Iterative Dominance and Sequential Bargaining

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Abstract

A new game theoretic analysis of finite horizon, complete information bargaining is advanced. The extensive form reflects an attempt to model unstructured negotiations, in which the negotiants can gain no artificial advantage from the details of the bargaining protocol. Conditions are identified under which the game is dominance solvable in the sense that iterative deletion of weakly dominated strategies selects a unique outcome. These conditions serve to preclude embedded static bargaining problems of the sort that have historically been deemed indeterminate, thereby ensuring that the dynamic problems analyzed will be resolvable without imposing any particular static theory.

JEL classification codes: C78, D71, D74.

Keywords: coalition, core, iterative dominance, temporal monopoly.

0. PRÉCIS

The following outline is intended to help the reader navigate this paper and process it in an efficient manner.

> §§1.1–1.2 supply the motivation for this work, sketching the history of the bargaining problem within economic theory and noting some objections to the dominant class of "temporal monopoly" models.
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> §1.3 lists three basic principles that guide the construction and analysis of our alternative model and offers an introduction to the mode of reasoning employed.
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> §2.1 defines the general "collective choice problem" made up of those aspects of the bargaining environment that are in principle observable.
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> §2.2 explains how we transform the collective choice problem into a well defined extensive form game.
>
> §2.3 constructs the normal form of this game (which is, formally, the object of our analysis).
>
> §2.4 discusses the application of iterative weak dominance in the present context and provides some essential notation.
>
> §§3.1–3.4 analyze our sequential bargaining game using a procedure of backward induction and identify sufficient conditions for it to be dominance solvable.
>
> §3.5 states, by way of summary, our main result (Theorem 1).
>
> §§4.1–4.2 show how the general theory plays out in the special cases of binary choice and bilateral surplus division.

1. INTRODUCTION

1.1. *An excursion in intellectual history*

The outstanding early successes of modern theoretical economics came in areas of the field where the influence on one individual's decision problem of the volition of others has the character of an impersonal and implacable force. It was an abiding theme of the Austrian School that the price system "direct[s the entrepreneur] to an invisible goal, the satisfaction of the distant unknown consumer of the final product" (von Hayek, 1988, p. 100), and in a perfectly competitive textbook economy this mediation by prices is complete: No single participant in a market need know anything about the other buyers and sellers except that in aggregate they are willing to transact at the prevailing rate; no two participants need ever meet face to face and engage in any sort of human interaction; and no indeterminacy regarding the allocation of resources remains once relative prices have been established.

Recognizing, as they did, the analytical advantages of impersonalization, theorists following in the tradition of Edgeworth (1881) took an understandably pessimistic view of their own ability to elucidate the highly personal activity of bilateral trade. Again, the comments of an Austrian (von Mises, 1966, p. 327) provide a representative statement of this appraisal.

> In an occasional act of barter in which men who ordinarily do not resort to trading with other people exchange goods ordinarily not negotiated, the ratio of exchange is determined only within broad margins. Catallactics,

> the theory of exchange ratios and prices, cannot determine at what point within these margins the concrete ratio will be established. All that it can assert with regard to such exchanges is that they can be effected only if each party values what he receives more highly than what he gives away.

More generally, it was felt that *all* negotiated decisions raise problems of group psychology beyond the competence of the economist, and it was with an admirable humility that these phenomena were largely surrendered to the other emergent disciplines of social science.

With this point of view well established, it is perhaps to be expected that the game theoretic approach to bargaining propounded in the landmark contribution of Rubinstein (1982) — building on earlier work by Nash (1953), Ståhl (1972), and others — would have generated great enthusiasm. Starting from the commonplace observation that any actual negotiation unfolds over time, Rubinstein considered how the outcome of a bilateral surplus division game would depend upon the differential delay costs or relative impatience of the players. And, surprising even himself (p. 99), he found that for a wide class of preferences quantifying these factors, the requirement of perfect equilibrium (Selten, 1965, 1975) is consistent with a unique allocation usually agreed upon at once.

The bargaining problem having been thus reclaimed as an object of study, it received no small amount of attention in the first two decades following the appearance of Rubinstein's paper, as economic theorists enriched his basic model to allow for such complications as a variable number of players (e.g., Chatterjee et al., 1993, Chae and Yang, 1994, and Krishna and Serrano, 1996); increased flexibility in the timing of events (e.g., Admati and Perry, 1987, Perry and Reny, 1993, and Sákovics, 1993); and, most importantly, incomplete information (see Kennan and Wilson, 1993, and Ausubel et al., 2002).[1] This literature has contributed substantially to a growing understanding of the causes of bargaining impasses and the extent to which the resulting inefficiencies can be ameliorated. Its models have been deployed in applications too numerous to mention and have triggered an avalanche of experimental studies (see Roth, 1995). And the prospects for further theoretical advances under the Rubinstein rubric continue to look bright (e.g., Yıldız, 2003).

1.2. *Objections to temporal monopoly*

Contemporaneously with these developments, however, there has persisted a subterranean dissatisfaction with the dominant framework for game theoretic modelling of negotiations; one in which a "recognition process" endows agents in turn with "temporal [i.e., temporary] monopoly" power over the candidate agreement. The consequences of this structure, memorably expounded by Kreps (1990, pp. 563–565), have been stated more succinctly by Yıldız (2003, pp. 793–794).

> [I]n the Rubinstein-Ståhl framework, the recognition process is the only source of bargaining power. In equilibrium, the recognized player at a given date extracts a (noninformational) rent, as he makes an offer that can be rejected only by destroying some of the [surplus]. These rents constitute the bargaining power: a player's continuation value is the present value of the rents he expects to extract when he is recognized in the future.

[1] Sutton (1986) summarizes the developments of the early 1980s, while Muthoo (1999) provides a comprehensive exposition of variations on the Rubinstein model.

While certainly enlightening, these observations are troubling to the extent that a correlate to the recognition process is difficult to identify in the economic, legal, and political settings where bargaining actually takes place. Generally speaking, any procedural guidelines given to a group of bargainers by a neutral party will be designed to *facilitate* rather than to artificially impede the making of offers.[2] If some physical constraint limits the speed of communication, then it is unclear why this reality should impinge only on the last person to make an offer and not on everyone else (noted by Sákovics, 1993). And even if we were to observe bargainers dutifully taking their turns to make offers and respond to those made by others, the unwarranted conclusion that they are not free to do otherwise would be in no way preferable to the less extravagant conclusion that they have adopted a conversational convention that keeps them all from talking at the same time.

It could be argued in defense of the temporal monopoly framework that if game theoretic methods are to be used to study bargaining, then it is necessary to write down a well defined negotiation game; and that the burden is on the critic to propose an alternative class of models that offers a more attractive interpretation while at the same time generating predictions as strong as those that emerge from Rubinsteinian game forms. But the capacity of an economic model to generate strong predictions is not in itself a virtue. And the heavy reliance on bargaining games governed by a recognition process seems to suggest that this feature has some unique appeal, when in fact the assumption of temporal monopoly has never been adequately defended. Thus history — painted, admittedly, in broad strokes — seems to present us with an uncomfortable choice between on the one hand the outright capitulation of the early price theorists in the face of the bargaining problem, and on the other the highly predictive models used by a majority of modern game theorists that distribute bargaining power through an effective but conceptually dubious mechanism.

