Macroeconomic Theory II: Investment

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the desired capital stock

- For a firm that can rent capital at a price of r_K, the firms' profits at a point in time are $\pi(k, X_1, X_2, \ldots, Xn) - r_k k$, where k is the amount of capital the firm rents and X_1 to X_n are exogenous variables.

- For a perfectly competitive firm, these would include the output price and input prices and the function therefore gives the firm's profits at the profit-maximising choice of inputs other than capital given k and the X variables.

- Assume that $\pi_K>0$ and $\pi_{KK}<0$ (subscripts indicate derivatives). The FOC is therefore:

$$(1) \quad \pi_K(k, X_1, X_2, .., X_n) = r_K$$

- that is, the firm rents capital up to the point where its marginal revenue equals its rental price.

the optimal capital stock

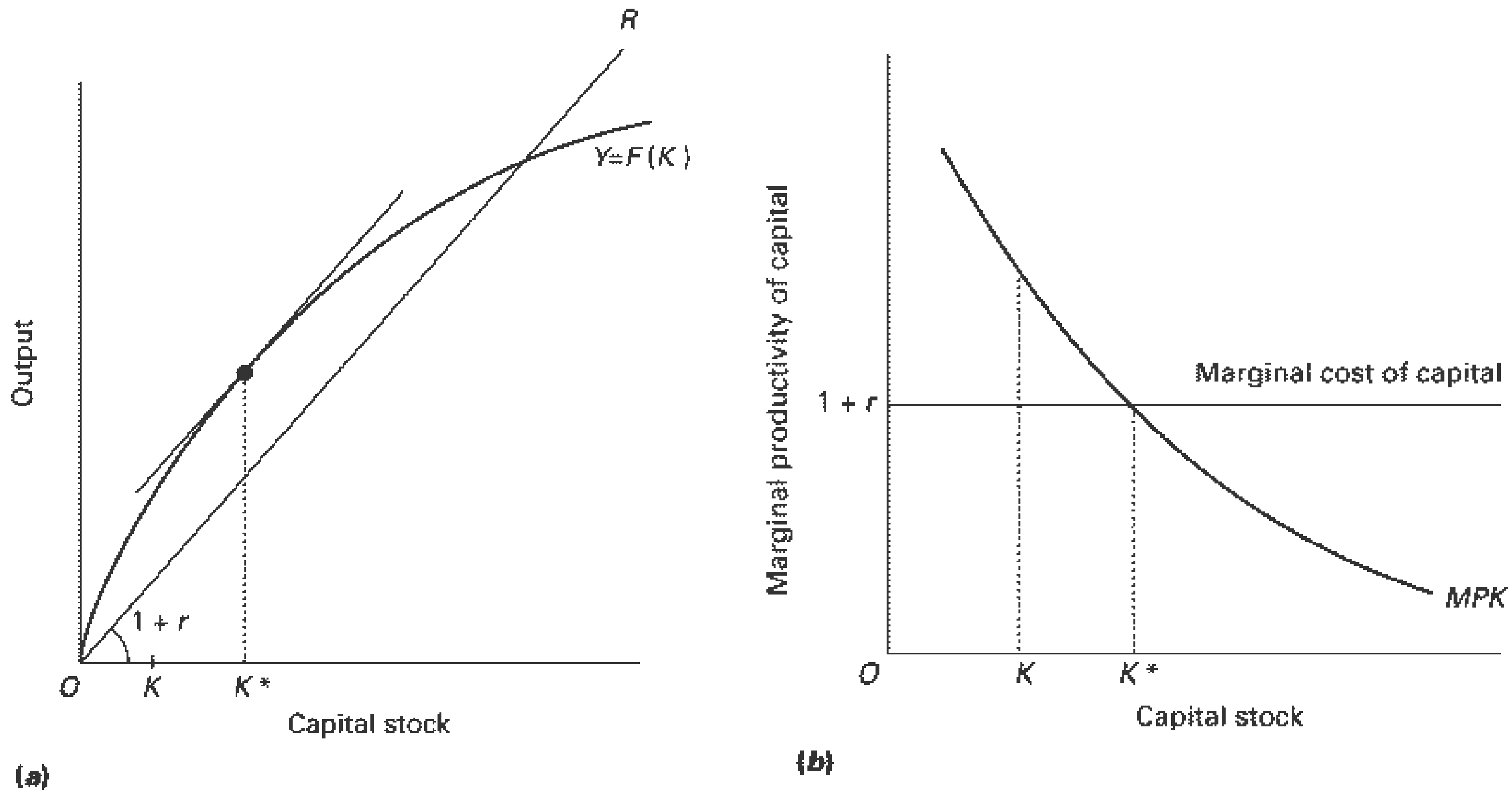


Figure 4.14. (a) The Optimal Capital Stock

Figure 4.14. (b) The Optimal Capital Stock

the user cost of capital

- Consider a firm that owns a unit of capital. Suppose that the real market price of that capital at time t is $p_K(t)$ and consider the firm's choice of selling the capital or continuing to use it. Keeping the capital has three costs to the firm. First, it forgoes interest on the proceeds if it sold the capital. Second, the capital is depreciating. Third, the price of capital goods may be changing. This yields:

(2) $$r_K(t) = r(t)p_K(t) + \delta p_K(t) - \dot{\rho}_K(t)$$

- In the presence of taxes though, the user cost may be rather different. Consider an investment tax credit where a firm's corporation tax is reduced by a fraction, *f*, of its investment spending and that taxable income is increased when capital goods are sold. This will change the effective price of capital and hence the user cost of capital (where τ is the corporation tax rate).

