sells only to consumers who previously bought from D. Also, $p_3^D = p_3^F + s = f(\tilde{q})$, so that the lowest-reservation-price consumers F sells to are willing to switch to F while the lowest-reservation-price consumers D retains are willing to stay with D (since the marginal consumers expect no benefit in period 4 from buying in period 3). Hence $p_4^D = p_4^F$ so that for all firms there exist consumers who expect to buy from them in period 4 and are willing to buy from them in period 3. Now consider D's alternative strategy of setting $q_3^D = q_2^D$. Since $f(q_2^D) \leqq f(\tilde{q})$, there would be no entry since no consumer with reservation price below $f(\tilde{q})$ can be profitable to a fringe firm even if served in both remaining periods, and Lemmas 1 and 0 (i) (applied to D) ensure that fringe entrants can attract no consumers with higher reservation prices than $f(q_2^D)$. It also follows that D would sell in period 3 to all the consumers that F would sell to in period 3 if D followed its previous (supposed-optimal) strategy. Thus D's period-3 profits would change from $q_3^D(f(\tilde{q}) - c)$ to $q_2^D(f(q_2^D) - c)$. Also its period-4 profits would be higher, since if D chose a period-4 output equal to that previously supposed plus the previous value of q_4^F, it would receive the same price as under the previous strategy. So for the previous strategy to be optimal we require that $q_3^D(f(\tilde{q}) - c) > q_2^D(f(q_2^D) - c)$. Since $q_3^D < \tilde{q}$ and $f(\tilde{q}) - c \geqq 0$ (from (B2) and the fact that $p_3^F + s = f(\tilde{q})$), we have $\tilde{q}(f(\tilde{q}) - c) > q_2^D(f(q_2^D) - c)$.

Now, given D's supposed-optimal strategy its period-2 customers gain nothing in periods 3 and 4 from buying in period 2 (since $p_3^D = p_3^F + s$ and $p_4^D = p_4^F$). Therefore, if $q_2^D \leqq q_1^D$, D's period-2 profits are $q_2^D(f(q_2^D) - c)$. However its period-2 profits from choosing a period-2 output of $\tilde{q}$ would be at least $\tilde{q}(f(\tilde{q}) - c)$ whatever consumers' expectations, since $\tilde{q} \leqq q_2^D \leqq q_1^D$. Also, its profits in periods 3 and 4 would then be higher than under its supposed-optimal strategy, because it could maintain its period-3 output at $\tilde{q}$ (earning $\tilde{q}(f(\tilde{q}) - c) \geqq q_3^D(f(\tilde{q}) - c)$, since there would then be no entry in period 3) and then choose its period-4 output equal to the sum of the period-4 outputs that it and F would choose if it followed its supposed-optimal strategy.

If, on the other hand, $q_2^D > q_1^D$, D's period-2 profits are $q_2^D(f(q_2^D) - c - s) < \tilde{q}(f(\tilde{q}) - c - s)$ since $\tilde{q} \leqq q_2^D$. In this case also, therefore, D does better to choose a period-2 output of $\tilde{q}$ (earning at least $\tilde{q}(f(\tilde{q}) - c - s)$ in that period), a period-3 output of $\hat{q}$, and a period-4 output equal to the sum of the period-4 outputs that it and F would choose if it followed its supposed-optimal strategy. Thus D's supposed-optimal strategy is not in fact optimal, which proves (A9). ||

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REFERENCES

AREEDA, P. and TURNER, D. F. (1975), "Predatory Pricing and Related Practices under Section 2 of the Sherman Act", *Harvard Law Review*, **88**, 697-733.

BAGWELL, K. (1987), "Introductory Price as a Signal of Cost in a Model of Repeat Business", *Review of Economic Studies*, **54**, 365-384.

BAIN, J. S. (1949), "A Note on Pricing in Monopoly and Oligopoly", *American Economic Review*, **39**, 448-464.

BANERJEE, A. and SUMMERS, L. H. (1987), "On Frequent-Flyer Programs and Other Loyalty-Inducing Economic Arrangements" (Working Paper, Harvard University).

