The role of income in money demand during hyper-inflation: the case of Yugoslavia [1]

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Abstract: During extreme hyper-inflations productivity tends to fall dramatically. Yet, in models of money demand in hyper-inflation variables such as real income has been given a somewhat passive role, either assuming it exogenous or to have a negligible role. In this paper we use an empirical methodology based on cointegrated vector autoregressions to analyse data from the extreme Yugoslavian episode to investigate the role of income. The analysis suggests that even in extreme hyper-inflation the monetary variables and real income are simultaneously determined. The methodology enables a description of the short term adjustment of the variables considered.
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1 Introduction

During extreme hyper-inflations productivity tends to fall dramatically. A 50% fall was observed for the German episode of the 1920s and a 70% fall was observed for the Yugoslavian episode of the 1990s. Yet, in models of money demand in hyper-inflation variables such as real income has been given a somewhat passive role. This has come about in various ways. One strand of the literature has followed the work of Cagan (1956), who assumed a money demand relation involving real money and inflation only, while the effect of real income is negligible. Another strand of literature lead by Calvo and Leiderman (1992) work with a utility maximising model in which the budget constraint involves income as an exogenous variable. In that kind of the model a money demand is derived from micro assumptions. In this paper we use an empirical methodology based on cointegrated vector autoregressions to analyse data from the Yugoslavian episode with a view to investigate the role of income. The analysis suggests that even in extreme hyper-inflation the monetary variables and real income are simultaneously determined. Two cointegrating relations are found. These take a form which gives support to the money demand schedules in the literature, yet, the short term dynamics of the system links the monetary variables and real income more intimately than typically assumed.

In this paper we consider the real income assumptions in an empirical analysis of monthly data from the extreme Yugoslavian hyper-inflation of the early 1990s. As in any statistical analysis it is important to show that the assumptions to dependence structure, functional form and parameter stability are satisfied to ensure that valid inferences

[1] Computations were done using OxMetrics (Doornik and Hendry, 2007) and CATS2.0 (Dennis, 2006).

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are drawn. Once the inferential assumptions have been justified it is then possible to draw on rich information of the cointegrated vector autoregressive model to learn about cointegrating relations as well as the short term dynamics. This statistical model allows all variables involved to be endogenous and drifting in a non-stationary fashion, while equilibria in terms of cointegrating relations and exogeneity are testable restrictions; see Juselius (2006, §2) for further discussion. By focusing on one episode it is possible to devote sufficient attention to these issues. The success in addressing these issues can be measured as follows. Many previous studies of extreme hyper-inflations, notably Cagan (1956), refrained from modelling the last months of the episodes where the inflation was worst. The presented empirical model not only goes to the end of the sample, but also forecasts well when estimated up to the point where previous analyses have stopped. The Yugoslavian episode of 1990s is particularly suited in these respects as it appears to be one of the most extreme episodes: it lasted two years, in which prices kept increasing without significant interruptions and ended up a staggering 10^{21}-fold higher at the end of the episode. The signal to noise ratio therefore appears to be higher than for instance for the German episode of the 1920s that ran for shorter time, that was less extreme and that was temporarily halted in early 1923. In addition, industrial production was collected systematically for Yugoslavia and the black foreign exchange market operated throughout the period. Finally, it is possible to draw on some empirical literature with analyses of the monetary variables from this episode, including Petrović and Vujošević (1996), Engsted (1998), Petrović, Bogetić and Vujošević (1999), and Nielsen (2008).

The analysis will involve the usual money and price series along with an exchange rate due to currency substitution and a measure for productivity. Four variables are constructed from these series. The four variables are the logarithm of nominal cash balances measured in German mark, $m_t - s_t$ and income measured as log real industrial production, i_t. Following, for instance Easterly, Mauro, Schmidt-Hebbel (1995) inflation is derived from the price index P_t as a cost of holding money $c_t = 1 - P_{t-1}/P_t$ as opposed to the conventional measure $\Delta \log P_t$. Nielsen (2008) argues that c_t is preferable for the Yugoslavian episode, and possibly many other episodes, as it has random walk behaviour, whereas $\Delta \log P_t$ has explosive behaviour. This argument goes against rational expectations models which explicitely exploit a random walk assumption for $\Delta \log P_t$ as in Sargent and Wallace (1973) and Sargent (1977). A similar measure, d_t, for the currency depreciation is constructed.

We argue that the role of income has to be analysed through a system of the four variables. For simplicity a linear cointegration analysis of these variables is used. This can encompass the Cagan-type money demand relation between $m_t - s_t$, c_t and d_t, while also capturing an inverse relationship between production and inflation. In addition the system analysis provides a possibility for testing exogeneity hypotheses through the short run dynamics. The rational expectations models assume that real income has negligible role. This assumption would be appropriate if (*i*) real money adjusts to Cagan-type money demand equation only and not to the productivity equations, and (*ii*) the conditional model for the monetary variables given income does not depend significantly on income. The assumption of the optimising models that income is exogeneous would seem appropriate if (*iii*) income is strongly exogenous, that is, it does not adjust to the monetary variables, and in particular not to the money demand equation. In the empirical analysis we find evidence against these three statements, that is particularly strong in relation to the first two statements. As for the third statement, income does appear to adjust to monetary variables, albeit the adjustment to the money demand equation is only marginally significant. Moreover, the residuals of the equations for the monetary variables are highly correlated. All in all this suggests that the monetary variables are simultaneously determined. This counters the very simple causality structure assumed in some rational expectations models.

The real income hypotheses are here formulated as restrictions on the econometric model so any lack of power due to the sample size should work in favour of the hypotheses, which are, however, not supported in the empirical analysis.

The real income assumption has previously been looked at by Michael, Nobay and Peel (1994) very much in the spirit of this paper, albeit through a univariate cointegration analysis. They used German data for real money, inflation, forward rates, and with real wages as measure of income. They found that wages did matter although the estimated parameters of the money demand function were very unstable. This is presumably due to a combination of several factors. First, the inflation was measured using the explosive $\Delta \log P_t$ rather than c_t. Secondly, the temporary halt of the German inflation in early 1923 is not modelled explicitly. Thirdly, the decline in productivity only appears late in the episode due to the recovery from the war economy.

The outline of the paper is that a theoretical framework followed is discussed in §2. The Yugoslavian data are introduced in §3, along with the relevant data transformation. While the details of the econometric analysis is left to an Appendix the final econometric model is presented in §4. §5 concludes.

2 The role of income in hyper-inflation models

In the hyper-inflation literature real economy variables have largely played a passive role. This comes about in two different ways, either assuming that these variables have a negligible effect or that they are exogenous to the monetary variables. We give an overview of these types of assumptions and then discuss how they could be tested in a linear cointegrated systems setup.

2.1 Models where real income is assumed to be negligible

Cagan (1956) gave an expectations based theory for money demand. He started by assuming that real income is negligible for the money demand. The validity of this real income assumption is the main issue of interest in this paper. Maintaining the real income assumption only two variables are of interest, $m_t = \log M_t$ and $p_t = \log P_t$, which are natural logarithms of the money stock and its deflator. In his equations 2 and 5 he set up continuous time equations linking the log real cash balances with the expected rate of change in prices:

$$m_t - p_t = -\alpha E_t - \gamma, \tag{2.1}$$

$$\dot{E}_t = \beta(\dot{p}_t - E_t). \tag{2.2}$$

Here, E_t is an adaptive expectation to the rate of change of prices, $\dot{p}_t = \partial p_t / \partial t$, which is governed by the differential equation (2.2). In his empirical analysis of monthly data Cagan discretised the model by replacing $\dot{p}_t$ by $\Delta p_t = p_t - p_{t-1}$.

With the advent of rational expectations Sargent and Wallace (1974) and Sargent (1977) re-analysed a discrete time version of the hyper-inflation equation (2.1), while maintaining the real income assumption. They showed how their model can be solved when $m_t - p_t$ and Δp_t are assumed to show random walk like behavior, so they are $\mathsf{I}(1)$, but not cointegrating. They applied this work to the classical episodes considered by Cagan. Later Taylor (1991) showed how the money demand equation (2.1) with rational expectations can be embedded in a cointegrating framework. He also applied it to the classical episodes. Taylor's approach was applied in an augmented version with the possibility of rational bubbles in the sense of Diba and Grossman (1988) in the work of Engsted (1993a,b, 2006). Recently, Marcet and Nicolini (2003, 2005) analysed a model showing that even if the assumption that agents are fully rational is replaced by a learning assumption constant seigniorage can be extracted with ever-increasing inflation and money supply.