(3) $$r_K(t) = \left[r(t) + \delta - \frac{\dot{p}_K(t)}{p_K(t)} \right] (1 - f\tau) p_K(t)$$

- Thus the investment tax credit reduces the user cost of capital and hence increases the desired capital stock.

adjustment costs

- Consider an industry with N firms. A typical firm's real profits at time t, neglecting investment costs, are proportional to its capital stock, k(t) and decreasing in the industry capital stock K(t); so they take the form $\pi(K(t))k(t)$, where $\pi'(.)<0$. The assumption that a firm's profits are proportional to its capital stock is appropriate when it has constant returns to scale, output markets are competitive and the supply of labour is elastic. In this case, a firm with twice as much capital will employ twice as much labour and hence have revenues, costs and profits that are twice as high.

- The key assumption is that firms face costs in adjusting their capital stocks. The adjustment costs are a convex function of the rate of change of the firm's capital stock. Specifically, the adjustment costs, $C(\dot{k})$, satisfy C(0)=0, C'(0)=0 and C''(.)>0. These imply that it is costly to increase or decrease the capital stock and that the marginal adjustment cost is increasing in the size of the adjustment.

- The purchase price of capital goods is constant and equal to 1; thus there are only internal adjustment costs. For simplicity, we assume that depreciation is zero, so that net investment equals gross investment.

a discrete-time problem

- These assumptions imply that the firm's profits at a point in time are π(K)k-I-C(I). The firm maximizes the present value:

$$\Pi = \int_{t=0}^{\infty} e^{-rt}[\pi(K(t))k(t) - I(t) - C(I(t))]dt$$

- Where we assume the real interest rate is constant. Each firm takes the path of the industry-wide capital stock K as given and maximizes accordingly.
- If we define q(t) as the value to the firm of an additional unit of capital at time t+1 in time-t dollars (the shadow price does not necessarily equal the market price), we obtain a Lagrangian

$$L = \sum_{t=0}^{\infty} \frac{1}{(1+r)^t}[\pi(K(t))k(t) - I(t) - C(I(t)) + q(k(t) + I(t) - k(t+1))]dt$$

the continuous time case

- In the continuous-time case, we need to solve a current-value Hamiltonian where k is the state variable, I is the control variable, and the shadow value of the state variable, q, is the co-state variable:

$$H(k(t),I(t)) = \pi(K(t))k(t) - I(t) - C(I(t)) + q(t)[I(t) - \dot{k}(t)]$$

- We can express q, the value of capital, in terms of the discounted future value of its marginal revenue products:

(4)
$$q(t) = \int_{\tau=t}^{\infty} e^{-r(\tau-t)}\pi(K(\tau))d\tau$$

- The derivative of the Hamiltonian with respect to the state variable equals the discount rate times the co-state variable minus the derivative of the co-state variable with respect to time.

(5)
$$\pi(K(t)) = rq(t) - \dot{q}(t)$$

- where $\pi(K(t))$ is the MRP of capital, $rq(t)$ is the user cost of capital and $\dot{q}(t)$ is the rate of appreciation of capital prices.

Tobin’s q

- Tobin’s q shows how an additional dollar of capital affects the present value of profits. Thus the firm wants to raise k if q is high and reduce it if k is low.
- A one unit increase in the firm’s capital stock increase the present value of profits by q and thus raises the value of the firm by q.
- If we set the price of a unit of capital to 1 then q must equal 1 in equilibrium.
- q is the ratio of the market value of a marginal unit of capital to its replacement cost. This is *marginal q*, which is harder to measure than *average q*.
- When adjustment costs are convex (diminishing returns in adjustment), marginal q is less than average q. It is more than twice as costly for a firm with 20 units of k to add 2 more than for a firm with 10 to add 1. Firm’s lifetime profits therefore rise less than proportionately with their capital stocks, so marginal q is less than average q.

adjustment of the capital stock

- The derivative of the Hamiltonian with respect to the control variable at each point in time is zero. Each firm invests to the point where the purchase price of capital plus the marginal adjustment cost equals the value of capital:

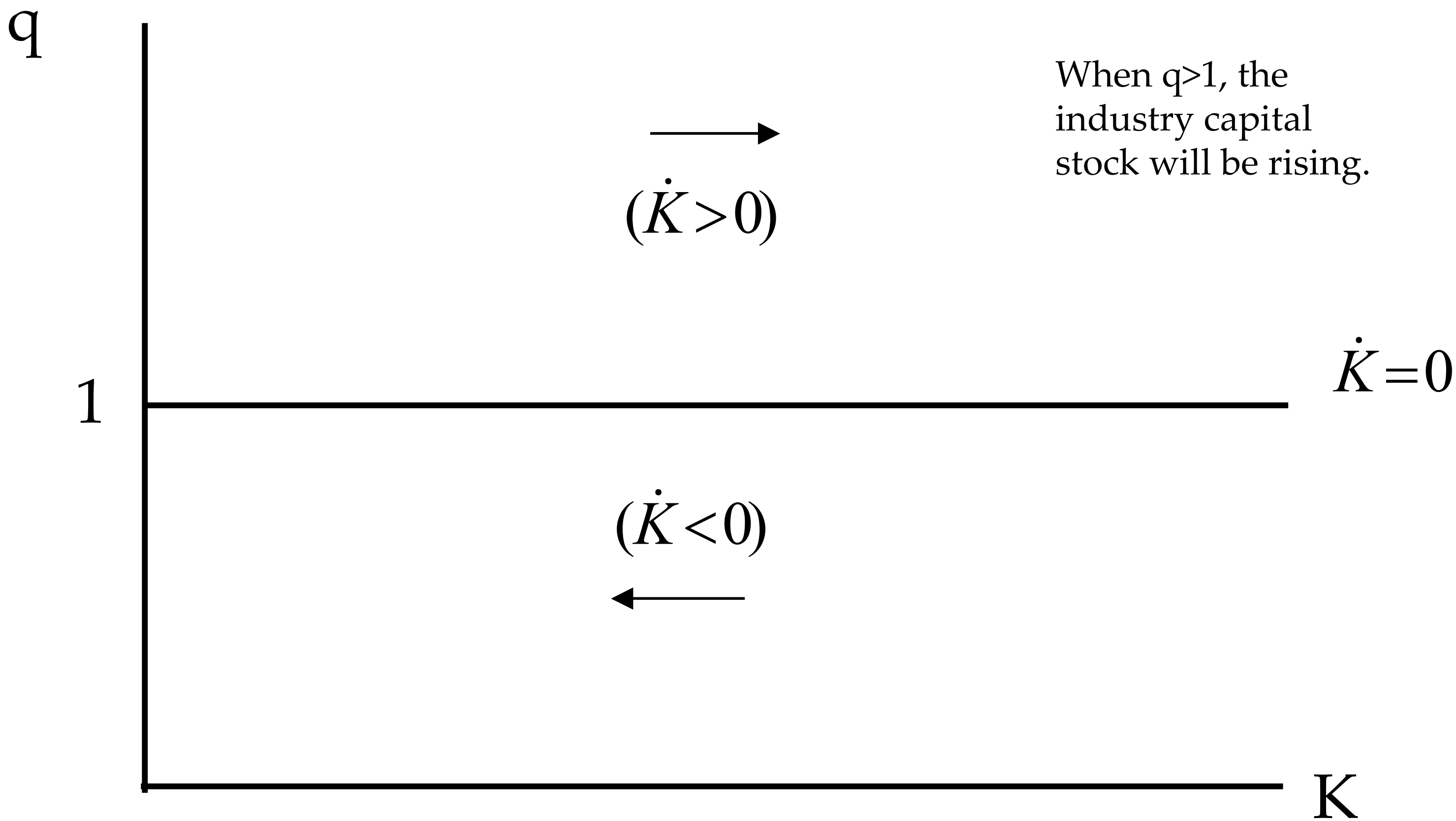
(6) $1+C'(I)=q$

- since $C'(I)$ is increasing in I, the condition implies that I is increasing in q. And since $C'(0)$ is zero, it implies that I is zero when q is 1. Finally, since q is the same for all firms, all firms choose the same value of I. Thus the rate of change of the aggregate capital stock, $\dot{K}$, is given by the number of firms, N, times the value of I that satisfies (6). Therefore

(7) $\dot{K}(t)=f(q(t))$ $\qquad$ $f(1)=0,\ f'(.)>0$

- this implies that K is increasing when q>1, decreasing when q<1 and constant when q=1.