1.3. *Preliminary remarks*

The purpose of this paper is to investigate the extent to which, in particularly simple (finite horizon, complete information) environments, Rubinstein's differential delay costs continue to constrain bargaining outcomes even in the absence of any temporal monopoly structure or other artificial protocol for negotiations.[3] We shall address this question using a game theoretic model, the construction and analysis of which will be guided by the following three basic principles.

1. Sequential (or "dynamic") bargaining is nothing more than repeated static bargaining with a variable, endogenously determined continuation outcome.

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[2] Even the federal mediator who in March of 2000 "laid out an aggressive weeklong schedule of proposals and counterproposals between Microsoft and the Department of Justice" (Brinkley, 2000) cannot be considered to have acted as a recognition process and deliverer of bargaining power in the sense described above by Yıldız. (Never mind the brevity of the scheduled negotiation.) For this to have been the case, he (namely Richard A. Posner) would have had to have been prepared to quash an improvement in one party's standing offer on the grounds that the concession was out of turn. But surely it is not the task of a mediator to engage in such despotism.

[3] Previous work involving models that are in one way or another "procedure-free" includes that of Perry and Reny (1993), Abreu and Gül (2000), and Smith and Stacchetti (2003).

2. Unstructured static bargaining is best implemented using simultaneous moves that represent outcome-relevant actions.

3. Dynamic considerations (e.g., delay costs) cannot resolve static problems, and thus a purely dynamic theory can be determinate only in bargaining problems with no nontrivial static aspects.[4]

While the first two principles will be reflected in the extensive form specified in Section 2.2, the third will appear in the form of conditions on the general collective choice problem (see Section 2.1) that lead to dominance solvability (see Section 2.4) of the associated sequential bargaining game (constructed in Section 2.3).

As an introduction to our mode of reasoning, consider the special case of two agents with conflicting preferences between two alternatives. Looking ahead to the horizon, we must rule out the possibility that at this point each agent prefers each alternative to the disagreement outcome, since such preferences would create a nontrivial static bargaining problem embedded within the overall dynamic problem at the deadline for agreement. As the potential agreement date moves backward away from the horizon, the possibility of avoiding delay costs will make the prospect of eventual disagreement relatively less attractive to both agents, and there are then two possibilities. Either the conflicting preferences are so strong that neither agent would ever be willing to accept his or her dispreferred alternative in order to avoid waiting for the disagreement outcome. Or there is some latest date (the "consensus point") at which *exactly one* of the two alternatives (the "consensus choice") is acceptable in this sense to *both* agents. Our conclusion will be that in the first case the negotiation should end in an impasse, while in the second the identified alternative should be agreed upon without delay.

The analysis in Section 3 will show that similar reasoning can be applied to any bargaining environment within the class under consideration. The main result obtained (i.e., Theorem 1 in Section 3.5) can be construed as simply establishing a set of sufficient conditions for a particular type of game to be dominance solvable. But under the suggested interpretation of our method, we can make a bolder claim as to the significance of this result: that for an environment within the designated class, its predictions *exhaust the implications of the dynamic structure of the problem*, and thus *any stronger predictions must be predicated in part on a theory of static bargaining.*

2. FINITE HORIZON SEQUENTIAL BARGAINING

2.1. *The collective choice problem*

A collection I of agents is charged at time 0 with the task of choosing one of the mutually exclusive alternatives in a set A. (Assume that both I and A are nonempty and finite.[5]) If no choice is made at or before a fixed deadline T

[4]Similarly, in settings with incomplete information, Kennan and Wilson (1993, pp. 50–55) stress the indeterminacy of incentive compatibility analysis (e.g., Myerson, 1979) as compared to the "fairly specific predictions" obtainable by imposing a rigid structure on negotiations and thereby "implicitly assign[ing] various monopoly powers to the parties".

[5]Where it is convenient to formulate A as an infinite set (e.g., in Section 4.2), it will often be possible to justify applying our results by appealing to a sequence of finite-alternative approximations.

then a situation of default arises, the consequences of which we shall represent by the symbol ω (where $\omega \notin A$). Hence the full set of possible outcomes is $X = (A \times [0,T]) \cup \{\langle \omega, T \rangle\}$, each element of which pairs substantive variables with a time value.

Externally imposed rules governing the conclusion of an agreement are encoded in a *potential function* $\phi : \mathbb{P}(I) \times A \to \{0,1\}$, where $\mathbb{P}$ represents the power set operator. An expression of the form $\phi(J,a) = 1$ indicates that the members of $J \subset I$ together have the authority (and thus the "potential") to implement $a \in A$, while $\phi(J,a) = 0$ indicates the absence of such authority. Logic dictates that there cannot exist disjoint $J, \hat{J} \subset I$ and distinct $a, \hat{a} \in A$ such that $\phi(J,a) = \phi(\hat{J},\hat{a}) = 1$ (the requirement of *coherence*). Given $a \in A$, we can also safely assume that the function $\phi(\cdot, a)$ is weakly increasing with respect to set inclusion (*monotonicity*) and that $\phi(I,a) = 1$ (*group sovereignty*).[6]

The behavior of agent i is consistent with a utility function $u_i : X \to \Re$ that is assumed to exhibit the following regularity properties.

[R1] There exists a binary relation $\succeq_i$ on A such that for each $a, \hat{a} \in A$ and $t \in [0,T]$ we have $u_i(a,t) \geqq u_i(\hat{a},t)$ if and only if $a \succeq_i \hat{a}$. ‖

[R2] For each $a \in A$, the function $u_i(a, \cdot)$ is strictly decreasing and continuous. ‖

Thus each agent's preferences among the alternatives are stable ([R1]), while time is valuable and the costs associated with delay accrue in small increments ([R2]). Note that under [R2] each function $u_i(a,\cdot)$ is a homeomorphism between the intervals $[0,T]$ and $[u_i(a,T), u_i(a,0)]$, with strictly decreasing inverse function $u_i(a,\cdot)^{-1}$.

2.2. *Extensive form specification*

Although our negotiants could in principle reach an agreement at any instant $t \in [0,T]$, game theoretic analysis of their predicament will be much simplified if we restrict attention to a sequence $\langle k \cdot T/n \rangle_{k=0}^{n}$ of $n+1$ evenly-spaced *decision points*. As the parameter n becomes large, this discretization will approximate the underlying continuous time variable to any desired degree of precision.

What gets decided at a decision point? The one question that can and must be conclusively settled at point $k \cdot T/n < T$ (resp., at point T) is that of whether an alternative will be chosen *at this moment* or whether instead the negotiation will continue on to point $(k+1) \cdot T/n$ (resp., whether instead default will occur). In view of the immutable rules for agreement enforced by the potential function, the sole influence that any single agent i can exert on this immediate decision lies in his option to join — or to refuse to join — a coalition with (including i) the authority to implement one of the alternatives. Naturally the agent cannot simultaneously join two coalitions in support of mutually exclusive alternatives; but he can refuse to lend his support to any alternative and recommend instead that the negotiation continue (resp., that default be allowed to occur).