For these models the linear structure of the variable $m_t - p_t$ and Δp_t is crucial. Unit root tests applied to a variety of data show that $m_t - p_t$ typically has random walk behaviour while Δp_t has explosive behaviour. The response of Cagan and many subsequent empirical analyses was to omit the last few months of the episode from the analysis. Even so, the explosive behaviour is present. This can for instance be seen from the unit root tests for classical hyper-inflationary episodes in Taylor (1991, Table 1), when judged against the test statistics against the explosive alternative and not only the stationary alternative. An alternative would be to measure inflation as a cost of holding money, $c_t = 1 - P_{t-1}/P_t$, so $c_t = 1 - \exp(-\Delta p_t)$. That measure could be relevant in a discrete time models in that the real value at time t of the growth of nominal money, $\Delta M_t = M_t - M_{t-1}$, is $(\Delta M_t)/P_t = M_t/P_t - (1 - c_t)M_{t-1}/P_{t-1}$. This idea was put forward by Calvo and Leiderman (1992). Empirically, it can make a rather big difference to use one rather than the other as shown by Easterly, Mauro and Schmidt-Hebbel (1995). Nielsen (2008, 2009) developed an econometric theory for co-explosive processes, which allows a more detailed systems based analysis of these issues. This shows that, at least for the Yugoslavian episode, log real money, $m_t - p_t$, can be thought of as a co-explosive relation which is $\mathsf{I}(1)$ while Δp_t is explosive without any $\mathsf{I}(1)$-component. Moving on to a system of $m_t - p_t$ and c_t permits a standard cointegration analysis without having to omit the last few observations.

The empirical findings in relation to the measurement of inflation are based on data at monthly frequency. The results may be less pronounced for less extreme hyper-inflations or for data measured at a higher frequency. This leaves a role for the rational expectations based models and a need to test the real income assumption. This was considered in an empirical analysis of the German episode by Michael, Nobay and Peel (1994). They made a univariate cointegration analysis of log real money on Δp_t and a measure for real wages. They analysed the German episode right to the end, but found that they needed two different augmented money demand for the first and the second part of the sample. This was presumably due to the choice of inflation, as will be discussed later on in connection with Figure 3. Our analysis is very similar in spirit to that of Michael, Nobay and Peel (1994). Apart from changing the measure of inflation we will, however, test the real income assumption through a multivariate analysis. The analysis is outlined in §2.3.

2.2 Models where income is assumed exogenous

A different strand of literature works with optimising models in which a representative agent maximises expectations to the utility of future consumption and real money balances as in the work of Calvo and Leiderman (1992) and Eckstein and Leiderman (1992). In these models the consumption is subject to a budget constraint which involves income as an exogeneous variable. With additional assumptions to the utility functions a money demand equation can be derived. Its functional form depends on the choice of utility functions and it has time varying coefficients depending on the utility function and the consumption.

It is instructive to look in further detail at the model of Calvo and Leiderman (1992). An representive agent has utility at time 0 given by

$$\mathcal{E}\sum_{t=0}^{\infty}\beta^t\{u(K_t)+v(M_t/P_t)\}. \tag{2.3}$$

Here $\mathcal{E}$ is a mathematical expectation, $\beta < 1$ is a discount factor, K_t and M_t/P_t are future values of consumption and real money per capita, and u and v are strictly concave utility functions. The representative agent maximises the utility (2.3) subject to a budget constraint involving real income, consumption as exogeneous variables, real money and possible transfers to and from the government as well as bonds. Assuming that the price level at time t is known at time $t-1$ and ignoring bonds this model can be solved giving the relation

$$v'(M_t/P_t) = u'(K_t)c_{t+1}. \tag{2.4}$$

Here v' and u' are derivatives of the utility functions and c_{t+1} is the cost of holding money one period ahead. In this derivation the exogeneity of income is important: solving a joint system including income as well would be much harder. Now, solving (2.4) for $m_t - p_t = log(M_t/P_t)$ gives the money demand

$$m_t - p_t = log[(v')^{-1}\{u'(K_t)c_{t+1}\}].$$

If $log[(v')^{-1}\{\cdot\}]$ is a linear function then the Cagan equation (2.1) is recovered albeit with the parameter α replaced by the function $u'(K_t)$, which could be close to constant, and the expectation E_t in (2.1) is replaced by the actual future value c_{t+1}.

While the solution of the Calvo and Leiderman model provides micro foundation for the Cagan equation it is complicated to model econometrically in general. Calvo and Leiderman (1992) and Eckstein and Leiderman (1992) made certain simplifying assumptions to for instance the involved utility functions, which allowed them to calibrate the models using generalized method of moments estimation. A related paper by Easterly, Mauro and Schmidt-Hebbel (1995) has an extensive empirical analysis of annual data from high inflation economies, in which the money demand schedule is modified so that consumption is replaced by real income, I_t, and the functional form of the money demand is changed so that inverse velocity, $M_t/(P_tI_t)$, rather than real money, is linked with inflation. This would be compatible with the Calvo and Leiderman model if the utility function v is actually time-varying and dependent on real income, which is assumed exogeneous.

2.3 Formulating testable hypotheses

We will now formulate an econometric framework in which the real income assumptions can be tested. As the real income assumptions concerns the interaction between the monetary variables and real income a simultaneous system has to be constructed, in which these assumptions can be formulated as hypotheses. In hyper-inflation dramatic changes are often seen in other variables than just real money and inflation. There may be a significant element of currency substitution, so some thought should be given to which variables to include. The cointegrated vector autoregression turns out to be useful in that the real income assumptions can be formulated as hypotheses. This econometric model has clear assumptions that can be tested. If it shown that the model fits the data we can have some confidence in the inferences drawn about the hypotheses of interest.

Cagan's money demand schedule can be viewed as a simplified version of a standard portfolio type model for a normal economy (Tobin, 1956). The idea is that money is used for transactions as well as being one of several assets in a portfolio. The agents would then maximize the return to their wealth subject to a given level of risk. Agents can hold different assets and switch among them simultaneously. In a normal economy the relevant assets are: domestic money, domestic bonds, foreign bonds and foreign money. When it comes to extreme hyper-inflation interest rates become increasingly redundant as the credit market dries up. At the same time currency substitution sets in so the relevant assets are: goods, money, and foreign currency. Due to the currency substitution and due to prices of goods having widely varying inflation rates and availability the log exchange rate measured in domestic currency, s_t, rather than a price index is taken as the relevant deflator of money. The depreciation rate is measured $d_t = 1 - \exp(-\Delta s_t)$ for the same reason as c_t is preferred over Δp_t. The long-run money demand function then reads as follows, assuming a log-linear form:

$$m_t - s_t = \psi + \psi_i i_t + \psi_d d_t - \psi_c c_t \tag{2.5}$$

If $\psi_i = \psi_d = 0$ this correspond to Cagan's money demand schedule. If $\psi_i = 0$ while $\psi_c > 0$, $\psi_d < 0$ this corresponds to the money demand schedule of Abel, Dornbusch, Huizinga and Marcus (1979). If ψ_c and ψ_d are positive then inflation, c_t, and depreciation, d_t, would have opposite impacts on real money. This is consistent with a portfolio view of the economy, in which agents move out of money if inflation is high, but out of foreign currency if the local currency depreciates.

