Macroeconomic Theory III: Competitive Equilibrium (Real) Business Cycles

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Michaelmas Term 2004

introduction

- Real business cycle models are *Walrasian* – they feature competitive markets, and have no externalities or other market failures.
- RBC models derive from extensions to the Ramsey model in two dimensions.
- First, they need a source of *real shocks*, usually either through technology or government spending.
- Second, they need a *propagation mechanism*, to explain why an initial shock persists for some period of time. In particular, earlier models such as Ramsey or Solow assume a constant supply of labour. Business cycles clearly feature procyclical labour inputs, however.

the economics of Robinson Crusoe

- Robinson Crusoe lives on his own on an island and has the choice between fishing (production), making nets (investment) and swimming (leisure). He faces the following shocks:
  - A big school of fish nearby (boom): production rises because productivity is high and Crusoe chooses less leisure.
  - A rain-storm (recession): production falls because productivity is low and Crusoe sits in his hut, possibly making nets (rise in investment), possibly teaching his parrot to speak.
  - Attack of the Cannibals (war-time boom): production (including defence) rises since he spends his whole time defending himself. Investment is crowded out since he has less time to spare to make nets.

why real business cycles?

- Until Kydland and Prescott's groundbreaking 1982 paper, economists tried to capture business cycles using disequilibrium models. Full-employment is considered an equilibrium: that is, as a situation where each worker's and each producer's preferences are satisfied and anything less than full-employment is a disequilibrium. The disequilibria is caused by different things in different models, such as money-illusion, imperfect competition or some form of nominal rigidity.
- In the RBC models, each stage of the business cycle is viewed as an equilibrium – the trough as well as the peak.
- This is not to say that workers prefer slumps to booms, just that slumps represent undesired, undesirable and unavoidable shifts in the constraints that people face, but that given those constraints, markets react efficiently and people achieve the best outcomes that circumstances permit.
- Of course, even some New Keynesian models such as those of coordination failures treat each possible outcome as an equilibrium. But those equilibria can be Pareto-ranked, whereas RBC models typically have a single, Pareto-optimal equilibrium.

dynamic properties of GDP

What would the stochastic process of GDP have to look like to generate a business cycle?

Random walk in output:
(1) $Y_t = a + Y_{t-1} + e_t$

autoregression about a trend:
(2) $Y_t = bY_{t-1} + ct + e_t$
with b around 0.95. Note the parameters a and c capture the growth trend.

Second-order autoregression:
(3) $Y_t = bY_{t-1} - dY_{t-2} + ct + e_t$
when the first lag has a positive coefficient and the second is negative, output can be 'hump-shaped'. Therefore, a model that produces an AR(2) output function can simulate a business-cycle.

production in a baseline RBC model

The production function is Cobb-Douglas:

(4) $Y_t = K_t^{\alpha}(A_t L_t)^{1-\alpha}$

Output is divided among consumption, investment and government purchases:

(5) $Y_t \equiv C_t + I_t + G_t$

Fraction δ of capital depreciates each period. Thus the capital stock in period t+1 is:

(6) $K_{t+1} = K_t + I_t - \delta K_t = K_t + Y_t - C_t - G_t - \delta K_t$

Labour and capital are paid their marginal products. Thus the real wage and the real interest rate in period t are

$$w_t = (1-\alpha)K_t^{\alpha}(A_t L_t)^{-\alpha} A_t = (1-\alpha)\left(\frac{K_t}{A_t L_t}\right)^{\alpha} A_t \qquad r_t = \alpha\left(\frac{A_t L_t}{K_t}\right)^{1-\alpha} - \delta$$

households in a baseline RBC model

The representative household maximizes the expected value of

(7) $$U = \sum_{t=0}^{\infty} e^{-\rho t} u(c_t, 1-\ell_t) \frac{N_t}{H}$$

u(.) is the instantaneous utility function of the representative member of the infinitely-lived household and ρ is the discount rate. N_t is the population and H is the number of households; thus N_t/H is the household size. Population grows exogenously at rate n:

(8) $\ln N_t = \bar{N} + nt$ $\qquad$ $N_t = e^{\bar{N}+nt}$ $\qquad$ where n<ρ

Thus the level of N_t is given by .

The instantaneous utility function, u(.) has two arguments. The first is consumption per household member, c. The second is leisure per member, which is the difference between the time endowment (normalized to 1) and the amount each member works. Since all households are the same, c=C/N and

$\ell = L/N$.

For simplicity, assume u(.) is log-linear in the two arguments:

(9) $u_t = \ln c_t + b\ln(1-\ell_t)$ $\qquad$ with b>0 (marginal disutility of labour).

technology in a baseline RBC model

To capture trend growth, the model assumes that technology grows at a constant rate subject to random disturbances.