The outcome-relevant actions available to an agent at any decision point are therefore to sign an agreement implementing some $a \in A$ or to abstain from

[6] Observe that in addition to anonymous requirements for agreement ranging from a simple majority to unanimous consent, potential functions can incorporate (non-anonymous) weighted or multiple majority requirements such as those used by the European Union, as well as individual or joint vetoes such as those available in the U. N. Security Council.

signing, the latter option indicated by the symbol α and the full set of actions by $A^\dagger = A \cup \{\alpha\}$. In view of our desire to avoid imposing artificial structure on the negotiation, considerations of symmetry strongly suggest modelling the agents' choices from this set as being game-theoretically simultaneous; indeed, any violation of simultaneity (as in Rubinstein's model, in which the agents alternate choosing first) would tend to arbitrarily allocate bargaining power in the form of permission to commit or to delay committing to an action. Thus our model can be described as a sequential, simultaneous signature game in which an alternative is chosen once a valid agreement authorizing the choice has been signed.

Similar "simultaneous offer" bargaining games have of course been studied in the past (e.g., by Nash, 1953, and Chatterjee and Samuelson, 1990), and the usual conclusion is that such games are plagued by "extremely large set[s] of subgame perfect ... equilibria" (Dekel, 1990, p. 301). But multiple equilibria per se will not be a concern here, as we shall be employing dominance rather than equilibrium principles. And moreover, this multiplicity of equilibria in sequential, simultaneous offer models can be interpreted as a manifestation of precisely the sort of embedded static bargaining problems which (it is claimed) are not resolvable by a purely dynamic theory.

2.3. *Normal form constructions*

Since nothing occurs before decision point 0, the associated set $\theta(0) = \{h^0\}$ of prior histories contains a single (vacuous) element. At any other decision point $k \cdot T/n$, the corresponding set $\theta(k \cdot T/n) = (A^\dagger)^{\{0,\ldots,k-1\}\times I}$ contains the possible records of past actions by the agents, and the full set of histories is then $\Theta = \bigcup_{k=0}^{n} \theta(k \cdot T/n)$.

A strategy $s(\cdot, i) \in (A^\dagger)^\Theta$ for agent i comprises a plan of action for each history, while a strategy profile $s = \langle s(\cdot, i)\rangle_{i\in I}^\top \in (A^\dagger)^{\Theta\times I}$ comprises a strategy for each agent or, equivalently, a plan of action for each history-agent pair. (Here the superscript $\top$ indicates transposition.) Given such a profile, the sequence $\langle h_k^*(s)\rangle_{k=0}^n$ (with $h_0^*(s) = h^0$) of *realized* histories can be constructed using the recursive definition

$$h_k^*(s) = \left[\langle s(h_0^*(s), i)\rangle_{i\in I}^\top, \ldots, \langle s(h_{k-1}^*(s), i)\rangle_{i\in I}^\top\right] \in \theta(k \cdot T/n), \tag{1}$$

and we can then define the point

$$\pi_a(s) = \inf\{k \cdot T/n : \phi\left(\{i \in I : s(h_k^*(s), i) = a\}, a\right) = 1\} \in [0, T] \cup \{+\infty\} \tag{2}$$

of earliest agreement on a particular $a \in A$; the point

$$\pi_{\min}(s) = \min_a \pi_a(s) \in [0, T] \cup \{+\infty\} \tag{3}$$

of earliest agreement on any alternative; the substantive result

$$\rho(s) = \begin{cases} \arg\min_a \pi_a(s) & \text{if } \pi_{\min}(s) \in [0, T] \\ \omega & \text{if } \pi_{\min}(s) = +\infty \end{cases} \tag{4}$$

of the negotiation (recall the coherence property of ϕ); the time value

$$\tau(s) = \min\{\pi_{\min}(s), T\} \in [0, T] \tag{5}$$

at which this result is decided; and the outcome

$$\psi(s) = \langle \rho(s), \tau(s) \rangle \in X \tag{6}$$

that eventually emerges. Finally, we can assemble both the normal form

$$\Game^{\mathrm{NF}} = \left\langle I, \left\langle (A^{\dagger})^{\Theta}, u_i \circ \psi \right\rangle_{i \in I} \right\rangle \tag{7}$$

and the "delegated" (or "agent normal"; Selten, 1975) form

$$\Game^{\mathrm{DF}} = \left\langle \Theta \times I, \left\langle A^{\dagger}, u_i \circ \psi \right\rangle_{\langle h, i \rangle \in \Theta \times I} \right\rangle \tag{8}$$

of our sequential bargaining game.

2.4. *Iterative weak dominance*

The idea that a weakly dominated strategy can be disregarded and effectively eliminated from a game has its roots in the normative analysis of statistical decision problems (e.g., Blackwell and Girshick, 1954). Repeated application of this idea — the principle of iterative weak dominance — has been described as a "powerful" yet "conceptually puzzling" procedure (Brandenburger and Keisler, 2003), and has in consequence generated both "widespread and fruitful applications" (Ewerhart, 2002) and penetrating theoretical investigations of its epistemic basis.

While the acknowledged strengths of iterative weak dominance as a solution concept include its capacity to generate both forward and backward induction equilibria (see Kohlberg and Mertens, 1986), it does suffer from the practical drawback that the output of the procedure can depend upon the order in which strategies are deleted. Fortunately a result due to Gretlein (1983, p. 113) (see also Marx and Swinkels, 1997) serves to mitigate this difficulty.

> [A]s long as players have strict preferences over the [outcomes] (of which there are a finite number), if they successively eliminate some subset of dominated strategies, . . . then the set of outcomes not eliminated will be the same no matter . . . which dominated strategies [are] eliminated at each stage.

The linear ordering hypothesis needed to guarantee this form of procedural independence will be satisfied in the game $\Game^{\mathrm{NF}}$ if for each $i \in I$ we supplement [R2] with the following technical assumption.

[T1] For each distinct $a, \hat{a} \in A$ and each $k, m \in \{0, \ldots, n\}$, we have

$$u_i(\omega, T) \neq u_i(a, k \cdot T/n) \neq u_i(\hat{a}, m \cdot T/n) .^{7} \parallel \tag{9}$$

[7] Observe that [T1] incorporates a requirement that the agent's preferences among the alternatives be discriminating (the second inequality for $k = m$), as well as a prohibition against coincidental indifference between otherwise unrelated outcomes (the remaining inequalities). The first of these assumptions is clearly the more demanding, so it is worth noting that discriminating preferences are not necessary for our conclusions except as a precondition for invoking Gretlein's result.

We can then demonstrate conclusively that our game is *dominance solvable* simply by exhibiting a particular strategy elimination procedure that selects a unique outcome (the *solution*).

