A Structural Estimation for the Effects of Uncertainty on Capital Accumulation with Heterogeneous Firms

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Job Talk 2009
Research Question

- What are the Effects of Uncertainty on a firm’s Investment Behaviour and Capital Accumulation?
  - Uncertainty
    - more than one possibility for future Demand/Productivity
  - short-run Investment Behaviour
    - Investment Rate; Capital Adjustment
  - long-run Capital Accumulation
    - Capital Stock Level; Capital-to-Output Ratio
Why Interesting and Important?

- **Uncertainty–Investment Dynamics**
  - implications for business cycle

- **Uncertainty–Capital Accumulation**
  - implications for economic growth
  - implications for economic development

- **Investment–Asset Returns**
  - how they are jointly determined
  - how they respond to exogenous shocks
Theoretical Literature

\[
MPK = \text{user cost of capital} \\
E[MPK(\sigma)] = E[\text{user cost of capital}(\sigma)] \\
E[MPK(\sigma)] = E[\text{Jorgenson user cost of capital}(\sigma)] + \text{adjustment cost of capital}(\sigma) \\
E[MPK(\sigma)] = E[(r(\sigma) + \delta) \cdot p_K] + \text{cost (wedge btw } p_K^+ \text{ and } p_K^-) + \text{cost (change in } K) \\
\]

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<thead>
<tr>
<th>↑ Jensen’s Inequality ↑</th>
<th>↓ risk-premium ↓</th>
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<th>↓ HAC Effect ↓</th>
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<td>BSW (2007)</td>
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<th>↑ Adjustment Cost Effect ↑</th>
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Guiying (Laura) Wu
Uncertainty and Capital Accumulation
Empirical Literature

- **Leahy and Whited (1996, JMCB)**
  - uncertainty-investment rate relationship
  - reduced-form sample splitting tests
  - irreversibility (Yes); HAC (No); discount rate (No)

- **Cooper and Haltiwanger (2006, RES)**
  - investment-rate distribution and dynamics
  - structural estimation for capital adjustment costs
  - evidence for irreversibility, quadratic and fixed costs

- **Bloom (2007, Econometrica)**
  - impact effect of uncertainty on investment rate
  - structural estimation for capital adjustment costs
  - irreversibility (Yes); discount rate (Little); HAC (off)
Why Little Evidence for HAC & Discount Rate Effects?

- **Uncertainty-Investment Rate is Insufficient for Identifying HAC and Discount Rate Effects.**
  - HAC effect might be a level but not a ratio effect.
  - Irreversibility dominates discount rate effect in short-run.

- **Reduced-Form Regression is Difficult to Disentangle Distinct Effects.**
  - Effects from different channels work simultaneously.
  - Effects from different channels work non-linearly.
Novelty of the Paper

1. Focus on Long-Run Uncertainty-Capital Accumulation
2. Develop a Structural Framework for Identification
3. Find Evidence for HAC and Discount Rate Effects
4. Highlight the Importance of Risk-Premium Component
5. Allow for Heterogeneity in Growth Rate & Firm Size
Route Map

1. An Investment Model under Uncertainty
2. Theoretical Propositions for Identification
3. Empirical Strategies for Identification
4. A Structural Estimation with SMM
5. Counterfactual Simulations
Dynamic Programming

- Net Revenue = Operating Profit - Adjustment Cost - Investment

\[ \Pi(Z_t, K_t; I_t) = \pi(Z_t, K_t; I_t) - G(Z_t, K_t; I_t) - I_t \]

↑

HAC Effect  Adj. Cost Effect

- Value Maximisation derived from Consumption Euler Equation

\[ V(Z_t, K_t) = E \left[ \sum_{s=1}^{\infty} \left( \frac{1}{1 + r_{t+1}} \right)^s \Pi(Z_{t+s}, K_{t+s}; I_{t+s}) \right] \]

= \max_{I_t} \{ \Pi(Z_t, K_t; I_t) + \frac{1}{1 + r_{t+1}} E_t [V(Z_{t+1}, K_{t+1})] \}