The equation (2.5) could be rearranged as

$$m_t - s_t = \psi + \psi_i i_t - (\psi_c - \psi_d) d_t + \psi_c (d_t - c_t). \tag{2.6}$$

If, as indicated, $\psi_c > \psi_d$ the interpretation would be that real money balances fall with d_t, but increase with the real depreciation rate $d_t - c_t$. Thus, if the currency depreciation is faster than the goods inflation the agents may demand more local currency. Any uncertainty over the actual inflation rate could show up in the black foreign exchange market, which is the only market that is widely followed by the agents in the economy. The depreciation rate will inevitably under-shoot or over-shoot the inflation. The difference, $d_t - c_t$, may therefore be taken as a measure of the inflation uncertainty. The possible effects inflation uncertainty may have on money demand in high/hyper inflation is discussed in Khan (1977), Blejer (1979), Bomberger and Makinen (1985) and Asilis, Honohan and McNelis (1993). The approach taken in those papers has been slightly different in that the uncertainty volatility has an impact that is either positive or negative, so that any expectation the agents may have to the sign of inflation changes would not be relevant. At a time where the government attempts to halt inflation the agents may, however, form strong expectations about the sign of the real depreciation rate. Another difference is that volatility measures often involve a moving average over a longish period whereas as measure like the real depreciation rate does not have any lag. The real depreciation rate could therefore show more clearly the instantaneous impact of government intervention on the inflation uncertainty.

The real income hypothesis, that the money demand does not respond to real income is in effect a statement about the system involving all the variables, $m_t - s_t$, c_t, d_t and i_t. Income would interact with inflation through various channels. For instance, inflation could increase the transactions costs in the economy which in turn would have a negative impact on productivity. Tomasi (1994) proposes a model in which the cross section price variability associated with high inflation is welfare decreasing; see also Driffill, Mizon, Ulph (1990) for an overview of the literature. A falling supply of goods could in turn induce inflation. The effect could very well be complicated, but for simplicity suppose it has a linear form

$$i_t = \rho - \rho_c c_t. \tag{2.7}$$

An implication is now, that if this income equation holds along with (2.5), then the money demand equation and the parameter ψ_i could be difficult to identify. In particular, adding ψ_i times the equation (2.7) to (2.5) would give an equation of the form

$$\begin{aligned} m_t - s_t &= (\psi + \psi_i \rho) + \psi_d d_t - (\psi_c + \psi_i \rho_c) c_t. \\ &= (\psi + \psi_i \rho) - (\psi_c + \psi_i \rho_c - \psi_d) d_t + (\psi_c + \psi_i \rho_c)(d_t - c_t). \end{aligned} \tag{2.8}$$

This shows that there is an inherent identification problem for the money demand equation as it could be of the form (2.5) or of the form (2.8). Succeeding in estimating a money demand equation without income from a system involving only monetary variables is therefore not evidence in favour of the real income assumption. So, the real income assumption has to be tested as a restriction on a simultaneous system involving both monetary variables and real income. This can be achieved within the framework of a cointegrated vector autoregression.

Suppose the available data are monthly time series of four-dimensional vector X_t composed of variables $m_t - s_t, c_t, d_t, i_t$. The cointegrated vector autoregression is a statistical model given by the equation

$$\Delta X_t = \alpha \beta' X_{t-1} + \Gamma \Delta X_{t-1} + \nu D_t + \varepsilon_t, \qquad t = 1, \ldots, T, \tag{2.9}$$

for some deterministic component D_t. The parameters are $\alpha, \beta \in \mathbb{R}^{4\times r}$ for some cointegrating rank r to be determined, $\Gamma \in \mathbb{R}^{4\times 4}$ and $\nu \in \mathbb{R}^4$. The innovations ε_t are assumed to be independent normal, $\mathsf{N}_4(0, \Omega)$-distributed. The notation α, β is chosen to be consistent with the cointegration literature and unrelated to the earlier uses of these letters. Working with this tight set of distributional assumptions ensures that test have reasonable finite sample properties and provides a benchmark for assessing if important properties of the data have been captured. Johansen (1988, 1995) has shown that the cointegrated vector autoregression can be interpreted through the Granger-Johansen representation

$$X_t = C \sum_{s=1}^{t} \varepsilon_t + \textit{stationary process} + \textit{deterministic process}.$$

The impact matrix C for the random walk has the property that $\beta' C = 0$. Thus β relates to a long-run behaviour of the process, while the other parameters, α, Γ, Ω, relate to the short-run behaviour of the process. Here, the notions of long-run and short-run relate to the sample and cannot necessarily be extrapolated outside the sample. A single equation analysis would provide valid inference if the cointegrating rank is one and the adjustment vector α has a particular structure, which we will discuss later. If these conditions are not satisfied, which is the case in the empirical analysis, then a single equation analysis will be inefficient relatively to a full system analysis. We will now formulate the various real income assumptions as hypotheses on the long-run and short-run parameters.

The assumption that real income is negligible can be tested by considering how the equations (2.5), (2.7) and (2.8) relate to the cointegrating vectors β. The negligibility assumption would be supported if the cointegrating rank is found to be unity and the cointegrating vector is found not to involve income as in (2.8). This amounts to an over-identifying restriction on the cointegrating vector β. If the rank is two the situation is more complicated. The two cointegrating vectors will generally be of the form (2.8) and (2.5). In that case the negligibility assumption would be supported if the monetary variables only adjust to the equation (2.8) and not to (2.5). This corresponds to α having a block triangular structure. In previous empirical analyses investigators have often considered a single equation with real money $m_t - s_t$ as regressand. In line with that the negligibility assumption would be supported in a weaker sense if real money only adjust to (2.8), while the other monetary variables may adjust to both (2.8) and (2.5). In the empirical analysis none of these hypotheses are supported.

The assumption that real money is exogenous can be tested through the short-run dynamics. Since real income is assumed exogeneous it can enter the money demand as in (2.5). So finding that (2.5) belongs to the cointegrating space is compatible with the exogeneity assumption. To make the notion of exogeneity operational in econometric terms it is useful to appeal to the notions of strong and weak exogeneity in Engle, Hendry and Richard (1983). Divide the vector X_t into the monetary components Y_t given by $m_t - s_t, c_t, d_t$ and the real income variable $Z_t = i_t$. The second order vector autoregression implies the joint density

$$\mathsf{f}(X_1, \dots, X_T | X_0, X_{-1}; \theta) = \prod_{t=1}^{T} \mathsf{f}(X_t | X_{t-1}, X_{t-2}; \theta),$$

for some parameter θ. Replacing X by Y, Z and conditioning gives

$$\mathsf{f}(X_1, \dots, X_T | X_0, X_{-1}; \theta) = \prod_{t=1}^{T} \mathsf{f}(Y_t | Z_t, X_{t-1}, X_{t-2}; \theta_y) \prod_{t=1}^{T} \mathsf{f}(Z_t | X_{t-1}, X_{t-2}; \theta_z),$$

where θ_y and θ_z together constitutes θ. In the optimising models exogeneity presumably mean that there is no feedback from the monetary variables Y into income Z, so

$$\mathsf{f}(Z_t | X_{t-1}, X_{t-2}; \theta_z) = \mathsf{f}(Z_t | Z_{t-1}, Z_{t-2}; \theta_z), \tag{2.10}$$

as in strong exogeneity. This corresponds to certain zero restrictions on the adjustment parameters α, Γ. Weak exogeneity is a weaker form in which only restrictions on α are imposed so

$$\mathsf{f}(Z_t | X_{t-1}, X_{t-2}; \theta_z) = \mathsf{f}(Z_t | Z_{t-1}, \Delta X_{t-1}; \theta_z). \tag{2.11}$$

In the case of two cointegrating relations yet a weaker form of exogeneity is if the real income only responds to the real income equation (2.7) and not to the money demand equation (2.8). This corresponds to a single zero restriction on α. In the empirical analysis neither strong or weak exogeneity is supported, whereas the weakest form of exogeneity is just marginally rejected.

Given the very limited support for the real income assumption it will be of interest also to consider the conditional system of the monetary variables Y given real income $Z = i$. This will show to what extent the contemporaneous value of real income matters for money demand. Indeed, in the empirical analysis real money is found to depend on real income.

Modelling the income fall in practice is going to be difficult if the relation between income and inflation is non-linear rather than linear as postulated in (2.7). Indeed, the subsequent analysis of the Yugoslavian data is going to show a rather stable money demand equation, but a somewhat more unstable income relation. The empirical analysis therefore needs to be backed up with some robustness analysis to check if the real income assumptions also fail under different specifications of the real income.