The procedure that we shall use to demonstrate dominance solvability will be one of backward induction with an allowance for multiple rounds of deletion at each decision point. Thus we shall be deleting actions from the delegated form of the game, with the understanding that deleting an action for delegate $\langle h, i\rangle$ amounts to simultaneously eliminating all remaining strategies of agent i that plan to take this action at history h. To facilitate record keeping as we carry out the induction, let $\Sigma^{\text{IWD}}[k \cdot T/n]$ denote the (common) dominance solution of the subgames proceeding from decision point $k \cdot T/n$ — so that the solution of the overall game $\eth^{\text{NF}}$ appears as $\Sigma^{\text{IWD}}[0]$. In keeping with this notation, write $\Sigma^{\text{IWD}}[(n+1) \cdot T/n] = \langle \omega, T\rangle$ to indicate that T is the deadline for agreement.

3. CONDITIONS FOR DOMINANCE SOLVABILITY

3.1. *Backward induction lemma*

Our analysis begins with the definition of a notion of collective acceptability of an alternative at a decision point.

Definition 1 Given a decision point $k \cdot T/n$, let $\Sigma^{\text{IWD}}[(k+1) \cdot T/n]$ exist. An alternative a^{v} is then said to be *viable* at $k \cdot T/n$ if there exists a $J^{\text{v}} \subset I$ such that both $\phi(J^{\text{v}}, a^{\text{v}}) = 1$ and $u_i(a^{\text{v}}, k \cdot T/n) \geqq u_i(\Sigma^{\text{IWD}}[(k+1) \cdot T/n])$ for each $i \in J^{\text{v}}$. ‖

Since, under complete information, outcome $\Sigma^{\text{IWD}}[(k+1) \cdot T/n]$ is the foreseeable consequence of failing to agree at $k \cdot T/n$, an alternative is viable at this point if the members of some coalition with the authority to implement it would be willing to do so were they all to conclude that no other agreement could be reached. We then say that the static bargaining problem arising at $k \cdot T/n$ is *trivial* if at most one alternative is viable at this decision point, and that it is *nontrivial* otherwise.

All of our conclusions about dynamic bargaining will follow from the simple observation that *trivial static problems are dominance solvable.* For purposes of backward induction it will be most convenient to formalize this fact recursively, using it to relate the solutions of subgames proceeding from consecutive decision points.

Lemma 1 *Given a decision point $k \cdot T/n$, let $\Sigma^{\text{IWD}}[(k+1) \cdot T/n]$ exist. If no alternative is viable at $k \cdot T/n$, then $\Sigma^{\text{IWD}}[k \cdot T/n] = \Sigma^{\text{IWD}}[(k+1) \cdot T/n]$. If a unique $a^v \in A$ is viable at $k \cdot T/n$, then $\Sigma^{\text{IWD}}[k \cdot T/n] = \langle a^v, k \cdot T/n\rangle$.*

In other words, if no alternative is viable at $k \cdot T/n$ then the dominance solution at this decision point is the continuation outcome, while any alternative that is uniquely viable at this point is agreed upon immediately. This elementary result (proved in the Appendix) lies at the heart of our theory, and leaves us only with the modest task of finding conditions under which the static bargaining problems embedded in a dynamic problem are all trivial.

3.2. *Disagreement after the consensus point*

As a first application of Lemma 1, we can state immediately a sufficient condition for dominance solvability at the horizon, since here the continuation outcome $\Sigma^{\text{IWD}}[(n+1)\cdot T/n] = \langle \omega, T \rangle$ is exogenously given.

Condition 1 (Terminal Solvability) *At most one alternative is viable at T.*

Proposition 1 *Let Terminal Solvability hold. If no alternative is viable at T, then $\Sigma^{\text{IWD}}[T] = \langle \omega, T \rangle$. If $a^v \in A$ is viable at T, then $\Sigma^{\text{IWD}}[T] = \langle a^v, T \rangle$.*

Suppose now (for the space of this paragraph) that no alternative is viable at T, in which case we know that there will be disagreement if this decision point is reached. We can use the continuation outcome $\Sigma^{\text{IWD}}[T] = \langle \omega, T \rangle$ to test the alternatives for viability at the previous decision point, and if again none is viable then Lemma 1 ensures that $\Sigma^{\text{IWD}}[(n-1)\cdot T/n] = \Sigma^{\text{IWD}}[T] = \langle \omega, T \rangle$ as well. This retrograde chain of equalities will extend until some alternative becomes viable with respect to the continuation outcome $\langle \omega, T \rangle$, an event that will occur (if at all) at the *latest* decision point $k \cdot T/n$ at which there exist an $a^{\text{v}} \in A$ and a $J^{\text{v}} \subset I$ jointly satisfying both $\phi(J^{\text{v}}, a^{\text{v}}) = 1$ and $u_i(a^{\text{v}}, k \cdot T/n) \geqq u_i(\omega, T)$ for each $i \in J^{\text{v}}$. As n becomes large, the latter point will approach the so-called "consensus point" defined as follows by way of two useful prior concepts.

Definition 2 The *acceptance point* of agent i for alternative a is defined by

$$\xi_i^a = \sup\{t \in [0,T] : u_i(a,t) \geqq u_i(\omega, T)\} \tag{10}$$

$$= \begin{cases} -\infty & \text{if } u_i(\omega,T) > u_i(a,0) \\ u_i(a,\cdot)^{-1} \circ u_i(\omega,T) & \text{if } u_i(a,0) \geqq u_i(\omega,T) \geqq u_i(a,T) \\ T & \text{if } u_i(a,T) > u_i(\omega,T) \end{cases} \tag{11}$$

(see Figure 1), the *viability point* of alternative a by

$$\Xi^a = \max_{\phi(J,a)=1} \min_{i \in J} \xi_i^a, \tag{12}$$

and the *consensus point* by $\Xi^{\max} = \max_a \Xi^a$; all of which take on values in the set $\{-\infty\} \cup [0,T]$. $\|$

The acceptance point ξ_i^a measures the appeal of alternative a to agent i using the space $\{-\infty\} \cup [0,T]$ of time values as a numerical scale. (Note that $a \preceq_i \hat{a}$ implies $\xi_i^a \leqq \xi_i^{\hat{a}}$, and that $0 \leqq \xi_i^a \leqq \xi_i^{\hat{a}} < T$ in turn implies $a \preceq_i \hat{a}$.) Similarly, the viability point Ξ^a measures the appeal of alternative a to the collectivity, equalling the acceptance point of the most skeptical member of the coalition with the authority to implement a that is least difficult to assemble. The consensus point is then simply the latest viability point, with $\Xi^{\max} = -\infty$ indicating that no alternative is ever viable and $\Xi^{\max} = T$ that some alternative is in fact viable at the horizon.