↑

D. R. Effect
Timing: $K_{t+1} = (1 - \delta) (K_t + I_t) \equiv (1 - \delta) \hat{K}_t$

where $\hat{K}_t$: productive capital stock; $\delta$: depreciation rate

Production: $Q_t = A_t L_t^{1-\beta} \hat{K}_t^\beta$

where $A_t$: productivity shocks; $0 < \beta < 1$

Demand: $Q_t = X_t P_t^{-\varepsilon}$

where $X_t$: horizontal demand shocks; $-\varepsilon < -1$
HAC Effect

Operating Profit and Output

- **Operating Profit:** Convexity

\[ \pi(X_t, A_t, \hat{K}_t) = \max_{L_t} P_t Q_t - wL_t \]
\[ = \max_{L_t} X_t^{\frac{1}{\varepsilon}} A_t^{\frac{\varepsilon}{\varepsilon-1}} L_t^{\frac{(\varepsilon-1)(1-\beta)}{\varepsilon}} \hat{K}_t^{\frac{(\varepsilon-1)\beta}{\varepsilon}} - wL_t \]
\[ = \text{const} \cdot X_t^{\gamma} (A_t^{\gamma})^{\varepsilon-1} \hat{K}_t^{1-\gamma} \]

- **Sales:** \[ Y_t = \gamma \varepsilon \cdot \pi_t \]

- **Labour:** \[ L_t = \left[ \left( \frac{\gamma \varepsilon - 1}{w} \right) \cdot \pi_t \right] \]
Stochastic Process

- **Demand Stochastic:**

\[
\begin{align*}
X_t &= \log X_t \\
X_t &= c_x + \mu_x t + \zeta_t^x \\
\zeta_t^x &= \rho_x \zeta_{t-1}^x + e_t^x = \zeta_0^x + \sum_{s=0}^{t-1} \rho_x^s e_{t-s}^x \\
\text{where } 0 < \rho_x < 1 \text{ and } e_t^x \sim_{i.i.d} N(0, \sigma_x^2)
\end{align*}
\]

- **Productivity Stochastic:**

\[
\begin{align*}
A_t &= \log A_t \\
A_t &= c_a + \mu_a t + \zeta_t^a \\
\zeta_t^a &= \rho_a \zeta_{t-1}^a + e_t^a = \zeta_0^a + \sum_{s=0}^{t-1} \rho_a^s e_{t-s}^a \\
\text{where } 0 < \rho_a < 1 \text{ and } e_t^a \sim_{i.i.d} N(0, \sigma_a^2)
\end{align*}
\]
Reparameterisation

- If $\rho_x = \rho_a = \rho$ and $e^x_t \perp e^a_t$, $Z_t$ encompasses $X_t$ and $A_t$

$$Z_t = X_t (A_t)^{e-1}$$

$$z_t = \log Z_t$$

$$z_t = c + \mu t + \zeta_t$$

$$\zeta_t = \rho \zeta_{t-1} + e_t = \zeta_0 + \sum_{s=0}^{t-1} \rho^s e_{t-s}$$

where $0 < \rho < 1$ and $e_t \sim N(0, \sigma^2)$
Combine $X$ and $A$ into $Z$

- **Measure of Overall Uncertainty**—$\sigma$
  \[ \sigma^2 = \sigma_X^2 + (\epsilon - 1)^2 \sigma_a^2 \]

- **Constant Proportion of Demand Uncertainty**—$\tau$
  \[ \sigma_X^2 = \tau \sigma^2 \]

- **Operating Profit after Reparameterisation**
  \[ \pi(Z_t, \hat{K}_t) = \begin{cases} 
  \text{const0} \cdot X_t^\gamma \hat{K}_t^{1-\gamma} & \text{if } \tau = 1 \\
  \text{const0} \cdot (A_t^\gamma)^{\epsilon-1} \hat{K}_t^{1-\gamma} & \text{if } \tau = 0 \\
  \text{const0} \cdot Z_t^\gamma \hat{K}_t^{1-\gamma} & \text{if } 0 \leq \tau \leq 1 
\end{cases} \]

- $\tau$: importance of HAC effect, estimated empirically
Three Forms of Adjustment Costs

- **Adjustment Cost Function:** Abel & Eberly (1994)

\[ G(Z_t, K_t; I_t) = -b_i I_t 1_{I_t < 0} + \frac{b_q}{2} \left( \frac{I_t}{K_t} \right)^2 K_t + b_f 1_{I_t \neq 0} \pi_t \]

\[ = \text{partial irreversibility} + \text{quadratic} + \text{fixed} \]

- **Parameter Restriction:**

\[ p_K^+ = 1, \quad p_K^- \leq p_K^+ \]
\[ 0 \leq b_i = (p_K^+ - p_K^-) / p_K^+ \leq 1 \]
\[ b_q \geq 0, \quad b_f \geq 0 \]

- **\( b = (b_i, b_q, b_f) \): estimated empirically**
Consumption-CAPM with Production

- **Fully-Diversification:** only aggregate shocks matter

  Craine (1989): \( r_{j,t+1} = \bar{r} - \frac{\text{cov}_t[SDF_{t+1}, MPK_{j,t+1}]}{E_t[SDF_{t+1}]} \)