adjustment of the capital stock



adjustment of q

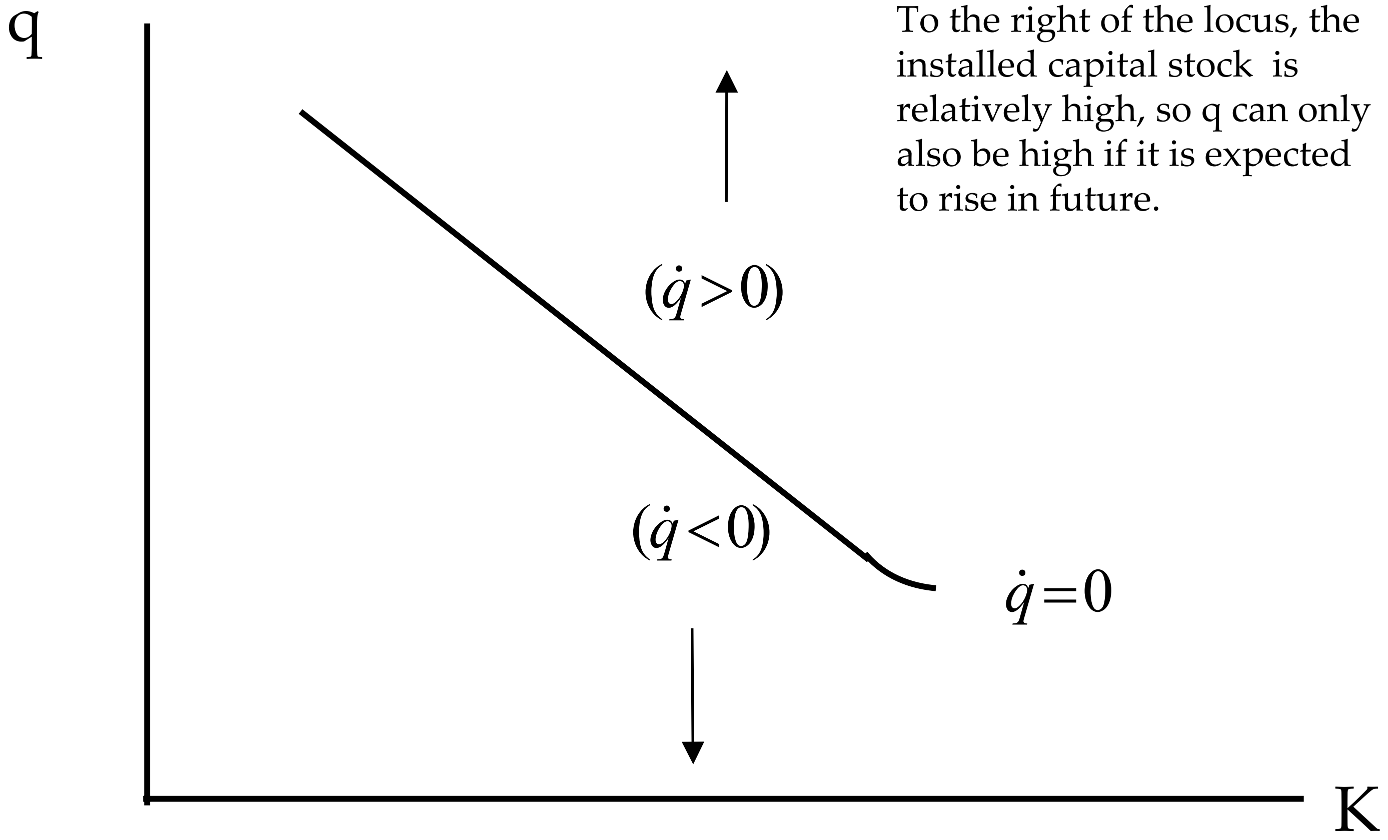
- Equation (5) states that the marginal revenue product of capital equals its user cost. Rewriting this as an equation for $\dot{q}$ yields

(8) $$\dot{q}(t) = rq(t) - \pi(K(t))$$

- this implies that q is constant when rq=π(K) or q=π(K)/r. Since π(K) is decreasing in K, the set of points that satisfy this condition is downward sloping in (K,q) space. In addition (8) implies that $\dot{q}$ is increasing in K, thus $\dot{q}$ is positive to the right of the $\dot{q} = 0$ locus and negative to the left.