Example 1 Let $I = \{1,2,3\}$ and $A = \{a,b,c\}$ and suppose that $\phi(J,\hat{a}) = 1$ if and only if $|J| \geqq 2$. If the nine acceptance points are ordered as

$$-\infty = \xi_1^b = \xi_1^c = \xi_2^c < 0 < \xi_3^a < \xi_3^b < \xi_2^a < \xi_2^b < \xi_1^a = \xi_3^c = T, \tag{13}$$

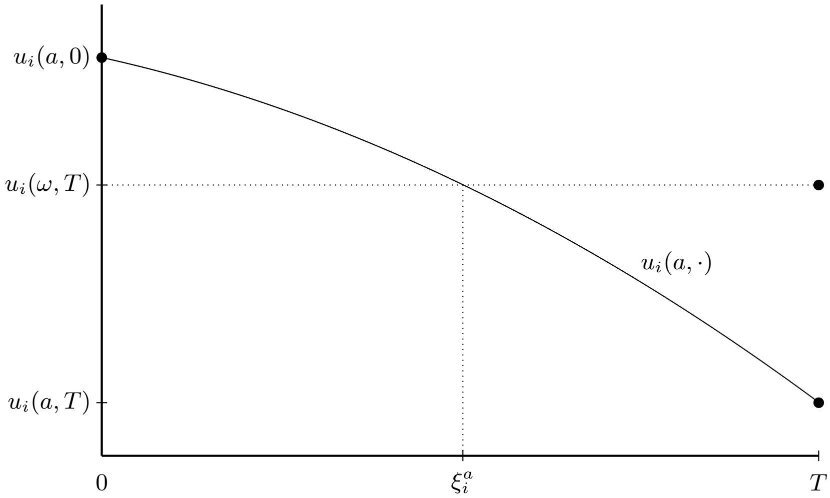

Figure 1: Graphical depiction of an acceptance point. When $u_i(\omega,T) \leqq u_i(a,0)$, the acceptance point ξ_i^a of agent i for alternative a is the last moment at which i weakly prefers agreement on a to eventual default. When $u_i(\omega,T) > u_i(a,0)$, in which case no such last moment exists, the acceptance point ξ_i^a takes on the value $-\infty$.

then the three viability points and the consensus point satisfy

$$-\infty = \xi_1^c = \xi_2^c = \Xi^c < 0 < \xi_3^b = \Xi^b < \xi_2^a = \Xi^a = \Xi^{\max} < T. \tag{14}$$

If we suppose, alternatively, that $\phi(J,\hat{a}) = 1$ if and only if both $|J| \geqq 2$ and $3 \in J$, then the ordering in Equation 13 instead yields

$$-\infty = \xi_1^c = \xi_2^c = \Xi^c < 0 < \xi_3^a = \Xi^a < \xi_3^b = \Xi^b = \Xi^{\max} < T. \tag{15}$$

Finally, we can use the concept of the consensus point to summarize our conclusions from inductive application of Lemma 1.

Proposition 2 *For each decision point $k \cdot T/n > \Xi^{\max}$, we have $\Sigma^{\text{IWD}}[k \cdot T/n] = \langle \omega, T \rangle$. In particular, if $\Xi^{\max} = -\infty$ then $\Sigma^{\text{IWD}}[0] = \langle \omega, T \rangle$.*

3.3. *Agreement at the (effective) consensus point*

If there is certain to be disagreement should the negotiation continue past the consensus point, what will occur at (or just before) the consensus point itself? Suppose for the moment that $\Xi^{\max} \geqq 0$, in which case at least one alternative must be viable with respect to the continuation outcome $\langle \omega, T \rangle$ at the latest decision point no later than $\Xi^{\max}$ — which is to say, at the "effective consensus point" $\lfloor \Xi^{\max} \cdot n/T \rfloor \cdot T/n$ (where $\lfloor \cdot \rfloor$ represents the floor operator). If this viable alternative happens to be unique, then Lemma 1 ensures that it will be selected at the decision point in question. But there could at this point be multiple viable alternatives and thus a nontrivial static bargaining problem; and to rule out this possibility we shall need a stronger solvability condition.

Condition 2 (CP Solvability) $\Xi^a = \Xi^{\hat{a}} = \Xi^{\max} \geqq 0$ *only if $a = \hat{a}$.*

Proposition 3 *CP Solvability implies Terminal Solvability.*

Note, however, that the difference in logical strength between CP and Terminal Solvability amounts to the relatively innocuous genericity-type assumption that two alternatives which are not viable at T do not both become viable at *precisely the same instant*; and so in practice (e.g., in Sections 4.1–4.2) it will generally suffice to verify only the weaker condition.

Our new condition enables us to identify a unique alternative that becomes viable as the agreement date crosses (moving backwards) the consensus point.

Definition 3 Let CP Solvability hold and $\Xi^{\max} \geqq 0$. The *consensus choice* is then defined by $a^{\max} = \arg\max_a \Xi^a$. ‖

A technical assumption requires that the discretization of time be sufficiently fine to disallow any additional alternative with viability point between the true and effective consensus points.

[T2] The parameter n satisfies $T/n < \min\{\Xi^{\max} - \Xi^a : \Xi^a < \Xi^{\max}\}$. ‖

And we can then apply Lemma 1 to legitimately conclude that the consensus choice will be selected in the event that the effective consensus point is reached.

Proposition 4 *Let CP Solvability hold. If* $\Xi^{\max} \geqq 0$*, then*

$$\Sigma^{\text{IWD}}\left(\lfloor \Xi^{\max} \cdot n/T \rfloor \cdot T/n\right) = \langle a^{\max}, \lfloor \Xi^{\max} \cdot n/T \rfloor \cdot T/n \rangle . \qquad (16)$$

In particular, if $0 \leqq \Xi^{\max} < T/n$ *then* $\Sigma^{\text{IWD}}[0] = \langle a^{\max}, 0 \rangle$.

3.4. *Agreement before the consensus point*

To complete the backward induction analysis of our sequential bargaining game, we must now investigate the agents' behavior at decision points that precede the effective consensus point. The static bargaining problem that arises at the first (i.e., the latest) such point differs from those we have previously considered in that it has continuation outcome $\langle a^{\max}, \lfloor \Xi^{\max} \cdot n/T \rfloor \cdot T/n \rangle$ rather than $\langle \omega, T \rangle$. Since time is valuable (recall [R2]) it follows immediately that $a^{\max}$ is viable at this point, and thus no other alternative can be viable if a nontrivial static problem is to be avoided. An evident necessary condition for this exclusion demands that the consensus choice be a member of the core of the coalitional game associated with the collective choice problem under consideration.

Condition 3 (Core Membership) *If CP Solvability holds and* $\Xi^{\max} \geqq 0$*, then for each* $a \in A \setminus a^{\max}$ *and each* $J \subset I$ *such that* $\phi(J, a) = 1$ *there exists an* $i \in J$ *such that* $a^{\max} \succ_i a$.

Example 2 The ordering of acceptance points in Equation 13 above implies the preferences $a \succ_1 b$; $a \succ_1 c$; $b \succ_2 a \succ_2 c$; and $c \succ_3 b \succ_3 a$. (Since $\xi_1^b = \xi_1^c = -\infty$, agent 1's preference between b and c is undetermined.) Supposing that $\phi(J, \hat{a}) = 1$ if and only if $|J| \geqq 2$, we have that $a^{\max} = a$ and $\phi(\{2, 3\}, b) = 1$ and hence Core Membership fails. Supposing, alternatively, that $\phi(J, \hat{a}) = 1$ if and only if both $|J| \geqq 2$ and $3 \in J$, we have that $a^{\max} = b$ and so Core Membership holds if and only if $b \succ_1 c$.