3 Yugoslavian Data

The Yugoslavian episode provides a good example for hyper-inflation studies for a number of reasons. It is the second longest and second highest recorded hyper-inflation in the 20th century. In addition, prices accellerate very smoothly, which gives a large signal to noise ratio. Finally, since the episode is relatively recent and happened in a country with a developed central administration a systematically constructed data series for productivity is available.

3.1 Institutional background

The institutional background for the extreme Yugoslavian hyper-inflation is described in Petrović and Vujošević (1996) and Petrović, Bogetić and Vujošević (1999). In short, the former Federal republic of Yugoslavia fell apart in 1991, reducing to the area of Serbia and Montenegro. The civil war started in the region and severe United Nations embargo on all international transactions was introduced in May 1992. Output and fiscal revenue sharply decreased. The fiscal deficit increased and a significant monetization of that deficit occurred. The monthly inflation rose above 50% in February 1992 and accelerated further, a price freeze was attempted in August 1993 and the inflation finally ended on 24 January 1994 with a currency reform after prices had risen by a factor of 1.6×10^{21} over 24 months and of 6.8×10^{21} over the full sample of 37 months. A currency substitution took place, but domestic currency, dinars, remained the primary mean of transactions throughout the episode.

All data considered relate to the Yugoslavia that remained after former Yugoslavia felt apart; that is Serbia and Montenegro. It may therefore not be unreasonable to assume the population was constant in this period so that m_t and i_t represents per capita money stock and per capita income.

3.2 The data

The empirical analysis is based on four monthly data series for the period 1990:12 to 1994:1. The series are plotted in levels in Figure 1 and in first differences in Figure 2.

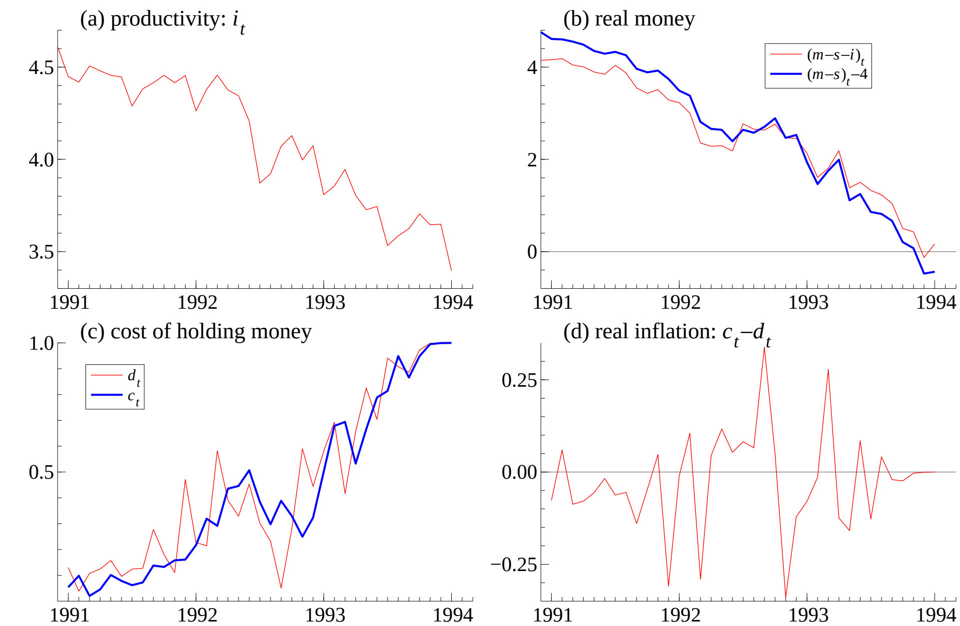

Figure 1: Data in levels.

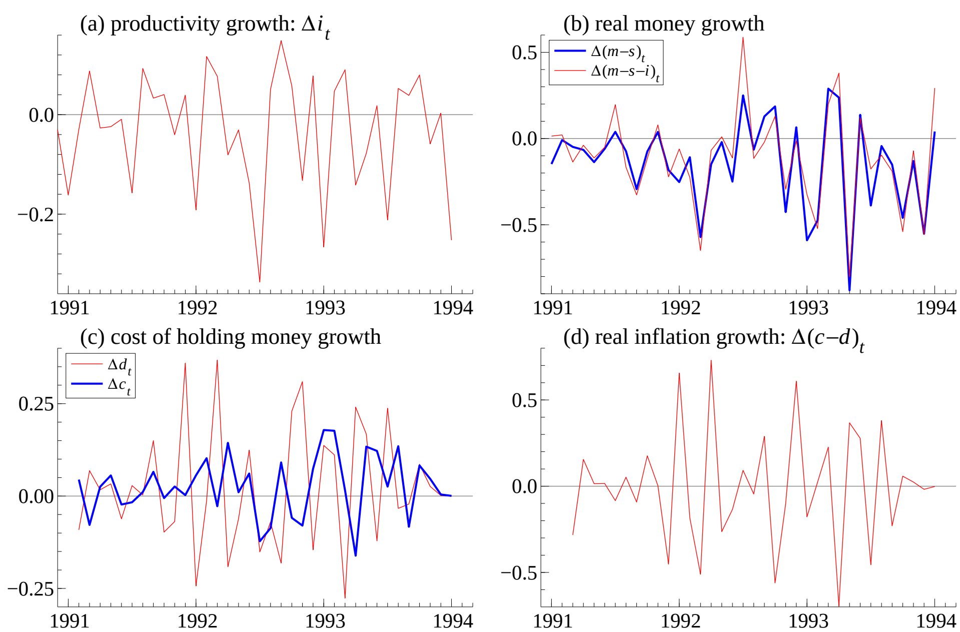

Figure 2: Data in differences.

The series are defined as follows.

Industrial production, $i_t = \log I_t$, plotted in Figure 1(a). It is an index of industrial production, obtained from the Yugoslav Statistical Office. As I_t is an index it measures the size of the economy in real terms. The productivity is seen to shrink by about 70% through this period.

Real money deflated by exchange rate, $m_t - s_t = \log M_t - \log S_t$, plotted in Figure 1(b). The money stock, M_t, is M1 money issued measured at the end of the month in million dinars. It was obtained from the Yugoslav Central Bank. The exchange rate, S_t, is a black market Dinars/German Mark rate. It was published daily in the news, thus being widely known. The data are sampled at the end of the month. The panel also shows the inverse volatility $v_t = m_t - s_t - i_t$. Both variables are falling dramatically with the increased turnover of money as the inflation increases. The fall in the real money stock is 99.4%, whereas the inverse volatility falls by an only slightly more modest 98.2%.

Cost of holding money, $c_t = 1 - P_{t-1}/P_t = 1 - \exp(\Delta p_t)$, where $p_t = \log P_t$, is plotted in Figure 1(c). A brief discussion of this choice of inflation is given in §3.3. The consumer price index, P_t, was collected on 22nd in the month and published in the newspapers. This is based on a basket of goods that have very different inflation rates in the hyper-inflation. As the inflation rises prices are increasingly prone to measurement error, so are not useful as a deflator of money. This measurement error has much less impact on c_t.

Depreciation rate of the Dinars/Mark currency $d_t = 1 - S_{t-1}/S_t$, is also plotted in Figure 1(c). It is more volatile than the cost of hold money.

The difference $c_t - d_t$ could be interpreted as the *rate of change of real exchange rate*, noting that German inflation was negligible over the considered period. It is plotted in Figure 1(d). The variable $c_t - d_t$ is seen to be very volatile, albeit with a distinct pattern. The volatility increases in the beginning and drops back towards the end of the episode, consistent with an interpretation that agents become increasingly uncertain about where the economy is going, but more certain once they realise the devastating nature of the inflation.

3.3 Measuring inflation

The depreciation of the currency is measured as d_t, rather than a log difference Δs_t. This is in line with the optimising models of Eckstein and Leiderman (1992) and Calvo and Leiderman (1992). Through an empirical analysis Easterly, Mauro and Schmidt-Hebbel (1995) noted that it makes a difference to use one or the other and preferred the former. Nielsen (2008) motivates that choice using the notion of co-explosive time series developed in Nielsen (2001, 2009). That analysis is summarised in Figure 3.