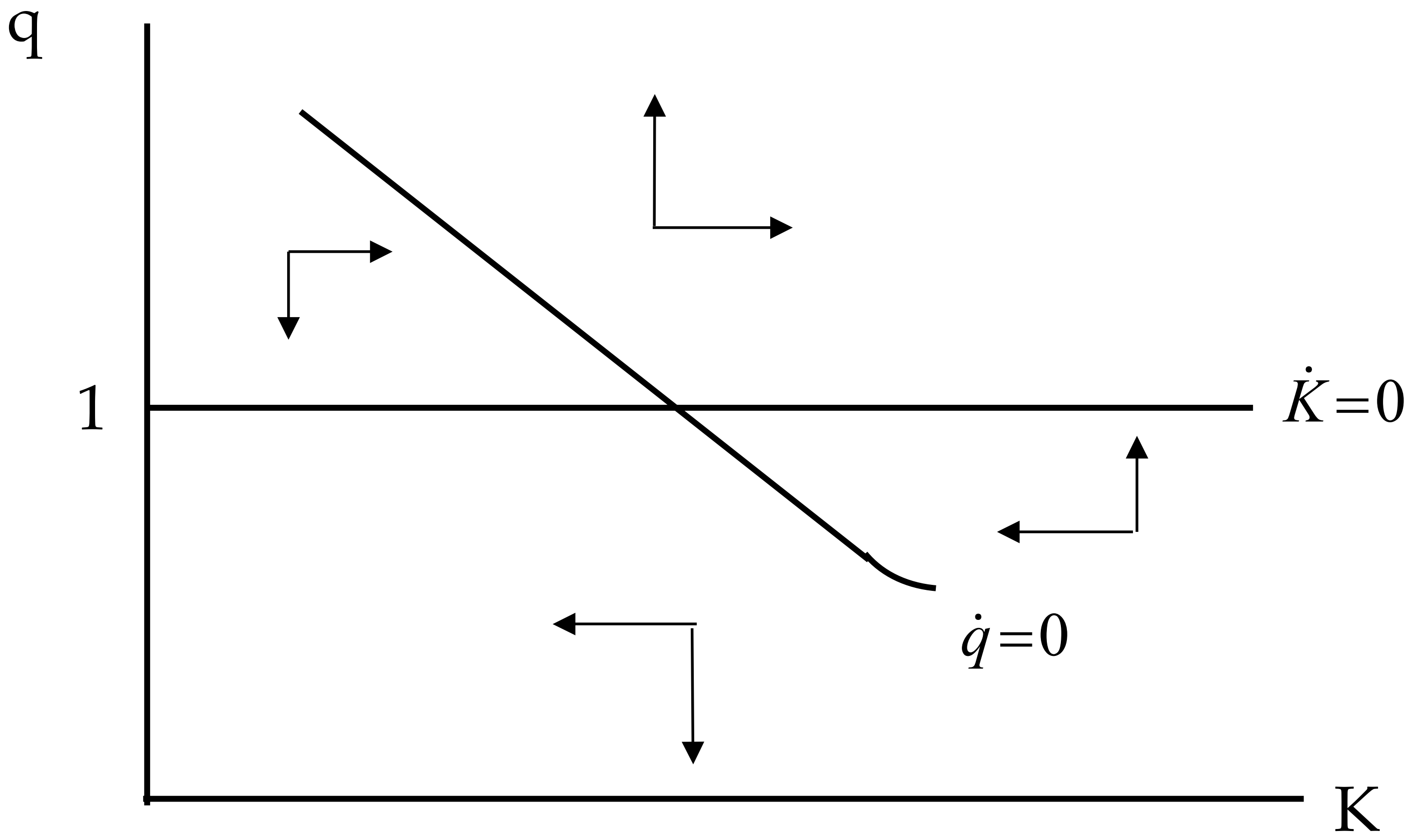
- To the right of the locus, K is high and therefore profits are low so q can only be high if it is expected to rise, so $\dot{q}$ is positive.