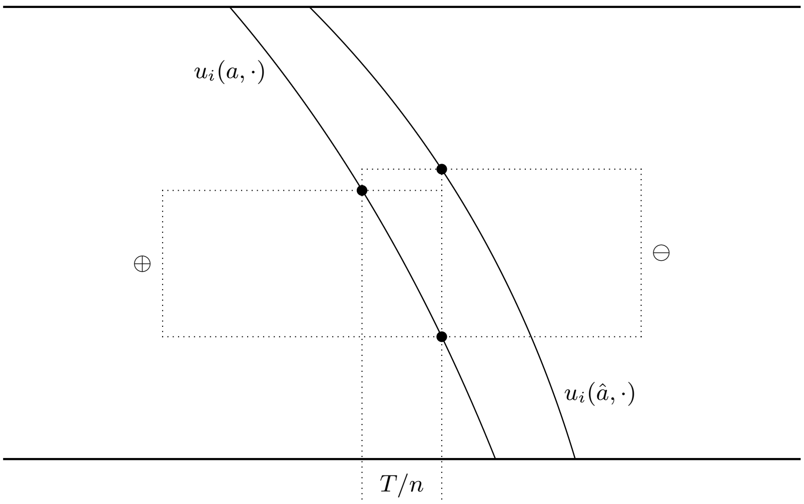

Figure 2: Assumption [T3]. The time interval (T/n) must be sufficiently short that an agent's loss (⊖) from replacing a better alternative ($\hat{a}$) with a worse alternative (a) is never outweighed by his gain (⊕) from reaching an agreement one decision point earlier.

While necessary, Core Membership alone is not quite sufficient to make the consensus choice uniquely viable at the decision point immediately preceding the effective consensus point. Although it does ensure that no coalition with the authority to replace $a^{\max}$ with a rival alternative would agree to do so were the point of implementation to remain fixed, it is possible that the gains from selecting the rival at the earlier decision point will be large enough to permit such a coalition to form. What is needed to prevent this is the following supplementary fineness assumption, which relates the time interval T/n to the variability of the agents' utility functions in their time arguments.

[T3] The parameter n satisfies

$$T/n < \min_{\hat{a} \succ_i a} \mathsf{MUC}_i^a \left[\min_{t \in [0,T]} u_i(\hat{a}, t) - u_i(a, t) \right] \tag{17}$$

(see Figure 2), where $\mathsf{MUC}_i^a[\cdot]$ returns the modulus of uniform continuity of the function $u_i(a, \cdot)$.[8] ‖

Together with this last technical assumption, Core Membership guarantees that the consensus choice will be selected if the negotiation reaches the first decision point before the effective consensus point. An identical situation then arises at the next earliest decision point, where once again we find that the consensus choice will be selected, and by induction we can show that this selection will in fact be made at *any* earlier decision point. Our final incremental result (proved in the Appendix) puts these facts on the record.

[8] Uniform continuity of $u_i(a, \cdot)$ follows from [R2] and the compactness of the domain $[0, T]$. In Equation 17, the existence and strict positivity of the minimum over $t \in [0, T]$ follow, respectively, from [R2] and the restriction $\hat{a} \succ_i a$; and strict positivity of the entire right-hand-side then follows from the finiteness of the sets I and A.

Proposition 5 *Let both CP Solvability and Core Membership hold. For each decision point $k \cdot T/n < \Xi^{\max}$, we have $\Sigma^{\text{IWD}}[k \cdot T/n] = \langle a^{\max}, k \cdot T/n \rangle$. In particular, if $\Xi^{\max} \geqq T/n$ then $\Sigma^{\text{IWD}}[0] = \langle a^{\max}, 0 \rangle$.*

3.5. *Summary of results*

By combining Propositions 2, 4, and 5, we reach the following conclusions about dominance solvability of the normal form game constructed in Section 2.3.

Theorem 1 *If both CP Solvability and Core Membership hold, then*

$$\Sigma^{\text{IWD}}[0] = \begin{cases} \langle a^{\max}, 0 \rangle & \text{if } \Xi^{\max} \geqq 0 \\ \langle \omega, T \rangle & \text{if } \Xi^{\max} = -\infty. \end{cases} \tag{18}$$

A verbal paraphrase may help to reinforce the content of the theorem.

> CP Solvability and Core Membership together are sufficient for ∂^{NF} to be dominance solvable. When these conditions hold, weak positivity of the consensus point is the criterion for agreement, which (if it occurs) is immediate and on the consensus choice.

Finally, it should be emphasized that the identified conditions are not *necessary* for dominance solvability, since the nontrivial static bargaining problems they serve to exclude can themselves be solvable — though in general, of course, they are not.

Example 3 Let $I = \{1, 2, 3\}$ and $A = \{a, b\}$; suppose that $\phi(J, \hat{a}) = 1$ if and only if $|J| \geqq 2$; and consider the static bargaining problem, arising at some decision point $k \cdot T/n$, in which

$$u_1(a, k \cdot T/n) > u_1(b, k \cdot T/n) > u_1(\Sigma^{\text{IWD}}[(k+1) \cdot T/n]) \tag{19}$$

$$u_2(a, k \cdot T/n) > u_2(b, k \cdot T/n) > u_2(\Sigma^{\text{IWD}}[(k+1) \cdot T/n]) \tag{20}$$

$$u_3(b, k \cdot T/n) > u_3(\Sigma^{\text{IWD}}[(k+1) \cdot T/n]) > u_3(a, k \cdot T/n). \tag{21}$$

Since both alternatives are viable, this problem is nontrivial. Nevertheless, we can eliminate weakly dominated action-agent pairs from the set $A^{\dagger} \times I$ in the sequence

$$\langle \alpha, 1 \rangle, \langle \alpha, 2 \rangle, \langle a, 3 \rangle, \langle \alpha, 3 \rangle, \langle b, 1 \rangle, \langle b, 2 \rangle \tag{22}$$

(recall from Section 2.2 that α represents "abstention"), and it follows that the problem is dominance solvable with solution $\langle a, k \cdot T/n \rangle$.

4. SPECIALIZED ENVIRONMENTS

4.1. *Binary choice problems*

One class of environments that all lead to dominance solvable bargaining games is that of binary choice problems in which at the horizon each agent ranks default strictly between the two potential agreements. Letting $A = \{a_0, a_1\}$, we have in this case that for each $i \in I$ there exists a $y \in \{0, 1\}$ such that $u_i(a_{1-y}, T) < u_i(\omega, T) < u_i(a_y, T)$, and moreover that I can be partitioned into the sets $F_0 = \{i \in I : a_0 \succ_i a_1\}$ and $F_1 = \{i \in I : a_1 \succ_i a_0\}$.

To check that Terminal Solvability holds, observe that for a given $y \in \{0,1\}$ we have that alternative a_y is viable at T if and only if $\phi(F_y, a_y) = 1$, with the latter equality implying that $\phi(F_{1-y}, a_{1-y}) = 0$ by coherence and hence that a_{1-y} is not viable at T. That Core Membership holds can be shown by contradiction: If it were to fail, then for $a_y = a^{\max}$ we would have that for some $J \subset I$ both $\phi(J, a_{1-y}) = 1$ and $a_{1-y} \succ_i a_y$ for each $i \in J$. It would follow that $J \subset F_{1-y}$; that $\phi(F_{1-y}, a_{1-y}) = 1$ by monotonicity; and hence that a_{1-y} is viable at T. But by definition $a^{\max} = a_y$ would then have to be viable at T as well, which would contradict Terminal Solvability.