- **Lack of Diversification:** idiosyncratic shocks also matter

  incomplete market–Angeletos & Calvet (2006):

  \( r_{j,t+1} = \bar{r} - \frac{\text{cov}_t[SDF_{j,t+1}, MPK_{j,t+1}]}{E_t[SDF_{j,t+1}]} \)

  imperfect information–Himmelberg, Hubbard & Love (2002):

  \( r_{j,t+1} = \bar{r} - \alpha \frac{\text{cov}_t[SDF_{j,t+1}, MPK_{j,t+1}]}{E_t[SDF_{j,t+1}]} - (1 - \alpha) \frac{\text{cov}_t[SDF_{t+1}, MPK_{j,t+1}]}{E_t[SDF_{t+1}]} \)

- **with Time Invariant Idiosyncratic Shocks:** \( r_j = \bar{r} + \theta \sigma_j \)

  \( \bar{r} \): risk-free interest rate; \( \theta \): estimated empirically
Route Map

1. An Investment Model under Uncertainty
2. Theoretical Propositions for Identification
3. Empirical Strategies for Identification
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5. Counterfactual Simulations
Effects of Uncertainty

- **Apply Mean-Preserving Spread:**
  - keeping $\mu$ constant, increasing $\sigma$

- **Two Measures of Capital Accumulation:**
  - Expected Capital Stock: $E\left[\hat{K}_t (\sigma)\right]$
  - Expected Capital Intensity: $E\left[\frac{\hat{K}_t (\sigma)}{Y_t (\sigma)}\right]$
Model Prediction

- **Frictionless Case:** analytical solution

\[ D.R. \quad HAC \]

\[ E \left[ \hat{K}_t^* (\sigma) \right] = const1 \cdot E [Z_t] \]

\[ E \left[ \hat{K}_t^* (\sigma) / Y_t (\sigma) \right] = const2 \quad \leftarrow \text{D.R. Effect} \]

where \( const1 = \left[ const0 \cdot (1 - \gamma) / \left( \frac{r + \delta}{1 + r} \right) \right]^{\frac{1}{\gamma}} \)

\[ const2 = \beta \left( 1 - \frac{1}{\varepsilon} \right) / \left( \frac{r + \delta}{1 + r} \right) \]

- **Friction Case:** numerical solution
  - **Adj. Cost Effect**
Summary

**Theoretical Propositions**

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<th>D.R.</th>
<th>HAC</th>
<th>Adj. Cost</th>
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<td>Prop1</td>
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<td>Prop2</td>
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<tr>
<td>Prop3</td>
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- $\partial E[\hat{K}_t(\sigma)] / \partial \sigma < 0$
- $\partial E[\hat{K}_t(\sigma) / Y_t(\sigma)] / \partial \sigma < 0$
- Parameters: if $\theta > 0$
- $\tau < 1$ if $\varepsilon > 2$
- $b_i, b_q, b_f$

- $I_t / K_t$: Prop4
- $\varepsilon > 2$
- $\rho, \mu, \sigma$

**Other Aspects of Economic Environment**

- Production: $\beta$; Demand: $\varepsilon$
- Stochastic Process: $\rho, \mu, \sigma$
Proposition 1—Discount Rate Effect

When $E[Z_t]$ is invariant to $\sigma$ and $G(Z_t, K_t; I_t) = 0$,

1. $\partial E \left[ \hat{K}_t^* (\sigma) \right] / \partial \sigma \begin{cases} < 0 & \text{if } \theta > 0 \\ \geq 0 & \text{if } \theta \leq 0 \end{cases}$ and

2. $\partial E \left[ \hat{K}_t^* (\sigma) / Y_t (\sigma) \right] / \partial \sigma \begin{cases} < 0 & \text{if } \theta > 0 \\ \geq 0 & \text{if } \theta \leq 0 \end{cases}$,

i.e. the effects of uncertainty on the expected capital stock and expected capital intensity depend on the sign of $\theta$.

Proof.

1. By comparative static analysis through $\text{const1}$;
2. By comparative static analysis through $\text{const2}$.
Proposition 2–HAC Effect

When \( \theta = 0 \) and \( G(Z_t, K_t; I_t) = 0 \),

1. \( \partial E \left( \hat{K}_t^* (\sigma) \right) / \partial \sigma \)
   \[= 0 \quad \text{if } \tau = 1 \text{ or } \varepsilon = 2 \]
   \[> 0 \quad \text{if } \tau < 1 \text{ and } \varepsilon > 2 \quad \text{and} \]
   \[< 0 \quad \text{if } \tau < 1 \text{ and } 1 < \varepsilon < 2 \]

2. \( \partial E \left( \hat{K}_t^* (\sigma) / Y_t (\sigma) \right) / \partial \sigma = 0, \)

i.e. the effects of uncertainty on the expected capital stock depend on the value of \( \tau \) and \( \varepsilon \), but uncertainty has no effect on the expected capital intensity through the HAC effect.