adjustment of q

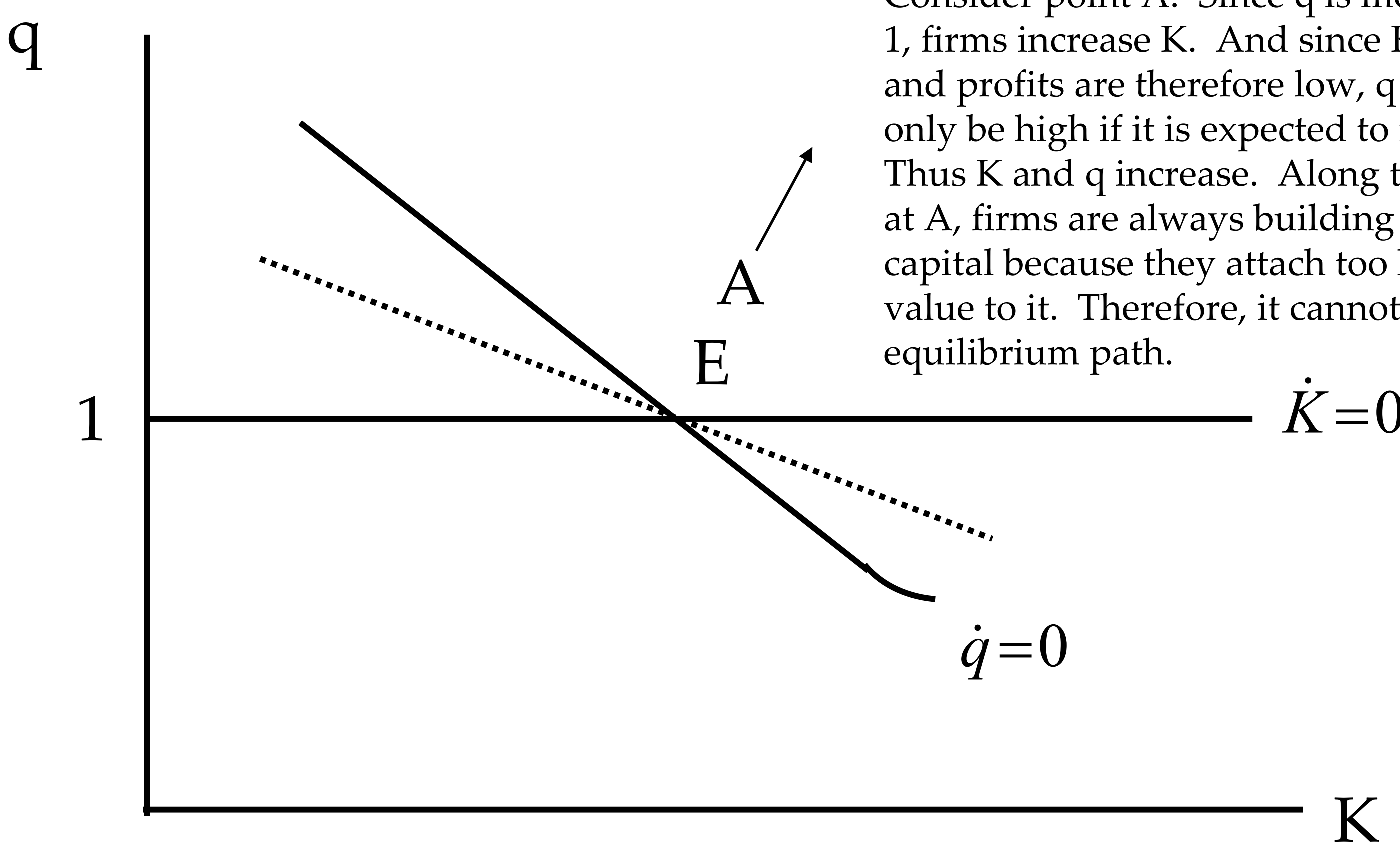


To the right of the locus, the installed capital stock is relatively high, so q can only also be high if it is expected to rise in future.

the phase diagram



the saddle path



Consider point A. Since q is more than 1, firms increase K. And since K is high and profits are therefore low, q can only be high if it is expected to rise. Thus K and q increase. Along the path at A, firms are always building up capital because they attach too high a value to it. Therefore, it cannot be an equilibrium path.