Since $\xi_i^{a_y} = T$ if and only if $i \in F_y$, we can order the agents $I = \{1, 2, \ldots, |I|\}$ so as to make $\xi_i^{a_0}$ weakly decreasing and $\xi_i^{a_1}$ weakly increasing in i. There will then exist both a smallest m_0 such that $\phi(\{i \in I : i \leqq m_0\}, a_0) = 1$ and a largest m_1 such that $\phi(\{i \in I : i \geqq m_1\}, a_1) = 1$, and we can write the viability points of the alternatives as $\Xi^{a_0} = \xi_{m_0}^{a_0}$ and $\Xi^{a_1} = \xi_{m_1}^{a_1}$. When

$$\Xi^{\max} = \max\{\xi_{m_0}^{a_0}, \xi_{m_1}^{a_1}\} \geqq 0, \tag{23}$$

our conclusion is that $a^{\max} = a_y$ if and only if $\xi_{m_y}^{a_y} > \xi_{m_{1-y}}^{a_{1-y}}$.

Example 4 Let $I = \{1,2,3,4,5\}$ and $A = \{a_0, a_1\}$ and suppose that for each $y \in \{0,1\}$ we have $\phi(J, a_y) = 1$ if and only if $|J| \geqq 4$. If the ten acceptance points are ordered as

$$\begin{aligned} -\infty = \xi_1^{a_1} = \xi_5^{a_0} < 0 < \xi_2^{a_1} < \xi_4^{a_0} < \xi_3^{a_0} < \cdots \\ \cdots < \xi_2^{a_0} = \xi_1^{a_0} = \xi_3^{a_1} = \xi_4^{a_1} = \xi_5^{a_1} = T, \end{aligned} \tag{24}$$

then we have $F_0 = \{1,2\}$ and $F_1 = \{3,4,5\}$; $m_0 = 4$ and $m_1 = 2$; $\Xi^{a_0} = \xi_4^{a_0}$ and $\Xi^{a_1} = \xi_2^{a_1}$; $\Xi^{\max} = \max\{\xi_4^{a_0}, \xi_2^{a_1}\} = \xi_4^{a_0} \geqq 0$; and $a^{\max} = a_0$.

A prototypical binary choice problem might be that faced by a jury charged with a decision to acquit or to convict, given certain (majority or supermajority) rules for reaching a verdict, and operating under the shadow of an explicit or implicit deadline at which it will be declared to be deadlocked. Other binary problems presumably arise in legislative settings, their defining feature being the impossibility of compromise between two competing points of view.

4.2. *Bilateral surplus division*

As a second application, we now specialize our theory to the bilateral surplus division problem considered by Rubinstein. In this setting we have $I = \{1,2\}$,

$$A = \{\langle a_1, a_2 \rangle \geqq \langle 0, 0 \rangle : a_1 + a_2 = 1\}, \tag{25}$$

and $\phi(J, a) = 1$ if and only if $J = I$. It is also useful to impose normalizations of the form $u_i(a, T) = a_i$ and constraints of the form $z_i \geqq 0$ for $z_i = u_i(\omega, T)$.

With these specializations, the acceptance point of agent i for alternative a becomes

$$\xi_i^a = \begin{cases} -\infty & \text{if } z_i > u_i(a, 0) \\ u_i(a, \cdot)^{-1}(z_i) & \text{if } u_i(a, 0) \geqq z_i \geqq a_i \\ T & \text{if } a_i > z_i \end{cases} \tag{26}$$

and the viability point of alternative a simplifies to $\Xi^a = \min\{\xi_1^a, \xi_2^a\}$. Terminal Solvability is satisfied if and only if $z_1 + z_2 \geqq 1$, while it can be shown that Core

Membership is always satisfied when (as in this case) the potential function calls for unanimous agreement.

If $u_i(a,t) = a_i + c_i(T-t)$ for $c_i > 0$, which is to say that agent i faces a constant delay cost, then we have

$$\xi_i^a = \begin{cases} -\infty & \text{if } z_i - c_i T > a_i \\ T - (z_i - a_i)/c_i & \text{if } z_i \geqq a_i \geqq z_i - c_i T \\ T & \text{if } a_i > z_i \end{cases} \tag{27}$$

(see Figure 3). When $\max\{z_1 - c_1 T, 0\} + \max\{z_2 - c_2 T, 0\} > 1$, we have that $\Xi^{\max} = -\infty$ and therefore anticipate the outcome $\langle \omega, T\rangle$. Otherwise $\Xi^{\max} \geqq 0$ and (provided that $z_1 + z_2 \geqq 1$) we anticipate the outcome $\langle a^{\max}, 0\rangle$ characterized by

$$T - (z_1 - a_1^{\max})/c_1 = T - (z_2 - a_2^{\max})/c_2, \tag{28}$$

with consensus choice

$$\langle a_1^{\max}, a_2^{\max}\rangle = \langle [c_1 - c_1 z_2 + c_2 z_1]/[c_1 + c_2], [c_2 - c_2 z_1 + c_1 z_2]/[c_1 + c_2]\rangle\,. \tag{29}$$

If, alternatively, $u_i(a,t) = a_i \exp[r_i(T-t)]$ for $r_i > 0$, which is to say that agent i discounts surplus exponentially, then (when $a_i \neq 0$) we have

$$\xi_i^a = \begin{cases} -\infty & \text{if } z_i \exp[-r_i T] > a_i \\ T - (1/r_i)\log[z_i/a_i] & \text{if } z_i \geqq a_i \geqq z_i \exp[-r_i T] \\ T & \text{if } a_i > z_i. \end{cases} \tag{30}$$

When $z_1 \exp[-r_1 T] + z_2 \exp[-r_2 T] > 1$ we have that $\Xi^{\max} = -\infty$, while otherwise $\Xi^{\max} \geqq 0$ and (again provided that $z_1 + z_2 \geqq 1$) the consensus choice is characterized by

$$T - (1/r_1)\log\left[z_1/a_1^{\max}\right] = T - (1/r_2)\log\left[z_2/a_2^{\max}\right]. \tag{31}$$

A. APPENDIX

Proof of Lemma 1 Call delegate $\langle h, i\rangle$ *decisive for* any alternative a that will be chosen at history h with agent i's assent but not without it; that is, for any a such that both

$$\phi(\{i\} \cup \{j \in I \setminus i : s(h,j) = a\}, a) = 1 \tag{32a}$$

and

$$\phi(\{j \in I \setminus i : s(h,j) = a\}, a) = 0. \tag{32b}$$

If $h \in \theta(k \cdot T/n)$, we can then construct the following normal form comparison of the actions a and α available to this delegate.

$s(h,i)$	history h realized and $\langle h,i\rangle$ decisive for a	history h not realized or $\langle h,i\rangle$ not decisive for a
a	$\langle a, k\cdot T/n\rangle$	$\psi(s_{\neg\langle h,i\rangle}, \alpha)$
α	$\Sigma^{\text{IWD}}[(k+1)\cdot T/n]$	$\psi(s_{\neg\langle h,i\rangle}, \alpha)$

Here, as usual, the expression $\langle s_{\neg\langle h,i\rangle}, \alpha\rangle$ denotes the strategy profile formed from s by replacing $s(h,i)$ with α.