Proof.

1. By \( E[Z_t] = \exp \left[ \zeta_0 + \mu t + \frac{(\varepsilon - 2)(1 - \tau)}{2(\varepsilon - 1)(1 - \rho^2)} \sigma^2 \right] \);
2. By linear homogeneity property of the model.
Proposition 3–Adjustment Cost Effect

**Proposition**

When $\theta = 0$ and $\tau = 1$,

1. $\partial E \left[ \hat{K}_t (\sigma) \right] / \partial \sigma \begin{cases} < 0 & \text{if } b_q > 0 \\ \geq 0 & \text{if } b_i > 0 \text{ or } b_f > 0 \end{cases}$ and

2. $\partial E \left[ \hat{K}_t (\sigma) / Y_t (\sigma) \right] / \partial \sigma \begin{cases} \leq 0 & \text{if } b_q > 0, b_i > 0 \text{ or } b_f > 0. \end{cases}$

**Proof.**

1. $b_q$: Abel (2002); $b_i$: Abel & Eberly (1999); $b_f$: CHP (1999); BSW (2007) provides numerical illustration.

2. If $e_t \geq 0$, $\partial \left[ \hat{K}_t (\sigma) / Y_t (\sigma) \right] / \partial \sigma \begin{cases} \leq 0; \end{cases}$ overall unclear.
Proposition 4–Investment Rate

Investment rate distribution and dynamics are sufficient for identifying capital adjustment costs.

Proof.

Partial Irreversibility Only

Quadratic Adjustment Costs Only

Fixed Adjustment Costs Only
Route Map

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Identification is from Cross-Sectional Variation

- **Heterogeneity in the Level of Uncertainty**  
  Necessary condition for identifying HAC & D.R. Effects

- **Heterogeneity in the Growth Rate**  
  Distinguish adjustment cost from stochastic process

- **Heterogeneity in the Firm Size**  
  Control for persistent difference in firm size
Uncertainty Heterogeneity

Assumption

**Uncertainty Heterogeneity:** The measure of overall uncertainty for firm $j$ is $\sigma_j$, where $\log \sigma_j \sim N(\mu_{\sigma}, \sigma^2_{\sigma})$.

- $\mu_{\sigma}, \sigma_{\sigma}$: estimated empirically
Assumption

Growth Rate Heterogeneity: The overall growth rate for firm $j$ is $\mu_j$, where $\mu_j \sim^d N(\mu, \sigma^2)$ and $\text{cov} (\mu_j, \sigma_j) = 0$.

- $\mu, \sigma$: estimated empirically
**Recall** \( z_t = \log Z_t = c + \mu t + \zeta_0 + \sum_{s=0}^{t-1} \rho^s e_{t-s} \)

**Assumption**

**Firm Size Heterogeneity**: The intercept in the stochastic process for firm \( j \) is \( \zeta_{0j} \), where \( \zeta_{0j} \overset{i.i.d.}{\sim} N \left( \mu_{\zeta_0}, \sigma^2_{\zeta_0} \right) \) and \( \text{cov} \left( \zeta_{0j}, \sigma_j \right) = 0 \), \( \text{cov} \left( \zeta_{0j}, \mu_j \right) = 0 \).

\( \mu_{\zeta_0}, \sigma_{\zeta_0} \): estimated empirically
model heterogeneity

Empirical Distribution
From Unobservable to Observable

- Demand/Productivity \( \{ Z_t \} \) is Not Observable.
- sales growth rate \( dy_{j,t} \) proxies changes in \( Z_{j,t} \)

\[
dy_{j,t} = \log(Y_{j,t}) - \log(Y_{j,t-1}) \\
= \log(Z_{j,t}) - \log(Z_{j,t-1}) \\
= \mu_j + \zeta_{j,t} - \zeta_{j,t-1} \\
\simeq \mu_j + e_{j,t}
\]

- WG 1\textsuperscript{st} and 2\textsuperscript{nd} moment of \( dy_{j,t} \) proxy \( \mu_j \) and \( \sigma_j \)

\[
Edy_j = \text{mean}_t(dy_{j,t}) = \mu_j \\
SDdy_j = \text{sd}_t(dy_{j,t}) \simeq \sigma_j
\]
Measurement Errors

**Assumption**

**Measurement Errors in Investment Rate:** Denote $i_{j,t}$ and $i_{j,t}^*$ as observed and true investment rate. Suppose $i_{j,t} = i_{j,t}^* \exp(e_j^I)$, where $e_{j,t}^I = e_{j,t}^{IT} + e_{j,t}^{IP}$, and $e_{j,t}^{IP} \overset{i.i.d.}{\sim} N(0, \sigma_{IP}^2)$, $e_{j,t}^{IT} \overset{i.i.d.}{\sim} N(0, \sigma_{IT}^2)$.