The idea is that graphs of the original time series m_t, p_t and s_t would show a strong exponential appearance. These series do, however, co-explode so that, for instance, $m_t - s_t$, has a random walk appearance as shown in Figure 1(b). Due to the explosiveness the choice of measurement of inflation is very important. Simply taking differences, as in $\Delta s_t = s_t - s_{t-1}$, preserves the exponential appearance as shown in Figure 3(a). Thus, the relationship between real money, $m_t - s_t$, and Δs_t is non-linear as shown in the cross plot in Figure 3(b) and is bound to be complicated to describe. Petrović and Mladenović (2000) suggested one possible non-linear specification linking $m_t - s_t$ with $(\Delta s_t)^\gamma$ for some $\gamma > 0$. For small inflation rates a Taylor expansion shows that d_t and Δs_t are quite similar. For large inflation rates they are very different in that d_t is bounded by one whereas Δs_t

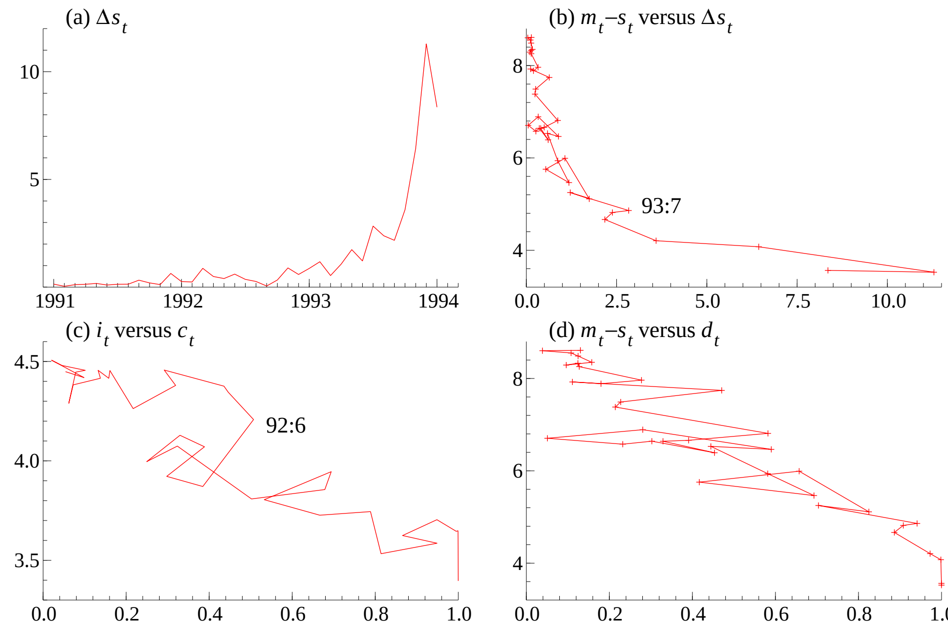

Figure 3: Measuring inflation: Differences of log prices versus cost of holding money.

is unbounded. Empirically, Nielsen (2008) found that d_t has a random walk appearance as shown in Figure 1(c). As d_t approaches its upper bound of unity the currency loses nearly its full value in one month. The measure d_t is quite possibly more relevant to the agents in the economy than Δs_t by telling more directly how fast money looses its value. It may even be forecastable as opposed to an exploding, non-linear measure as Δs_t. The cross plot in Figure 3(d) shows a relationship between $m_t - s_t$ and Δs_t that is close to linear.

The non-linear appearance of the cross plot in Figure 3(b) illustrates a problem encountered by Cagan and much of later empirical literature. The only way to construct a linear relationship between $m_t - s_t$ and Δs_t would be to stop the analysis about 1993:7 (as in Petrović and Mladenović, 2000) or starting it about 1993:5. A similar picture could be drawn for data from German episode of the 1920s, so Cagan and many later empirical studies of the German episode stop their analysis in 1923:7 rather than at the end of the inflation 1923:11. An exception is the analysis of Michael, Nobay and Peel (1994) which essentially proposes two linear regimes.

3.4 The inverse relation between income and inflation

The central variable in this analysis of the real income assumption is the productivity measure i_t. This is falling dramatically throughout the episode as seen in Figure 1(a). It shows a strong seasonality with dips in January and July, coinciding with the Orthodox Christmas and the summer break. With this short sample the seasonality pattern cannot be captured well, but this turns out not to matter that much for the overall conclusions.

In the beginning the productivity falls are relatively modest. The productivity fall only accelerates after the introduction of the UN embargo in 1992:6. This observation is drawn out more clearly in the cross plot of i_t against the depreciation c_t in Figure 3(c).

This behaviour gives some challenges to the empirical analysis. While the graph shows as clear inverse relationship it is not clearly linear. The sample is too short for modelling a non-constant slope between i_t and c_t. A softer option would be to allow the intercept to change value at the introduction of the embargo. Compared with a constant intercept this does not make all that much differences for the final conclusions. Thus, we have settled for a standard linear cointegration analysis of the variables $(m_t - s_t, c_t, d_t, i_t)$.

4 Empirical model for Yugoslavian data

The cointegration analysis sketched in §2.3 is carried out on the Yugoslavian data. Starting with a analysis of the specification the hypotheses are tested in turn. Only the main results are discussed, whereas the details of the analysis are left to the Appendix.

4.1 Vector autoregressive specification and cointegration rank

A second order autoregression of the type (2.9) is fitted to the vector X_t consisting of the variables $m_t - s_t, c_t, d_t, i_t$. The model includes a constant as well as a dummy variable taking unity value in 92:12 and zero otherwise. While the specification appears to be quite robust as documented in the Appendix, it is useful to discuss aspects of the fit relating to the second half of 1992.

The main difficulty in the specification is to match the sample variation in the second half of 1992. This starts with the UN embargo 92:6. The full impact of the embargo was spread out over the next half year in part because of the annual holiday period in July and Government attempts unsuccessfully to halt the inflation. Great volatility can be seen in the real depreciation rate $d_t - c_t$ in Figure 1(c), and also in the cross plot of i_t versus c_t in Figure 3(b). Due to the short sample we choose to present a model that does not impose any particular structure for this period.

Towards the end of the tumultuous second half of 1992 presidential elections were held in 92:12. These were won by the incumbent, S. Milosević, against a strong democratic opponent. In the run up to the elections the reported figures for prices are low relative to the exchange rate as seen in Figure 1(c). It is a possibility that the reported statistics distorted the picture of the economy in the month prior to the elections. As being already in power it was possible for S. Milosević to influence the reported statistics if not the economy itself prior to the elections. Choosing between various dummy specifications a dummy taking unity value at 92:12 gives the best fit of the model. So this specification should give the most reliable inference. Using other specifications does, however, not substantially alter the conclusions about the real income assumptions.

We also explored the seasonality in the income variable as evident in Figure 1(a). While the sample is too short for proper seasonal adjustment a regressor like Δy_{t-12} is available and could be included. This improves the fit of the income equation, but does not otherwise alter the conclusions.

It is interesting to compare the model with the model of three monetary variables $m_t - s_t, c_t, d_t$ presented by Nielsen (2008). Whereas that model needs three lags to achieve proper specification the present model for the system extended by i_t only needs two lags. The fit in terms of the order of magnitude of the estimated covariance matrix, Ω, is, however, more or less the same. So the difference in modelling is not so much in terms of the level of fit, as it is in terms of increasing the potential for economic interpretation, in

		$m-s$	i	c	$c-d$	1
unidentified β'		1	−1.36	2.44	4.56	−2.45
		0.31	1	2.60	1.72	−7.06
identified β'	$e_{Cagan,t}$	1	0	4.20 (3.0)	4.83 (6.1)	−8.46 (−4.0)
	$e_{income,t}$	0	1	1.29 (2.9)	0.21 (0.2)	−4.42 (−2.9)

Table 1: Cointegrating vectors. First panel shows the first two unidentified eigenvectors. Secon panel shows just-identified cointegrating vectors. Signed likelihood ratio statistics are reported in parantheses and are asymptotically standard normal.