Supposing now that a is not viable at $k \cdot T/n$, let $J \subset I$ be such that $\phi(J,a) = 1$. By the definition of viability at a decision point, there must exist an $i \in J$ such that

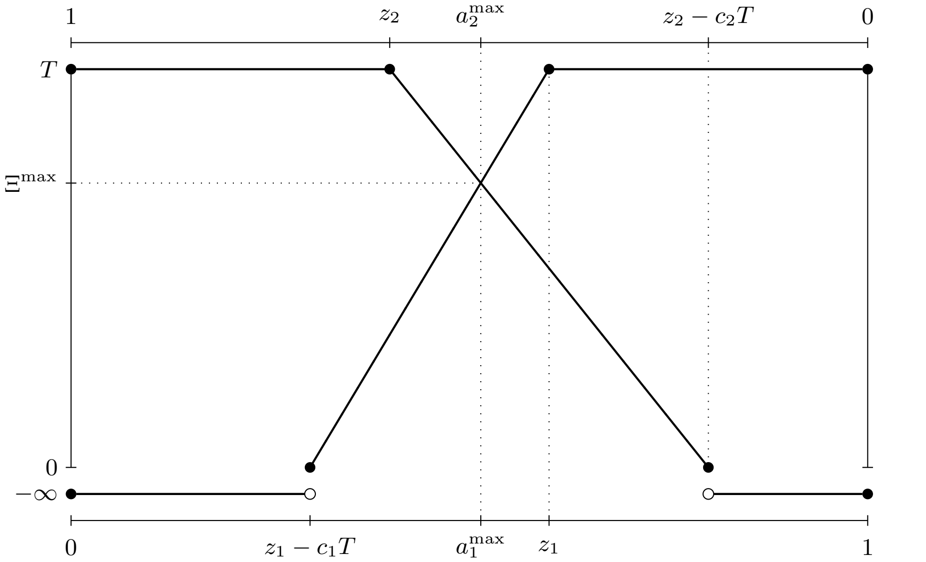

Figure 3: Bilateral surplus division with constant delay costs. Agent 1's surplus allocation is measured on the lower scale, agent 2's (complementary) allocation on the upper scale, and time on the vertical dimension. Considered as a function of a, the viability point Ξ^a is the lower envelope of the acceptance points ξ_1^a and ξ_2^a. The consensus choice $a^{\max} = \langle a_1^{\max}, a_2^{\max} \rangle$ achieves the maximum $\Xi^{\max}$ of this viability point function, a.k.a. the consensus point.

$u_i(a, k \cdot T/n) < u_i(\Sigma^{\text{IWD}}[(k+1) \cdot T/n])$, and the above comparison then confirms that α weakly dominates a for delegate $\langle h, i \rangle$. Eliminating all such dominated actions (by varying $a \in A$ and $i \in J \subset I$), we find that only an alternative that is viable at $k \cdot T/n$ could be chosen at a history in $\theta(k \cdot T/n)$; and thus, if no such alternative exists, that $\Sigma^{\text{IWD}}[k \cdot T/n] = \Sigma^{\text{IWD}}[(k+1) \cdot T/n]$.

If a unique $a^{\text{v}} \in A$ is viable at $k \cdot T/n$, then no other alternative can possibly be chosen at any $h \in \theta(k \cdot T/n)$ and so the full normal form decision problem facing delegate $\langle h, i \rangle$ can be written as follows.

$s(h, i)$	history h realized and $\langle h, i \rangle$ decisive for a^{v}	history h not realized or $\langle h, i \rangle$ not decisive for a^{v}
a^{v}	$\langle a^{\text{v}}, k \cdot T/n \rangle$	$\psi(s_{\neg\langle h,i \rangle}, \alpha)$
$a \neq a^{\text{v}}$	$\Sigma^{\text{IWD}}[(k+1) \cdot T/n]$	$\psi(s_{\neg\langle h,i \rangle}, \alpha)$
α	$\Sigma^{\text{IWD}}[(k+1) \cdot T/n]$	$\psi(s_{\neg\langle h,i \rangle}, \alpha)$

The viability of a^{v} at $k \cdot T/n$ means that there exists a $J^{\text{v}} \subset I$ such that both $\phi(J^{\text{v}}, a^{\text{v}}) = 1$ and $u_i(a^{\text{v}}, k \cdot T/n) \geqq u_i(\Sigma^{\text{IWD}}[(k+1) \cdot T/n])$ for each $i \in J^{\text{v}}$, and [R2] and [T1] together imply that each of these inequalities is in fact strict. But then we have that a^{v} is iteratively dominant for delegate $\langle h, i \rangle$ and thus that $s(h, i) = a^{\text{v}}$ for each $i \in J^{\text{v}}$; that

$$\phi(\{i \in I : s(h, i) = a^{\text{v}}\}, a^{\text{v}}) \geqq \phi(J^{\text{v}}, a^{\text{v}}) = 1 \tag{33}$$

(recall the monotonicity property of ϕ); that a^{v} is chosen at each history in $\theta(k \cdot T/n)$; and hence that $\Sigma^{\text{IWD}}[k \cdot T/n] = \langle a^{\text{v}}, k \cdot T/n \rangle$.

Proof of Proposition 5 In view of Proposition 4, we can adopt the hypothesis $\Sigma^{\text{IWD}}[(k+1) \cdot T/n] = \langle a^{\max}, (k+1) \cdot T/n \rangle$ and prove the result by induction. Since under this hypothesis $a^{\max}$ is clearly viable at decision point $k \cdot T/n$, it will suffice (by Lemma 1) to demonstrate that a given $a \in A \setminus a^{\max}$ is not viable at this point.

For each $J \subset I$ such that $\phi(J, a) = 1$ there exists by Core Membership an $i \in J$ such that $a^{\max} \succ_i a$, and thus

$$u_i(a^{\max}, (k+1) \cdot T/n) - u_i(a, (k+1) \cdot T/n) > 0 \tag{34}$$

by [R1]. From [T3] we have

$$\begin{aligned} T/n &< \mathsf{MUC}_i^a \left[\min_{t \in [0,T]} u_i(a^{\max}, t) - u_i(a, t) \right] && (35) \\ &\leqq \mathsf{MUC}_i^a \left[u_i(a^{\max}, (k+1) \cdot T/n) - u_i(a, (k+1) \cdot T/n) \right], && (36) \end{aligned}$$

and therefore

$$u_i(a, k \cdot T/n) - u_i(a, (k+1) \cdot T/n) < u_i(a^{\max}, (k+1) \cdot T/n) - u_i(a, (k+1) \cdot T/n). \tag{37}$$

But then

$$u_i(a, k \cdot T/n) < u_i(a^{\max}, (k+1) \cdot T/n) = u_i(\Sigma^{\text{IWD}}[(k+1) \cdot T/n]) \tag{38}$$

and hence a is not viable at $k \cdot T/n$.

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