**Assumption**

**Measurement Errors in Sales:** Denote $Y_{j,t}$ and $Y_{j,t}^*$ as observed and true sales. Suppose $Y_{j,t} = Y_{j,t}^* \exp(e_j^Y)$, where $e_{j,t}^Y = e_{j,t}^{YT} + e_{j,t}^{YP}$, and $e_{j,t}^{YP} \overset{i.i.d.}{\sim} N(0, \sigma_{YP}^2)$, $e_{j,t}^{YT} \overset{i.i.d.}{\sim} N(0, \sigma_{YT}^2)$.

- $\sigma_{IP}$, $\sigma_{IT}$, $\sigma_{YP}$, $\sigma_{YT}$: estimated empirically
Aggregation

**Assumption**

**Aggregation:** Each firm is made of \( m \) plants, where \( m \geq 1 \). For plant \( i \) of firm \( j \) in period \( t \), the law of motion for \( Z_{i,j,t} \) is given by

\[
\begin{align*}
  z_{i,j,t} &= \log Z_{i,j,t} \\
  z_{i,j,t} &= c_j + \mu_j t + \zeta_{i,j,t} \\
  \zeta_{i,j,t} &= \rho \zeta_{i,j,t-1} + e_{i,j,t} = \zeta_{0j} + \sum_{s=0}^{t-1} \rho^s e_{i,j,t-s}
\end{align*}
\]

where \( 0 < \rho < 1 \) and \( e_{i,j,t} \overset{i.i.d.}{\sim} N(0, \sigma_j^2) \)

- \( m \): pinned down empirically
## Route Map

1. An Investment Model under Uncertainty
2. Theoretical Propositions for Identification
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4. A Structural Estimation with SMM
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## Parameters

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<td>$\theta$</td>
<td>$r = \bar{r} + \theta \sigma$</td>
</tr>
<tr>
<td><strong>HAC Effect</strong></td>
<td>$\tau$</td>
<td>$\sigma_x^2 = \tau \sigma^2$</td>
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<tr>
<td><strong>Adjustment Costs</strong></td>
<td>$b_q$</td>
<td>quadratic adjustment costs</td>
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<td></td>
<td>$b_i$</td>
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<td>fixed adjustment costs</td>
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<td><strong>Production and Demand</strong></td>
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<td>capital share in production function</td>
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<td></td>
<td>$\varepsilon$</td>
<td>demand elasticity with respect to price</td>
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<td><strong>Stochastic Process</strong></td>
<td>$\rho$</td>
<td>serial correlation of shocks</td>
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<tr>
<td></td>
<td>$\mu_\mu$</td>
<td>mean of $\mu$, where $\mu$ is the growth rate</td>
</tr>
<tr>
<td></td>
<td>$\sigma_\mu$</td>
<td>standard deviation of $\mu$</td>
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<td></td>
<td>$\mu_{1\sigma}$</td>
<td>mean of log($\sigma$), where $\sigma$ measures the level of uncertainty</td>
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<td></td>
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<td>standard deviation of log($\sigma$)</td>
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<td></td>
<td>$\mu_{\zeta_0}$</td>
<td>mean of $\zeta_0$, where $\zeta_0$ is the intercept</td>
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<td></td>
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<td>standard deviation of $\zeta_0$</td>
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<tr>
<td><strong>Measurement Errors</strong></td>
<td>$\sigma_{IT}$</td>
<td>sd of transitory measurement errors in investment rates</td>
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<tr>
<td></td>
<td>$\sigma_{IP}$</td>
<td>sd of permanent measurement errors in investment rates</td>
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<tr>
<td></td>
<td>$\sigma_{YT}$</td>
<td>sd of transitory measurement errors in sales</td>
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<td></td>
<td>$\sigma_{YP}$</td>
<td>sd of permanent measurement errors in sales</td>
</tr>
</tbody>
</table>
Simulated Method of Moments

- **SMM Estimator:** (Gourieroux & Monfort, 1996)

\[
\hat{\Theta} = \arg \min_{\theta} L = \left( \hat{\Phi}^D - \frac{1}{H} \sum_{h=1}^{H} \hat{\Phi}_h^S (\Theta) \right)' \Omega \left( \hat{\Phi}^D - \frac{1}{H} \sum_{h=1}^{H} \hat{\Phi}_h^S (\Theta) \right)
\]