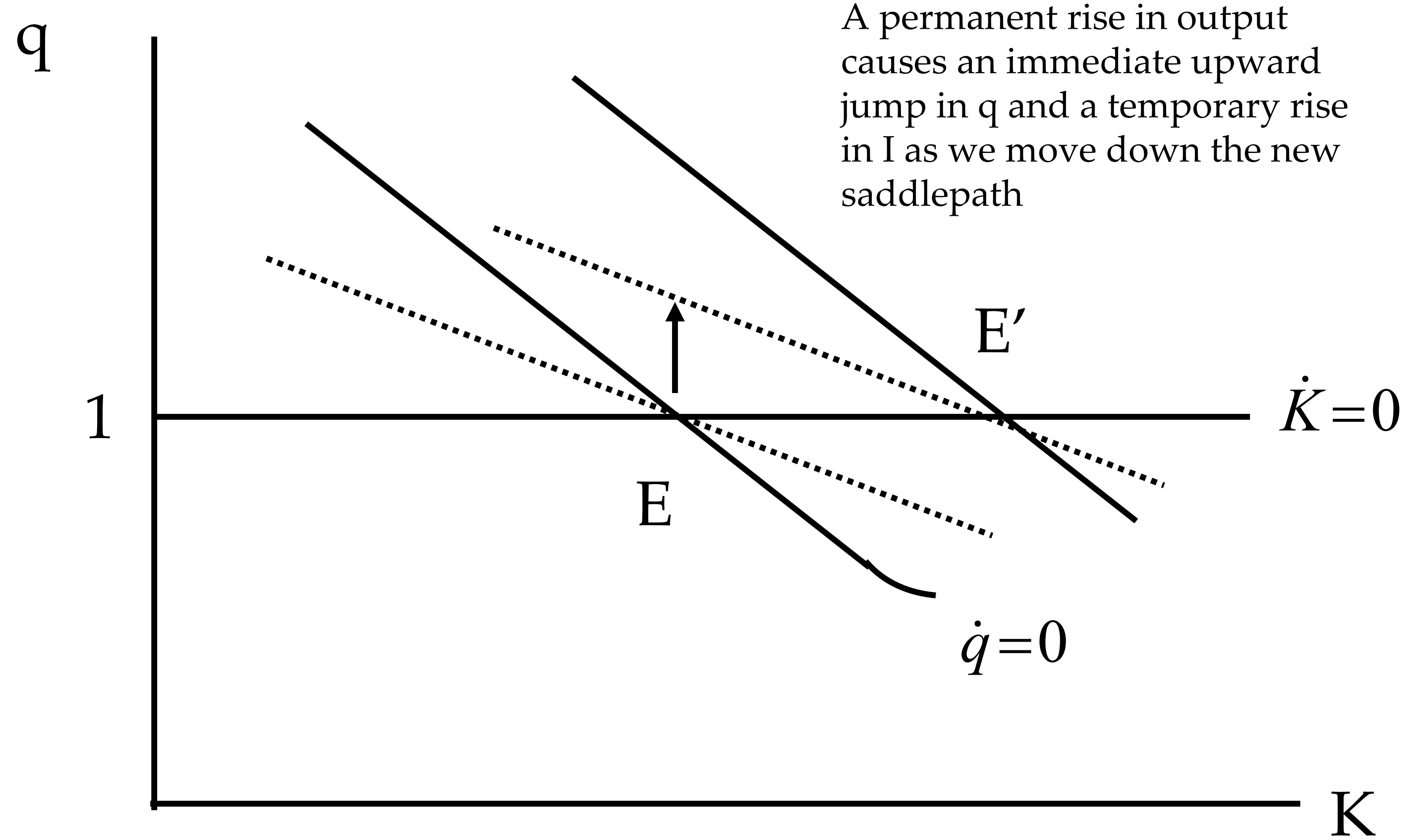
an investment bubble

- Consider a point A (to the NE of E). Since q is more than 1, firms increase their K stocks; so K is rising. And since K is high and profits are therefore low, q can only be high if it is expected to rise, thus q is also rising. Thus K and q move up in the diagram.
- Along the path starting at A, the firms is always building up capital because it attaches too high a value to capital. Therefore, it cannot be an equilibrium.
- At some stage, therefore, the value of q must fall immediately to the saddlepath, at which point the capital stock will start to contract.

investment dynamics

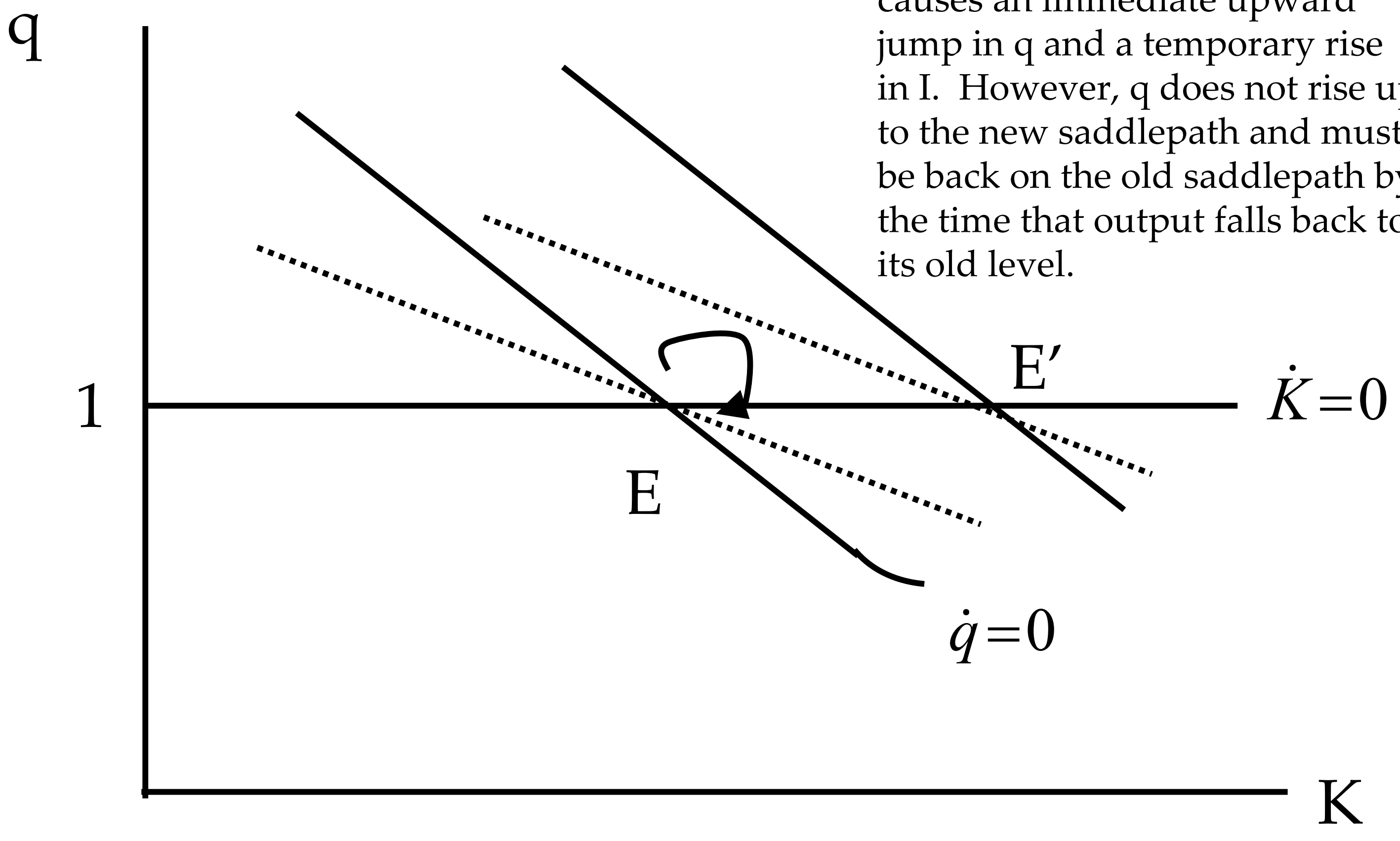
- an increase in output raises the demand for the industry's product.
  - since the capital stock cannot adjust instantly, the market value of capital rises and this induces investment until the value of capital returns to normal
  - the same is true if output is expected to rise in future
  - investment is higher when output has recently risen than when it has been high for a while (the *accelerator)*
- a fall in the interest rate lowers the cost of capital
  - this raises the market value of the capital stock and induces a rise in investment