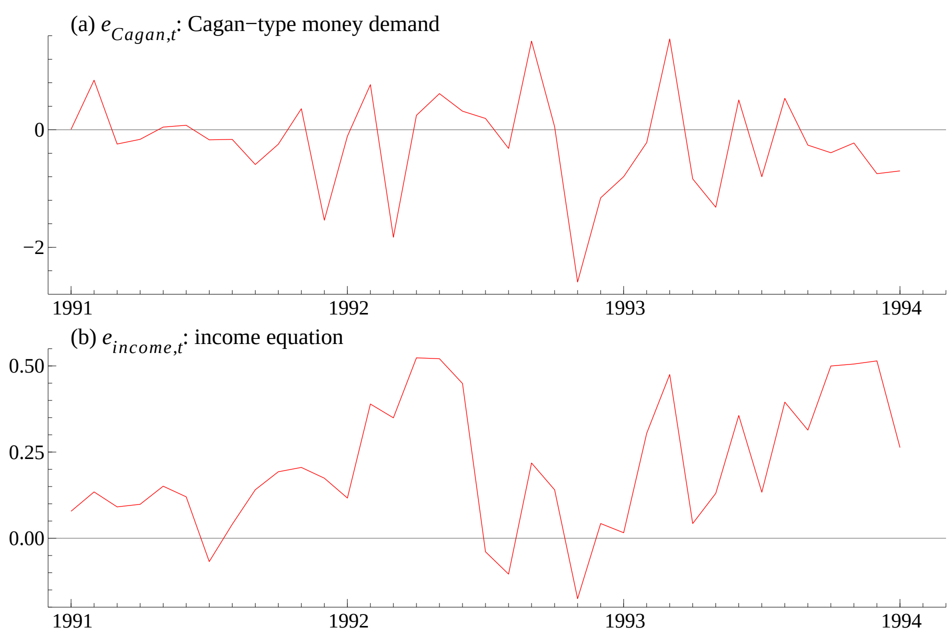

Figure 4: Cointegrating relations.

that the longest lags have been eliminated by extending the information set. This is in line with the recommendations of Juselius (2006).

Having established that the specification of the model cannot be rejected a cointegration analysis can be carried out using the method of Johansen (1995). The details of the analysis are reported in the Appendix. The cointegration rank is quite clearly determined to be two.

Unidentified and just identified cointegrating vectors are reported in Table 1. As the cointegration analysis is invariant to linear transformations of the vector X_t it is equivalent to express the results in terms of the vectors $m_t - s_t, i_t, c_t, d_t$ and $m_t - s_t, i_t, c_t, c_t - d_t$. The just-identified vectors match the money demand equation (2.5) and the income equation (2.7). Indeed, the coefficient to the real depreciation, $d_t - c_t$, is not significant in the income equation. The money demand equation is quite similar to that obtained by Nielsen (2008) when analysing a smaller system without income.

The two just-identified cointegrating vectors of Table 1 are plotted in Figure 4. Panel (a) shows the Cagan-type money demand and is denoted $e_{Cagan,t}$. The swings in the money demand relation in (a) match those of the variable $c_t - d_t$, shown in Figure 1(d).

This supports the idea that $c_t - d_t$ measures the uncertain of the agents in the economy over relative spead of depreciation of the currency and of the rise in prices. The similarity of the money demand and $c_t - d_t$ suggests that the adjustment of the economy to the equilibria happens nearly instantly. Figure 4(b) shows the income equation, denoted $e_{income,t}$. The income relation is correlated with the second unrestricted cointegrating relation. Accordingly it has a more drifting appearance than the money equation. Broadly speaking there is a tendency towards two trending periods with a shift at the time of the embargo.

4.2 Testing that real income is negligible

As outlined in §2.3 the hypotheses that real income is negligible can be tested by imposing restrictions on the cointegrating vectors and on the adjustment vectors.

It is interesting first to contemplate the outcome, had the cointegration rank been chosen as unity. In that case the single cointegrating vector would have been estimated by the first of the unidentified two cointegrating vectors reported in the first panel of Table 1. This vector is more akin to the money demand specification with income (2.5) than the specification without income (2.8). Indeed, the likelihood ratio test for the hypothesis that the coefficient to income is -1 is $\chi^2_1 = 0.7$, $\mathsf{p} = 0.42$, whereas the test for the coefficient being zero is $\chi^2_1 = 7.9$, $\mathsf{p} = 0.005$. Thus, if the rank had been found to be one, this would have been evidence against the negligibility assumption.

Working now with a rank of two as suggested by the rank tests just-identified cointegration vectors are reported in Table 1. These vectors are only identified up to taking linear combinations, so as discussed in §2.3 it is not possible to test whether income should be included in the money demand or not. Instead we turn to the adjustment matrix α and test the negligibility assumption through various restrictions.

The likelihood ratio test for the hypothesis that none of the monetary variables, $m_t - s_t, c_t, d_t$, adjust to $e_{income,t}$ is $\chi^2_3 = 18.3$, $\mathsf{p} < 0.001$. The test for the hypothesis that only $m_t - s_t$ does not adjust to $e_{income,t}$ is $\chi^2_1 = 10.2$, $\mathsf{p} = 0.001$. If anything real money adjusts to the difference $e_{Cagan,t} - e_{income,t}$, which is the money demand equation involving velocity (2.5). The test for real money only adjusting to that cointegrating vector is inconclusive, $\chi^2_1 = 3.7$, $\mathsf{p} = 0.06$. Thus, when the rank is chosen as two as suggested by the rank statistic, this also gives evidence against the negligibility assumption.

In order to investigate the role of real income further a conditional system is analysed of the monetary variables, $m_t - s_t, c_t, d_t$, given real income, i_t. We look at c_t, d_t since the residuals from these variables are less correlated than those of for instance $c_t, c_t - d_t$. The analysis is done by regressing the differenced variables $\Delta(m_t - s_t, c_t, d_t)$ on Δi_t, the estimated cointegrating vectors, $e_{Cagan,t-1}$ and $e_{income,t-1}$ of Table 1, the lagged differenced variables $\Delta(m_{t-1} - s_{t-1}, y_{t-1}, d_{t-1}, c_{t-1} - d_{t-1})$, and the dummy $D_{92:12}$. This gives a total of 24 regression parameters. Imposing 11 restrictions with test $\chi^2_{11} = 6.1$, $\mathsf{p} = 0.87$ results in the restricted model reported in Table 2.

The simultaneous equations system shows that real money and to a lesser extent the depreciation rate adjust significantly to both Δi_t and the income equation $e_{income,t-1}$, whereas the impact of these variables in the equation for c_t is more marginal. The sign of the coefficient to Δi_t in the conditional real money equation is negative suggesting that the mainly negative productivity growth has a positive effect on real money corresponding to a dampening effect on velocity. This, in turn, has a dampening effect on inflation through

$$
\begin{aligned}
\widehat{\Delta(m-s)}_t &= \underset{(-3.1)}{-0.73}\,\Delta i_t + \underset{(2.1)}{0.42}\Delta i_{t-1} + \underset{(8.7)}{0.46} e_{Cagan,t-1} - \underset{(-6.2)}{0.92}\, e_{income,t-1} \\
&\quad \underset{(-6.3)}{-1.15}\,\Delta(m-s)_{t-1} - \underset{(-2.0)}{0.75}\,\Delta c_{t-1} + \underset{(4.1)}{0.66} D_{92:12} \\
\widehat{\Delta c}_t &= \underset{(1.6)}{+0.11}\Delta i_t - \underset{(-8.6)}{0.12}\, e_{Cagan,t-1} + \underset{(1.6)}{0.16}\Delta(c-d)_{t-1} - \underset{(-3.5)}{0.18}\, D_{92:12} \\
\widehat{\Delta d}_t &= \underset{(2.5)}{+0.21} e_{income,t-1} + \underset{(3.2)}{0.27}\Delta(m-s)_{t-1}
\end{aligned}
$$

$$
\mathsf{corr} = \begin{pmatrix} 1 & -0.54 & 0.80 \\ -0.54 & 1 & 0.52 \\ 0.80 & 0.52 & 1 \end{pmatrix}, \qquad \mathsf{sdv} = \begin{pmatrix} 0.221 \\ 0.053 \\ 0.145 \end{pmatrix}
$$

Table 2: Restricted simultaneous equations system taking cointegrating vectors as given and conditioning on Δi_t. t-statistics in parentheses.

the money demand equation. The contemporaneous relation of real money and real income is reassuring in that agents' demand for money should, in the short run, rely on the current economic activity rather than solely on the past activity. This contemporaneous effect is harder to find in normal economies, see Hendry and Santos (2007) for a discussion.