- **Global Specification Test:**

\[
OI = \frac{NH}{1 + H} L \sim \chi^2 \left[ \text{dim} (\hat{\Phi}) - \text{dim} (\Theta) \right].
\]

- **Local Identification:**

\[
\sqrt{N} \left( \hat{\Theta} - \Theta^* \right) \overset{D}{\rightarrow} N (0, W (H, \Omega^*))
\]

\[
W = \left( 1 + \frac{1}{H} \right) \left( E \left[ \partial \Phi^S (\hat{\Theta}) / \partial \Theta \right] \Omega^* E \left[ \partial \Phi^S (\hat{\Theta}) / \partial \Theta' \right] \right)^{-1}
\]
Dataset

- U.K. Manufacturing Firms from Datastream
- Unbalanced Panel for 672 firms during 1972 to 1991

Key Variables
- Investment \( (l_{j,t}) \)
- Capital stock \( (K_{j,t}) \)
- Sales \( (Y_{j,t}) \)
- Operating Profit \( (\pi_{j,t}) \)
## Moments-Identification

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<tr>
<th>Symbol</th>
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<th>Informativeness</th>
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<tr>
<td>$\text{corr}(EK_j, SDdy_j)$</td>
<td>corr. btw. BG capital stock and measure of uncertainty</td>
<td>$\tau, \theta, b$</td>
</tr>
<tr>
<td>$\text{corr}(EKY_j, SDdy_j)$</td>
<td>corr. btw. BG capital intensity and uncertainty</td>
<td>$\theta, b$</td>
</tr>
<tr>
<td>$\text{prop}(i_{j,t} &lt; -0.01)$</td>
<td>proportion of negative investment rates</td>
<td>$b_i$</td>
</tr>
<tr>
<td>$\text{prop}(</td>
<td>i_{j,t}</td>
<td>&lt; 0.01)$</td>
</tr>
<tr>
<td>$\text{prop}(i_{j,t} &gt; 0.20)$</td>
<td>proportion of investment spikes</td>
<td>$b_f, b_q$</td>
</tr>
<tr>
<td>$\text{corr}(i_{j,t}, yk_{j,t})$</td>
<td>corr. btw. investment rates and log sales-to-capital ratio</td>
<td>$b_q, b_i, \sigma_{IT}, \sigma_{IP}, \sigma_{YT}, \sigma_{YP}$</td>
</tr>
<tr>
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<td>serial correlation of investment rates</td>
<td>$b_q, \rho, \sigma_{\mu}, \sigma_{IT}, \sigma_{IP}$</td>
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<td>mean of profit-to-sales ratio</td>
<td>$\beta, \varepsilon$</td>
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<td>$\beta, \varepsilon, \theta$</td>
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### Estimates

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<td>0.024</td>
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<td>0.12% of total sales</td>
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Comparison with Literature

**Similar Estimates for Capital Adjustment Costs**

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### Specification Tests

**Restricted Models Fit the Data Worse**

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## Number of Plants

### Trade-off btw more Smoothness and Better Proxies

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Robustness Tests

- Relatively Robust to Choice of $\bar{\tau}$ and Moments

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<td>-7.654</td>
<td>-7.674</td>
<td>-8.000</td>
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<td>$\mu_{\zeta_{0}}$</td>
<td>1.837</td>
<td>1.754</td>
<td>1.723</td>
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<tr>
<td>$\sigma_{\zeta_{0}}$</td>
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<td>0.271</td>
<td>0.203</td>
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<td>$\sigma_{\zeta}$</td>
<td>0.286</td>
<td>0.262</td>
<td>0.444</td>
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<td>$\sigma_{\zeta}$</td>
<td>0.061</td>
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<td>0.051</td>
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<tr>
<td>$\sigma_{\gamma}$</td>
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<td>0.506</td>
<td>0.697</td>
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<tr>
<td>$O_{I}$</td>
<td>165</td>
<td>194</td>
<td>1131</td>
</tr>
<tr>
<td>degree of freedom</td>
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<td>1</td>
<td>18</td>
</tr>
</tbody>
</table>
Route Map

1. An Investment Model under Uncertainty
2. Theoretical Propositions for Identification
3. Empirical Strategies for Identification
4. A Structural Estimation with SMM
5. Counterfactual Simulations
Uncertainty-Investment Dynamics

- **Quantity of Interest**
  - **Expected Investment Rate:** $E \left[ \frac{I_t}{K_t} (\sigma) \right]$
  - how it responds to demand/productivity shocks $e_t$