a permanent rise in output



a temporary rise in output

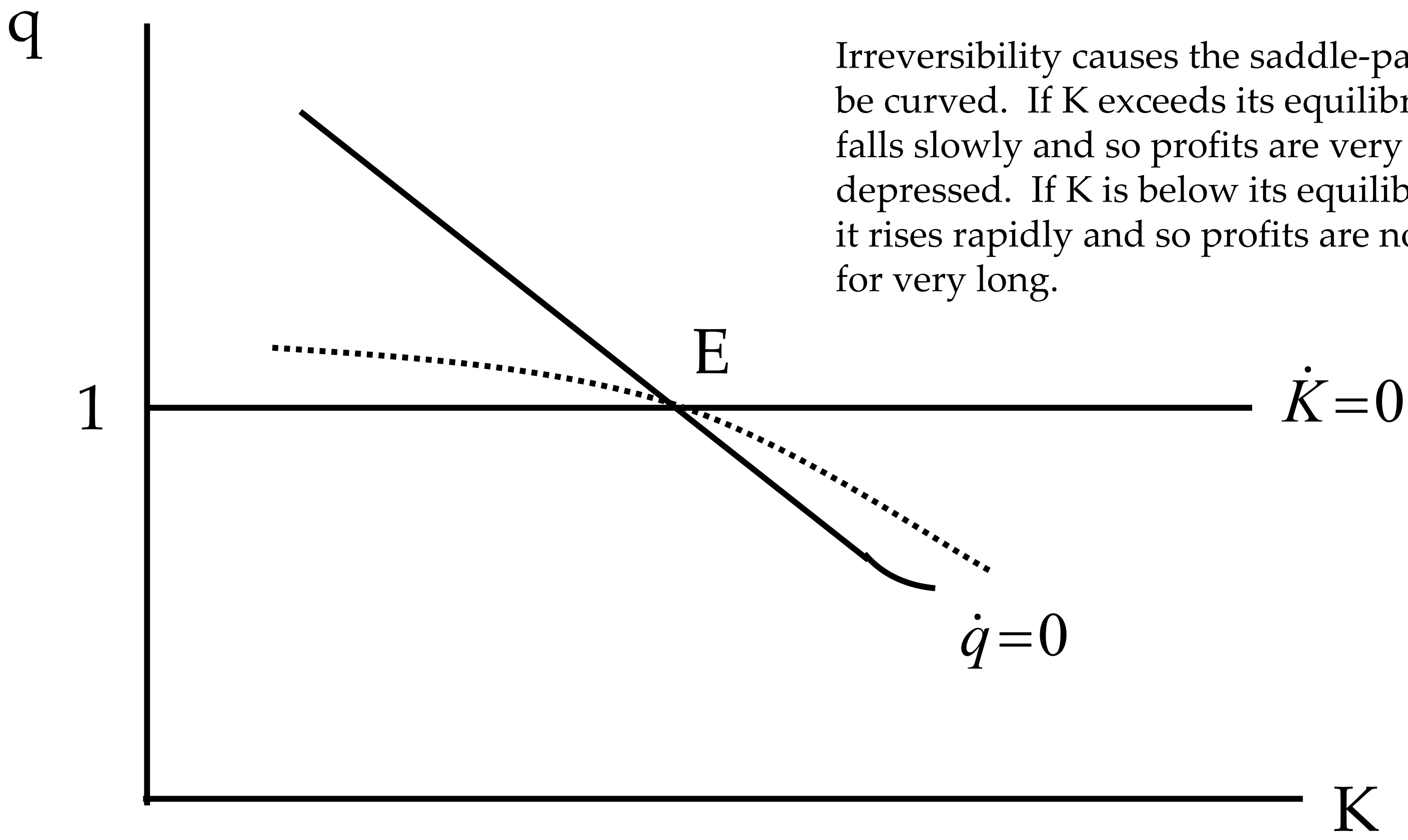
A temporary rise in output causes an immediate upward jump in q and a temporary rise in I. However, q does not rise up to the new saddlepath and must be back on the old saddlepath by the time that output falls back to its old level.



investment and irreversibility

- when adjustment costs are symmetric, uncertainty does not affect investment
- but sometimes it is more costly to reduce the physical capital stock than to increase it (asymmetry leads to *irreversibility*)
- when investment is irreversible, there is an option value to waiting rather than investing
- if a firm does not invest, it retains the possibility of keeping its capital stock low
- if it invests, it commits itself to a high capital stock

irreversibility and the saddle path



Irreversibility causes the saddle-path to be curved. If K exceeds its equilibrium, it falls slowly and so profits are very depressed. If K is below its equilibrium, it rises rapidly and so profits are not high for very long.