The correlations of the monetary variables, $m_t - s_t, c_t, d_t$, are high. As in the money demand relation the pairwise correlations of $m_t - s_t$ with c_t and d_t, respectively, have opposite sign. Both real money and the depreciation rate adjust to the lagged growth in real money, which indicates an importance of real money in the dynamics of hyper-inflation. This counters the idea that nominal money should be a driver of hyper-inflation, but then nominal money is of course not allowed to play a role of its own in this analysis. This is a further indication that the monetary variables variables are determined simultaneously as opposed to a causal ordering.

4.3 Testing that real income is exogenous

We now turn to the assumption that real income is exogenous. In order to solve the optimising model as done by Calvo and Leiderman (1992) it is presumable required that real income is strongly exogeneous, so that the distribution of income given past values of income and monetary variables does not depend on the monetary variables. As this does not appear to hold we focus on weaker forms for exogeneity.

Real income is weakly exogenous if it does not adjust to the cointegrating relations. With a cointegrating rank of two this corresponds to two zero restrictions on the matrix α. In context of the system with just-identified cointegrating vectors, but no further restrictions on adjustment parameters, the test for that hypothesis is $\chi^2_2 = 13.8$, $\mathsf{p} = 0.001$. So strong exogeneity, which amounts to imposing further 3 zero restrictions on the matrix Γ, would also be rejected. This provides strong evidence against weak and strong exogeneity, which is needed for solving the optimising problems.

In order to investigate the exogeneity of real income further a regression is conducted of real income on the estimated cointegrating vectors, $e_{Cagan,t-1}$ and $e_{income,t-1}$ of Table 1, the lagged differenced variables $\Delta(m_{t-1} - s_{t-1}, y_{t-1}, d_{t-1}, c_{t-1} - d_{t-1})$, and the dummy $D_{92:12}$. This gives a total of 7 regression parameters. Imposing 4 restrictions with test

$\chi^2_4 = 1.2$, $\mathsf{p} = 0.88$. results in the restricted model

$$\widehat{\Delta i}_t = \underset{(2.5)}{0.07} e_{Cagan,t-1} - \underset{(-4.1)}{0.31} e_{income,t-1} - \underset{(-2.6)}{0.24} \Delta(m-s)_{t-1}, \qquad \widehat{\sigma} = 0.103, \qquad (4.12)$$

with t-statistics reported below coefficients. This equation indicates how income depends on the monetary variables.

The equation (4.12) shows that real income adjusts to both cointegrating vectors. The adjustment to the money demand relation is, however, only marginally significant at a 5% level. In this regression the cointegrating vectors are taken as given with appeal to the super-consistency of estimators of cointegrating vectors. An alternative test would be to test a single zero restriction directly in the adjustment matrix α, which would result in a test of $\chi^2_1 = 2.8$, $\mathsf{p} = 0.10$. The same inconclusive outcome would be reached in a model with a dummy at 92:11 rather than at 92:12.

Even if it were concluded that real income does not adjust to the money demand equation $e_{Cagan,t}$, it would still adjust to the levels of inflation through the income relation, and to the lagged changes in real money. Thus, the evidence appear to be against the exogeneity assumption in the optimising model, albeit perhaps not quite as strongly as it was seen for the negligibility assumption.

5 Conclusion

Most past analyses of hyper-inflations are based on the assumption that real income plays a secondary role for money demand in hyper-inflations. A detailed analysis of data from the Yugoslavian hyper-inflation show that this does not seem to be the case at least for that episode. The monetary variables are all found to be simultanously determined among them selves and simultaneously with income. This counters the assumption of the Cagan-type model that real income is negligible and the assumption of the optimising models that real income is exogeneous. In addition it appears that simple causality assumptions that are helpful in solving theoretical models are not supported empirically.

In summary, the research suggests that theoretical models of hyper-inflation should incorporate the real economy in a more active way than done at present. In the Yugoslavian episode, like in previous empirical studies of other hyper-inflations, exchange rates are found to be important due to currency substitution. This should presumable also be reflected in theoretical models.

Further reseach has to be made to see if these conclusions also would hold for other episodes for which data are perhaps less informative. With the experience from the relative long and smooth Yugoslavian episode it may be possible to analyse other episodes in greater detail than has been done before.

Test	$m_t - s_t$	y	d_t	$c_t - d_t$	Test	system
$\chi^2_{normality}(2)$	4.0 [0.14]	3.6 [0.17]	1.1 [0.57]	3.1 [0.22]	$\chi^2_{normality}(8)$	8.0 [0.43]
$\mathsf{F}_{ar,1-3}(3,22)$	1.7 [0.19]	0.3 [0.83]	2.7 [0.07]	1.8 [0.18]	$\mathsf{F}_{ar,1-3}(48,40)$	0.9 [0.66]
$\mathsf{F}_{arch,1-3}(3,19)$	0.2 [0.92]	0.2 [0.89]	0.0 [1.00]	0.1 [0.97]		

Table 3: All tests are asymptotically valid regardless of the location of the parameters. For the normality tests see Engler and Nielsen (2007), for autoregressive tests see Nielsen (2006), for ARCH test see Doornik and Hendry (2007). p-values in square brackets.

$r = 4$	1.03, $0.61 \pm 0.20i$, $0.07 \pm 0.20i$, $-0.24 \pm 0.62i$, -0.34
$r = 2$	1, 1, 0.92, $-0.22 \pm 0.62i$, $-0.29 \pm 0.19i$, 0.02

Table 4: Characteristic roots when r is unrestricted and when $r = 2$ is imposed.

Appendix: details of empirical analysis

First, consider the specification of the second order vector autoregression in §4.1. A constant, but no linear trend is included: the strongly trending behaviour is attributed to random walks as linear trends have no obvious economic interpretation. The lag length is chosen so as to ensure a parsimonious empirical model which also passes specification tests.

Formal mis-specification tests are reported in Table 3. Interpreting these in the usual way indicates that the model is well specified. Graphical tests for mis-specification, which are not reported here, include Q-Q-plots for normality and residual correlograms, and are likewise supportive of the model. Note that the usual asymptotic theory is valid for general autoregressions with stationary, unit, as well as an explosive root. This has been proved for the test for autocorrelation in the residuals, see Nielsen (2006a,b), and for Q-Q plots for normality by Engler and Nielsen (2007). Some of the test statistics are reported in an F-form as advocated by Doornik and Hendry (2007) in an attempt to deal with finite sample issues for these tests. The fit is remarkably good. All tests are accepted at the 5% level, and all but one are accepted at the 10% level. Changing the dummy at 92:12 to a dummy at 92:11 or removing the dummy altogether would introduce slightly autocorrelated errors.

The recursive plots in Figure 5 are mainly supportive of the model. The one-step ahead Chow test for $c_t - d_t$, (row 2, column 4) indicates a slight problem in 92:9, which is in the initial phase of the UN embargo. The out-of-sample forecasts for the last 6 months in Figure 5(m-p) also indicate stability of the model. Previous analyses of these data have typically omitted these last 6 observations, which can be forecast with this model.

The first row of Table 4 shows the estimated characteristic roots. The slightly explosive root of 1.03 turns out not to be significantly different from unity.

Secondly, turn to the cointegration analysis of §4.1. The techniques of Johansen (1995) are used. In the cointegration analysis the constant is restricted to the cointegrating space.

Table 5 give a quite clear indication of a cointegrating rank of two when using the asymptotic critical values. The Bartlett-corrected values do not change this conclusion.