- **Counterfactual Simulations**
  - Keeping all else Constant and Varying $\sigma$
Uncertainty-Investment Dynamics

The diagram illustrates the relationship between uncertainty and investment dynamics. The $E[I_t/K_t]$ values are shown for different levels of uncertainty, with markers indicating $2.0 \times \sigma$, $1.0 \times \sigma$, and the scenario without adjustment costs. The x-axis represents the level of uncertainty, $e_t$, ranging from -1 to 1, and the y-axis shows the expected value of investment, $E[I_t/K_t]$. The graph highlights how investment dynamics change with varying levels of uncertainty.
Uncertainty-Capital Accumulation

- **Quantities of Interest**
  - **Average Capital Stock:** \( E \left[ \hat{K}_t (\sigma) \right] = \frac{\text{sum} [\hat{K}_t (\sigma)]}{N} \)
  - **Aggregate Capital Intensity:** \( \frac{\text{sum} [\hat{K}_t (\sigma)]}{\text{sum} [Y_t (\sigma)]} \)
  - how they vary with level of uncertainty \( \sigma \)

- **Nested Control Experiments**
  1. setting \((\theta, \tau, b)\) at estimated level
  2. setting \(\theta = 0\): discount rate effect off
  3. setting \(\theta = 0\) and \(\tau = 1\): discount rate & HAC effects off
Guiying (Laura) Wu
Uncertainty and Capital Accumulation

HAC + Adj. Cost Effect

![Graphs showing the relationship between average capital stock and aggregate capital intensity with varying levels of uncertainty.

- Graph 1: Average capital stock versus average level of uncertainty.
- Graph 2: Aggregate capital intensity versus average level of uncertainty.

The graphs illustrate the impact of adjusting costs on capital accumulation under uncertainty. The x-axis represents the average level of uncertainty, while the y-axis shows the respective capital measures. The line indicates an increasing trend as uncertainty increases, highlighting the cost-effectiveness in adjusting capital in response to uncertain conditions.
Discount Rate + HAC + Adj. Cost Effect

![Graph showing the relationship between average capital stock and average level of uncertainty. The graph plots the average capital stock (E[K]) against the average level of uncertainty (σ). As the average level of uncertainty increases, the average capital stock decreases. The graph also shows the relationship between aggregate capital intensity (sum[K]/sum[Y]) and average level of uncertainty (σ), with a similar decreasing trend.]
Evidence for Three Forms of Adjustment Costs
Evidence for HAC and Discount Rate Effect
Negative Effects of Uncertainty on Capital Accumulation

(−) Discount Rate >> (+) HAC > (−) Adj. Cost
Related Works

- **Add Aggregated Shocks in addition to Idiosyncratic Ones**
  - *Uncertainty, Investment and Asset Returns: Variance v.s. Covariance*

- **Study the Effects of Financing Constraint**
  - *A Structural Estimation for a Financing Constraint Model with Financial Assets and Heterogeneous Firms*

- **Build a Bridge between Structural and Reduced**
  - *An Auxiliary Investment-Finance Model with Binding Functions Derived from a Structural Estimation*
**Consumer Intertemporal Optimisation**

**Consumption Euler Equation**

- **Gross Rate of Return for firm j:**

  \[ 1 + r_{j,t+1} = \frac{V_{j,t+1} + \Pi_{j,t+1}}{V_{j,t}} \]

- **Consumption Euler Equation:**

  \[ U'(C_{j,t}) = (1 + \vartheta)^{-1} E \left[ U'(C_{j,t+1}) \left(1 + r_{j,t+1}\right) \right] \]

  \[ = (1 + \vartheta)^{-1} E \left[ U'(C_{j,t+1}) \left(\frac{V_{j,t+1} + \Pi_{j,t+1}}{V_{j,t}}\right) \right] \]
Value Maximisation

- **Rearrange and Solve Recursively Forward:**

\[
V_{j,t} = E \left[ \sum_{s=1}^{\infty} \left( \frac{(1 + \vartheta)^{-s}}{U'(C_{j,t+s})} \right) \Pi_{j,t+s} \right]

= E \left[ \sum_{s=1}^{\infty} \left( \prod_{l=1}^{s} \frac{1}{r_{j,t+l}} \right) \Pi_{j,t+s} \right]

\geq E \left[ \sum_{s=1}^{\infty} \left( \frac{1}{1 + r_{j,t+1}} \right)^s \Pi_{j,t+s} \right] \text{ if } r_{j,t+s} \sim r_{j,t+1}, \ \forall s > 1
\]
Horizontal v.s. Vertical