BILS, M. (1985), "Pricing in a Customer Market" (Working Paper, University of Rochester).

CASSADY, R. (1963), "*Price Warfare in Business Competition: A Study of Abnormal Competitive Behavior*" (Occasional Paper No. 11, Bureau of Business and Economic Research, Graduate School of Business Adminstration, Michigan State University).

CONRAD, C. A. (1983), "The Advantage of Being First and Competition between Firms", *International Journal of Industrial Organization*, **1**, 353-364.

DOYLE, C. (1986), "Intertemporal Price Discrimination, Uncertainty and Introductory Offers", *Economic Journal (Supplement)*, **96**, 71-82.

FARRELL, J. and SHAPIRO, C. (1988), "Dynamic Competition with Switching Costs", *Rand Journal of Economics*, **19**, 123-137.

FUDENBERG, D. and TIROLE, J. (1986), "A Theory of Exit in Duopoly", *Econometrica*, **54**, 943-960.

FUDENBERG, D. and TIROLE, J. (1986), "A 'Signal-Jamming' Theory of Predation", *Rand Journal of Economics*, **17**, 366-376.

GASKINS, D. W. (1971), "Dynamic Limit Pricing: Optimal Pricing under Threat of Entry", *Journal of Economic Theory*, **3**, 306-322.

GREEN, E. J. and PORTER, R. H. (1984), "Noncooperative Collusion under Imperfect Price Information", *Econometrica*, **52**, 87-100.

KLEMPERER, P. D. (1983), "Notes on Consumer Switching Costs and Price Wars" (Working Paper, Graduate School of Business, Stanford University).

KLEMPERER, P. D. (1986), *Markets with Consumer Switching Costs* (Unpublished PhD Thesis, Stanford University).

KLEMPERER, P. D. (1987a), "Markets with Consumer Switching Costs", *Quarterly Journal of Economics*, **102**, 375-394.

KLEMPERER, P. D. (1987b), "The Competitiveness of Markets with Switching Costs", *Rand Journal of Economics*, **18**, 138-150.

MAILATH, G. J. (1984), "Welfare Implications of Differential Information in a Dynamic Duopoly Model" (Working Paper, Princeton University).

MATTHEWS, S. A. and MIRMAN, L. J. (1983), "Equilibrium Limit Pricing: The Effects of Private Information and Stochastic Demand", *Econometrica*, **51**, 981-996.

MILGROM, P. and ROBERTS, J. (1982), "Limit Pricing and Entry under Incomplete Information: An Equilibrium Analysis", *Econometrica*, **50**, 443-459.

ORDOVER, J. A. and WILLIG, R. D. (1981), "An Economic Definition of Predation: Pricing and Product Innovation", *Yale Law Journal*, **91**, 8-53.

RIORDAN, M. H. (1975), "Imperfect Information and Dynamic Conjectural Variations", *Rand Journal of Economics*, **16**, 41-50.

SALONER, G. (1984), "Dynamic Equilibrium Limit-Pricing in an Uncertain Environment" (Working Paper 342, Massachusetts Institute of Technology).

SCHMALENSEE, R. (1982), "Product Differentiation Advantages of Pioneering Brands", *American Economic Review*, **72**, 349-365.

SHILONY, Y. (1977), "Mixed Pricing in Oligopoly", *Journal of Economic Theory*, **14**, 373-388.

STIGLER, G. J. (1964), "A Theory of Oligopoly", *Journal of Political Economy*, **72**, 44-61.

VARIAN, H. R. (1980), "A Model of Sales", *American Economic Review*, **70**, 651-659.

VON WEIZSÄCKER, C. C. (1984), "The Costs of Substitution", *Econometrica*, **52**, 1085-1116.

WILLIAMSON, O. (1977), "Predatory Pricing: A Strategic and Welfare Analysis", *Yale Law Journal*, **87**, 284-340.

ZACHAU, U. (1985), "Strategic Price and Quantity Signalling in a Duopoly Game", (Working Paper, Oxford University).