(10) $\ln A_t = \bar{A} + gt + \tilde{A}_t$

where $\tilde{A}$ reflects the effects of the shocks. $\tilde{A}$ is assumed to follow a *first-order autoregression*:

(11) $\tilde{A}_t = \rho_A \tilde{A}_{t-1} + \varepsilon_{A,t}$ where $-1<\rho_A<1$

where the ε_A terms are *white-noise* disturbances – a series of mean zero shocks with no serial correlation. (11) shows that the random component of lnAt, $\tilde{A}_t$,equals fraction ρ_A of its previous period's value plus a random term. If ρ_A is positive, this means that the effects of a shock gradually disappear over time.

government in a baseline RBC model

An alternative driving variable for the model is the amount of government purchases.

The trend rate of per capita government purchases equals trend growth rate of technology, otherwise they would become arbitrarily large or small. Thus,

(12) $\ln G_t = \bar{G} + (n+g)t + \tilde{G}_t$

(13) $\tilde{G}_t = \rho_G \tilde{G}_{t-1} + \varepsilon_{G,t}$ where $-1<\rho_G<1$

where the ε_G terms are also white-noise disturbances and are uncorrelated with the ε_A disturbances.

intertemporal substitution of labour

- The two most important differences between this model and the Ramsey model are the inclusion of leisure in the utility function and the introduction of autoregressive technology or government purchases.
- Because of the logarithmic form of the utility function (9), the intertemporal elasticity of substitution of leisure is 1. A rise in wages today causes an increase in labour supply today relative to supply tomorrow.
- A rise in the interest rate raises relative labour supply today as well. Intuitively, a rise in the interest rate increases the attractiveness of work today and saving today relative to tomorrow.

household optimization under uncertainty

- The household's optimization problem also differs from the Ramsey model because it faces uncertainty about the path of future wages and interest rates (due to autoregressive technology and government purchase shocks).
- Because of this uncertainty, the choices of consumption and leisure at any point depend upon all shocks up to that date, so the household does not choose deterministic paths for consumption and labour supply.
- With uncertainty we can derive a Euler equation relating current consumption to expectations concerning consumption and interest rates in the next period. Consider a household that reduces consumption by a small amount today and uses its consequent greater wealth to fund higher consumption tomorrow. If the household is behaving optimally, a small change of this type must leave utility unchanged. Hence,

(14) $$\frac{1}{c_t} = e^{-\rho} E_t \left[\frac{1}{c_{t+1}} \left(1 + r_{t+1} \right) \right]$$

tradeoffs between labour and consumption

- The household chooses consumption and leisure at each date. A second first-order condition (to go with the Euler equation in consumption) relates its current consumption and labour supply.
- Specifically, imagine the household raising its labour supply today by a small amount and using the income to increase its consumption today. If the household is behaving optimally, its expected utility must be unchanged.
- At this point, the ratio of consumption to leisure is an increasing function of the wage and a decreasing function of the marginal disutility of labour, b:

  (15) $$\frac{c_t}{1-\ell_t} = \frac{w_t}{b}$$

solving the model

- Basic problem is to maximize utility (7) subject to the production function (4), the output identity (5), the capital stock (6) and the time endowment.
- This yields a set of first-order conditions which characterize market equilibrium. As we have seen, the two most important of which are: the equation which equates the marginal utility of consumption to its shadow price and one which equates the marginal disutility of labour to its marginal product.
- The solution focuses on two variables, labour supply per person and the fraction of output that is saved. The basic strategy is to rewrite the equations of the model in log-linear form, substituting (1-s)Y for C whenever it appears.

a simple model

- Given explicit forms for the utility and production functions, it is possible to solve for the time paths of consumption, capital and labour.
- In order to obtain a specific solution we assume (McCallum, 1989) that capital fully depreciates, utility is log-linear and the production function is Cobb-Douglas.
- In this case we find that there is a constant optimal saving rate and that labour supply is also constant:

$$\hat{s} = \alpha e^{n-\rho} \qquad \ell_t = \frac{1-\alpha}{(1-\alpha)+b(1-\hat{s})} \equiv \hat{\ell}$$

- Despite household's desire to substitute their labour supply intertemporally, movements in either technology or capital have offsetting impacts on the relative-wage and interest rate effects on labour supply. An improvement in technology raises current wages relative to expected future wages and hence raises labour supply. But it also raises the amount saved and hence lowers the expected interest rate, which reduces labour supply. In this specific case, these effects exactly balance.

dynamics of output I

When labour and saving are constant, we can examine the dynamics of output in the following way:

The production function implies

(16) $\ln Y_t = \alpha \ln K_t + (1-\alpha)(\ln A_t + \ln L_t)$

we know that $K_t = \hat{s} Y_{t-1}$ and $L_t = \hat{\ell} N_t$; thus

(17) $\ln Y_t = \alpha \ln \hat{s} + \alpha \ln Y_{t-1} + (1-\alpha)(\ln A_t + \ln \hat{\ell} + \ln N_t)$

since only Y_{t-1} and A_t are not deterministic in the model, we can re-write this as:

(18) $\ln \tilde{Y}_t = \alpha \ln \tilde{Y}_{t-1} + (1-\alpha)\tilde{A}_t$

where $\tilde{Y}_t$ is the difference between $\ln Y_t$ and the value it would take if lnA_t equalled $\bar{A} + gt$ at each period.

dynamics of output II

Note that since (18) holds each period, it implies that

(19) $\ln \tilde{Y}_{t-1} = \alpha \ln \tilde{Y}_{t-2} + (1-\alpha)\tilde{A}_{t-1}$

or

(20) $\tilde{A}_{t-1} = \dfrac{1}{1-\alpha}(\tilde{Y}_{t-1} - \alpha \tilde{Y}_{t-2})$

and since (11) states that $\tilde{A}_t = \rho_A \tilde{A}_{t-1} + \xi_{A,t}$ we can substitute these two equations into (18) we obtain

(21)
$$\begin{aligned}
\tilde{Y}_t &= \alpha \tilde{Y}_{t-1} + (1-\alpha)(\rho_A \tilde{A}_{t-1} + \varepsilon_{A,t}) \\
&= \alpha \tilde{Y}_{t-1} + \rho_A(\tilde{Y}_{t-1} - \alpha \tilde{Y}_{t-2}) + (1-\alpha)\varepsilon_{A,t} \\
&= (\alpha + \rho_A)\tilde{Y}_{t-1} - \alpha\rho_A \tilde{Y}_{t-2} + (1-\alpha)\varepsilon_{A,t}
\end{aligned}$$

Thus, departures of log output from its normal path follow a *second order autoregression* – that is, output is a linear combination of its two previous values plus a white-noise disturbance.

a hump-shaped cycle

- This can lead to a hump-shaped response to shocks.
- Consider a shock of $1/(1-\alpha)$ to ε_A when $\alpha=1/3$ and $\rho_A=0.9$. This raises output by 1 in the first period (1-α times the shock), 1.23 in the next ($\alpha+\rho_A$ times 1), 1.22 in the following ($\alpha+\rho_A$ times 1, minus α ρ_A times 1) then 1.14, 1.03, 0.94…
- Note that the persistence of shocks is being driven by ρ_A. Unfortunately, since saving and labour are constant, this version of the model is not very realistic.

a general method

- Papers in this general area cannot be solved analytically. Instead, they are usually solved numerically: parameter values are chosen and the model's quantitative implications for the variances and correlations of macroeconomic variables are discussed.
- An alternative, recommended by Campbell (1994) is to take first-order Taylor approximations of the equations of the models in the logs of the relevant variables around the model's balanced growth paths in the absence of shocks, and then look at the properties of these approximate models. Campbell also emphasizes that you should look at the impulse response functions rather than just the variances and correlations.
- In the case of the earlier model, when depreciation is less than 100%, investment and employment respond more to shocks. When depreciation is not complete, a rise in technology raises the marginal product of capital and hence makes it optimal for households to save more. Since saving is temporarily high, we know that the interest rate must be higher. But a higher interest rate raises current labour supply. So, investment and employment respond more to shocks.

problems with RBC models I

- The intertemporal substitution of labour
  - Some economists argue that the intertemporal substitution of labour is not an important phenomenon since desired employment is not very sensitive to the real wage and interest rate. It is the unemployment rate that fluctuates over the business cycle. Why would so many people chose to work zero hours in recessions? Studies of labour supply (such as Ball 1990) suggest that expected changes in real wages lead to only small hours responses.
- The Solow residual
  - Although the Solow residual does fluctuate significantly over time, it is hard to believe that the year on year measured changes represent technology changes rather than changes in labour and capital utilisation. As we saw, we for RBC models to generate plausible cycles we need a high degree of persistence. Empirical evidence for the UK suggests a coefficient of lagged adjusted quarterly TFP of around 0.25, which means that after four quarters less than one per cent of the shock survives. Indeed, how are we to interpret a negative technology shock?
- Neutrality of money
  - Evidence does not support the neutrality of money (which is important in RBC models). Romer & Romer (1989) look at occasions where the FOMC has tightened money without any major change in conditions and find that employment and income fall.

problems with RBC models II

- Flexibility of wages and prices
  - Most of the evidence is that wages and prices are adjusted infrequently and that there are particular downward rigidities.
- Do the models fit the facts?
  - K&P stress that RBC models are unrealistic in that they aim only to capture certain features of the data rather than a complete explanation. RBC models try to explain the relationships among a number of series using just a technology shock. Millard, Scott and Sensier conclude from simulations of six different RBC models that none can give a coherent account of UK labour market developments. They all understate the volatility and persistence of employment and especially of unemployment. There is also usually too high a correlation between unemployment and wages.
- Is a representative agent model appropriate?
  - While representative agent models can be useful in modelling growth and investment, are they really appropriate for modelling business cycles? One of the most striking features of business cycles is how outcomes differ across agents, for example, the unemployed are different from the employed.