- **Horizontal Demand Shocks:** \( Q_t = X_t P_t^{-\varepsilon} \)
  - possibility of identifying HAC
  \[
  \pi(X_t, A_t, \hat{K}_t) = const0 \cdot X_t^\gamma (A_t^\gamma)^{\varepsilon-1} \hat{K}_t^{1-\gamma}
  \]

- **Vertical Demand Shocks:** \( P_t = X_t Q_t^{-1/\varepsilon} \)
  - no possibility of identifying HAC
  \[
  \pi(X_t, A_t, \hat{K}_t) = const0 \cdot (X_t^\gamma)^{\varepsilon} (A_t^\gamma)^{\varepsilon-1} \hat{K}_t^{1-\gamma}
  \]

- **Which One is more Sensible?**
  - It depends–Klemperer & Meyer (1986)
Convexity of Profit Function under Perfect Competition

\[ \pi(P) = P \cdot Q - w \cdot L \]

\[ \pi'(P) = P \cdot \bar{Q} - w \cdot \bar{L} \]

Note: from Varian (1992), page 42
SMM–Intuition

Structural Model
DGP

Guess Structural Parameters (Θ)

Observe Empirical Dataset with N*T

Simulate H Datasets with N*T

Estimate a set of Empirical Moments \( \hat{\Phi} \)

Estimate same set of Simulated Moments
\[ \frac{1}{H} \sum_{h=1}^{H} \hat{\Phi}_h(\Theta) \]

Θ*

YES

MATCH?

NO

\( H \)
Jacobian Matrix for Binding Functions

\[
J = \frac{\partial \Phi^S(\Theta)}{\partial \Theta} = \begin{bmatrix}
\frac{\partial \Phi^S_1(\Theta)}{\partial \Theta_1} & \frac{\partial \Phi^S_2(\Theta)}{\partial \Theta_2} & \cdots & \frac{\partial \Phi^S_{19}(\Theta)}{\partial \Theta_{19}} \\
\frac{\partial \Phi^S_1(\Theta)}{\partial \Theta_1} & \frac{\partial \Phi^S_2(\Theta)}{\partial \Theta_2} & \cdots & \frac{\partial \Phi^S_{19}(\Theta)}{\partial \Theta_{19}} \\
\vdots & \vdots & \ddots & \vdots \\
\frac{\partial \Phi^S_1(\Theta)}{\partial \Theta_1} & \frac{\partial \Phi^S_2(\Theta)}{\partial \Theta_2} & \cdots & \frac{\partial \Phi^S_{19}(\Theta)}{\partial \Theta_{19}} \\
\end{bmatrix}
\]

- **Necessary Condition for Local Identification:**
  - \( J \) is of full row rank.
Numerical Procedures

1. detrend time trend \( \exp(\mu t) \) and intercept \( \exp(\tilde{\zeta}_0) \)
2. solve the model
   - loop over \( \sigma = \exp(\log \sigma_1, \log \sigma_2, \log \sigma_3) \)
   - loop over \( \mu = (\mu_1, \mu_2, \mu_3) \)
   - loop over \( \tilde{Z}_t = (\tilde{Z}_1, \ldots, \tilde{Z}_9) \) and \( \tilde{K}_t = (\tilde{K}_1, \ldots, \tilde{K}_{200}) \)
3. interpolate for \( \tilde{I}_t = h(\tilde{Z}_t, \tilde{K}_t) \)
4. simulate data for plant \( i \) of firm \( j \) in period \( t \)
5. plant-level data are aggregated into firm-level data
6. recover time trend \( \exp(\mu_j t) \) and intercept \( \exp(\tilde{\zeta}_{0j}) \)
7. add measurement errors
8. calculate moments
Nuisance Parameters

Lemma

Denote $\Gamma_j = \Gamma(\bar{r}_j, \delta_j, \beta_j, \varepsilon_j, w_j, t_j)$. If $\text{cov} (\Gamma_j, \sigma_j) = 0$, the effect of imposing common value for $(\bar{r}, \delta, \beta, \varepsilon, w, t)$ on the dispersion of $E \left[ \hat{K}_t (\sigma) \right]$ can be accounted for by adjusting $\sigma_{\zeta_0}$; the effect of choosing arbitrary value for $(w, t)$ and the unit of measurement on the level of $E \left[ \hat{K}_t (\sigma) \right]$ can be accounted for by adjusting $\mu_{\zeta_0}$.

- Impose $\bar{r} = 0.065$, $\delta = 0.08$
- Estimate $\beta$ and $\varepsilon$
- Choose arbitrary $w$, $t$ and unit of measurement