Table 4 shows the estimated characteristic roots when the rank is restricted to be 2. The slightly explosive root then disappears. One root is located somewhat close to

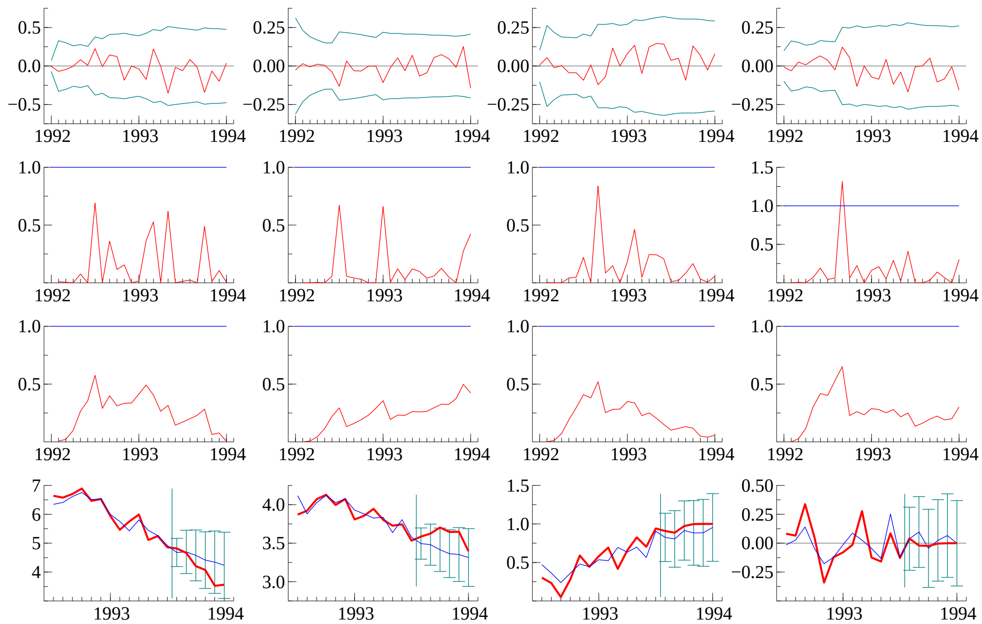

Figure 5: Row 1 shows recursive 1-step ahead residuals. Row 2 shows 1-step ahead Chow tests, normalised so 1%-critical value is one. Row 3 shows break point Chow tests, normalised the same way. Row 4 shows dynamic out-of-sample forecasts for the last 6 periods with thin blue, and actual data with bold red. Columns 1,2,3,4 are for the variables $m_t - s_t, i_t, d_t, c_t - d_t$, respectively.

the unit circle, but the rank test in Table 5 suggests that the rank should not be lower. Likewise an I(2) analysis rejects the possibility of I(2)-ness. That analysis is, however, done in a somewhat different model with a linear trend but no dummy in order to exploit the test of Rahbek, Kongsted and Jørgensen (1999).

The properties of the cointegrating vectors $(\beta', \beta_c')'$ and of the adjustment vectors α are investigated in Table 6. This establishes that none of the variables can be excluded from the cointegrating vectors, that none of the variables are stationary on their own, and that none of the variables are weakly exogeneous. This is done for the variables $(m_t - s_t, i_t, d_t, c_t - d_t)$, as well as for $v_t = m_t - s_t - i_t$ and c_t, which are linear combination thereof. The decision about weak exogeneity is, however, marginal for velocity, v_t, and for depreciation, d_t, but clear for real money, $m_t - s_t$, for income, i_t, and for real inflation, $c_t - d_t$; thus it seems reasonable not to impose any weak exogeneity restrictions.

r	likelihood	LR	p-value	LR^*	p*-value
0	98.46	98.7	[0.00]	85.5	[0.00]
1	126.36	42.9	[0.01]	36.3	[0.04]
2	140.99	13.7	[0.32]	7.7	[0.85]
3	145.13	5.4	[0.25]	2.2	[0.74]
4	147.83				

Table 5: Cointegration rank tests. LR is the usual likelihood ratio test statistic of Johansen (1995), LR^* is the Bartlett-type-corrected statistic of Johansen (2002).

	v_t	$m_t - s_t$	i_t	d_t	$c_t - d_t$	c_t	constant
Exclusion	16.3 [0.00]	16.3 [0.00]	9.0 [0.01]	10.5 [0.01]	34.1 [0.00]	34.1 [0.00]	9.5 [0.01]
Stationarity	9.0 [0.01]	10.0 [0.01]	12.7 [0.00]	9.4 [0.01]	9.3 [0.01]	9.1 [0.01]	
Weak Exogeneity	5.7 [0.06]	17.3 [0.00]	13.8 [0.00]	4.6 [0.10]	13.9 [0.00]	19.4 [0.00]	

Table 6: All tests are χ^2_2. p-values in square brackets.

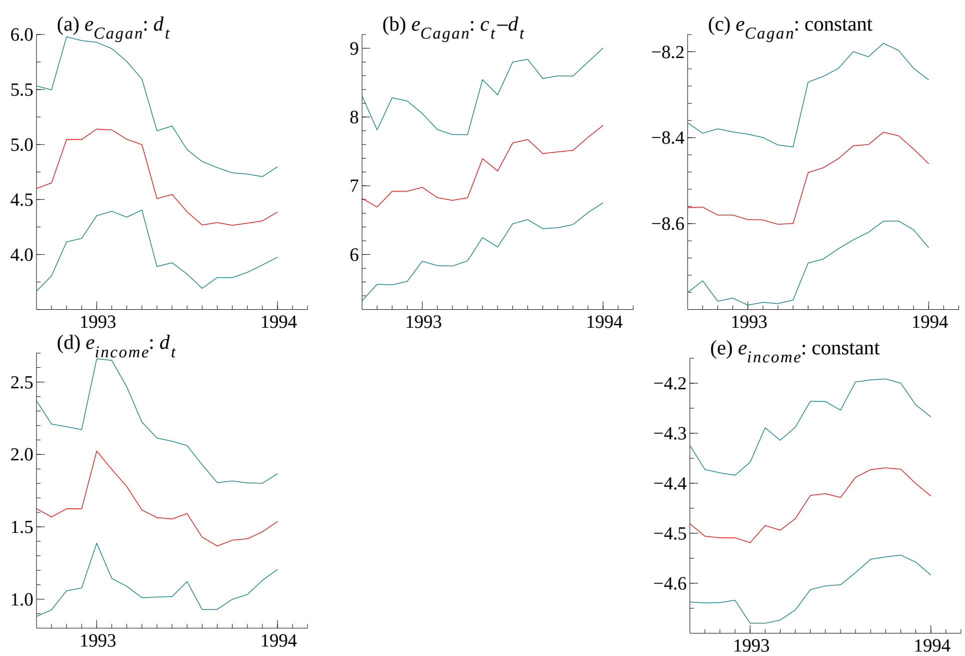

Figure 6: Recursive plots of coefficients in over-identified (β', β_c'), where coefficient to $c_t - d_t$ is zero in income relation. 95%-confidence bands given.

Figure 6 shows recursive estimates for the coefficients of the cointegrating vectors, (β', β_c'). Here the coefficient to $c_t - d_t$ is restricted to zero in the income relation. These plots, starting at 92:9, indicate stability of the cointegrating relations. Going further back some instability is evident - this is likely to be a consequence of instability as well as poor finite sample properties of the confidence bands. The recursive estimates for the model where the income relation is unrestricted show a slight instability in the period 92:9 to 93:3, but the effect is so small that it is likely to be driven by poor finite sample properties of the confidence bands. Overall, the recursive plots point to quite stable cointegration parameters, albeit there is evidence of some instability right in the middle of the sample from 92:7 to 93:3, which is the period right after the introduction of the UN embargo.

Recursive plots for the adjustment coefficients α were also considered, although not shown here. Both forward expanding and backward expanding plots were constructed. The coefficients generally appear stable although also indicating potential problems in the period 92:7 to 93:2. While it would be preferable to model the period after the introduction of the UN embargo and the elections in a better way, it is not so obvious how to do that. The dummy at 92:12 goes some way in doing that.

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